INTRODUCTION

Precise measurements of co-ordinates of an acoustic wave's points of reflection from bed of a water region in which the measurements are under execution are an essence of hydrographic survey. Demand on high accuracy of depth measurements, determination of position and its co-relation with the depth results from a necessity of ensuring high accuracy of produced charts and electronic maps, as well as estimating volume of dredged spoils. The first area of concern is connected with provision of safety for vessels and the second one is of financial measure resulting from contracts concluded on dredging works and their labour consumption.

Achievement of the depth measurements' high accuracy and of the position determination with a use of satellite techniques does not guarantee achievement of the hydrographic survey's high accuracy, although the measurements are not easy. They are accompanied by many processes and phenomena which decrease the accuracy and reliability if not taken into consideration. In general, factor having impact on the measurements' accuracy may be divided into two basic groups: of those resulting from dynamics of sounding vessel and of those that are related to environment.

As a result of interferences, the following actions are connected with a moving sounding vessel:

- pitching, resulting from wavy motion, wind as well as change of the vessel's speed,
- rolling, mainly resulting from wavy motion and wind,
- yawing, i.e. change of the sounding vessel's vertical position, resulting from impact of waves.

Environmental factors are connected with the acoustic wave propagation in water during performance of the depth measurements by means of echo-sounder as a basic device used in under-water surveys. Change of initial parameters of water: temperature and salinity, influencing its density, results in changes of propagation speed of the acoustic wave what has impact on accuracy of the depth measurements. These changes are of spatial and time-domain character: the acoustic wave's propagation speed changes according to a depth function and at the time of performance of the sounding surveys. Moreover, trajectory of acoustic ray is not rectilinear for rays deflected from perpendicular as a result of occurrence of the acoustic wave's speed changes. Such a ray covers longer distance to result in false measurement of the depth. Also, there is a co-relation error of location of the positioning system and acoustic wave's point of reflection from the sea bed there - even if devices compensating pitching and rolling have been used.

The following factors may be listed along with those mentioned above:

- instrumental error of time measurement being a base for operation of echo-sounder, on which the user – hydrographer has no influence and which is an accuracy measure of the measuring device,
- arrangement of the echo-sounder's sending and receiving transducers; such a variant used to be applied in earlier models of echo-sounders,
- slope of the sea bed, having impact on accuracy of the depth measurements performed with a use of transducers represented by broad obtuse angle of direction characteristics; this phenomenon occurs during performance of measurements by means of navigation echo-sounders equipped with transducers cheaper than those provided for hydrographic echo-sounders,
- frequency of the acoustic wave used for surveys; application of the acoustic wave's various frequencies results in its reflection from layers of beds of different densities and different depth indications.

However, analysis of those phenomena and factors is not a key of this study. **Development of method of precise determination of water region's co-ordinates for various applications is a target of this elaboration – with selected factors (dynamics of the sounding vessel and environmental factors)** taken into consideration. In spite of possibility of application motion sensors available on the market (of very high price – more expensive than professional single-beam echo-sounders), an alternative method of determining pitching and rolling has been developed – introduced in a model of precise determining of acoustic ray's reflection co-ordinates – for a ray characterized by non-linear trajectory. It is an alternative for accelerometers used in hydrography and magnetometers with floating cores available on the market. The elaboration covers those areas of concern which has not been solved in a complex way so far and which regard:

- analysis of vertical sound speed distribution's impact on accuracy of sounding surveys and the distribution's changes,
- spacial-time modelling of the sound speed distributions in water,
- development of method of pitching and rolling determination, with a method of two points applied and with a use of two receivers of radio-navigation system,
- development of method of determining acoustic ray's trajectory,
- development of method of determining reflection point's co-ordinates of acoustic wave sent by echo-sounder, as a synthesis of the above-said and solved areas of concern.

Main theses of the study may be presented as the following:

- the vertical sound speed in water has an impact not only on accuracy of the depth measurement but also on trajectory of the acoustic ray, resulting in its non-linear propagation,
- having positions of two points known (in their specific locations), there is a possibility of determining pitching and rolling with an accuracy comparable with performance of the motion sensors,
- application of the elaborated methods for selected factors determining accuracy of the hydrographic surveys shall influence accuracy of created charts, safety of navigation and efficiency in searching under-water objects in a significant way.

The study contains four chapters. Genesis of a problem – hydrographic surveys in the light of regulations of the International Maritime Organization (IMO) and of International Hydrographic Organization (IHO), required accuracies of the depth measurements and position determination for the needs of hydrography – is presented in the first chapter. Reference systems used in hydrography and in this method are defined. Also, methods of measurements of the sound speed in water, acoustic climate of the South Baltic Sea and inland water regions as well as its impact on the depth measurements' accuracy are presented. Results of the sound speed measurements done in the Bay of Gdansk, their 24-hour and seasonal changes and own measurements performed in the inland water regions, all available in literature, have been used. A concept of a hydrographic system based on single-beam echo-sounder and two-antenna positioning system is presented.

Spacial-time modelling of the sound speed distributions in water, serving determination of a local sound speed, depth and acoustic ray's trajectory, are presented in the second chapter. Own method of modelling of spatial and time-domain sound speed distributions in water is presented – with a use of known surface modelling methods. Equipotential surfaces of constant sound speed in water have been taken for the modelling. Thanks to such a model using local measurements of the vertical sound speed in water, there is a possibility of determining the local vertical sound speed in a place of the depth measurement performance – of determining position of point of the acoustic wave reflection from the sea bed.

Eikonal equation is used in the third chapter to describe trajectory of the acoustic ray emitted by the echo-sounder’s transducer. Different cases of the acoustic ray’s emission are considered: at no movement disturbances, when sensors are placed, or not, in the sounding vessel's symmetry plane. Method of determining spatial orientation of the sounding vessel, using two receivers of the radio-navigation system, has been elaborated. The method of determining the acoustic ray’s trajectory has been used to determine the acoustic wave’s reflection point in case of occurrence of the movement disturbances and refraction. A global model of the EGM96 geoid has been used in the hydrographic surveys.

Methodology of performance of geodesic bathymetric measurements is presented in the fourth chapter: Preparation of the sounding, calibration of the measuring system, execution of the bathymetric measurements and determination of co-ordinates of the acoustic wave’s points. Analysis of model of the sound speed spatial distribution in water (global interpolation) and of its local distribution (local interpolation) has been performed.

1 GENESIS OF A PROBLEM

1.1 REQUIREMENTS IN RESPECT TO ACCURACY FOR HYDROGRAPHIC SURVEYS

1.1.1 HYDROGRAPHIC SURVEYS IN THE LIGHT OF REGULATIONS OF THE INTERNATIONAL MARITIME ORGANIZATION

The International Maritime Organization (IMO) does recommend performance of hydrographic surveys serving rise of safety of navigation in its regulations. There are no strictly defined requirements regarding accuracy of position calculation there, nor of depth measurements. Recommendations concerning recommendations for hydrographic services are given in the following documents:

- Resolution A.958(23),
- SOLAS 1974 (International Convention for the Safety of Life at Sea),
- Declaration of the safety of navigation and emergency capacity in the Baltic Sea area (Copenhagen Declaration).

Resolution A.958 (23) has been passed during 23-rd IMO Assembly and its stipulations, with reference to the SOLAS requirements (chapter V, rule 9), do appeal to member countries for establishments of Hydrographic Offices in consultation with International Hydrographic Organization (IHO), for provision of hydrographic information in accordance with procedures covered by guidelines of the Resolution A.706(17), for promotion of ECDIS and ENC electronic charts' usage, as well as it encourages hydrographic services of particular countries to undertake close inter-cooperation.

Implementation of the SOLAS rule V/9 has been passed during 78th Session of the Maritime Safety Committee (MSC/Circ. 1118 of 27 May 2004). Circular of Appendix contains and IHO's Note, addressed to coastal countries and concerning improvement of cooperation between those countries' hydrographic services within a range of IHO, which appeals for fulfilment of respective obligations according to stipulations of the SOLAS V/9 and Resolution A.958(23).

Mentions about introduction of additional activities targeted on assurance of the hydrographic service improvement and promotion of electronic navigation charts, by means of performance of repeated hydrographic surveys for ships' routes and sea ports, are said in the Declaration of the safety of navigation and emergency capacity in the Baltic Sea area (Copenhagen Declaration), accepted on 10 September

2001 in Copenhagen by Extraordinary Summit of the Helsinki Commission while attended by ministers – the Participating States were asked to elaborate diagrams of hydrographic surveys for ships' routes and sea ports to ensure that safe navigation would not be threatened with inappropriate source of information. The surveys should be performed according to a standard meeting at least requirements of the last IHO's S-44. The diagram should be elaborated mutually by hydrographic services responsible for respective areas not later than by the end of the year 2002, so that the implementation process could start by the year 2003.

1.1.2 HYDROGRAPHIC SURVEYS IN THE LIGHT OF REGULATIONS OF THE INTERNATIONAL HYDROGRAPHIC ORGANIZATION

Requirements regarding accuracy in determining position and depth measurement are covered by [IHO 1998]. These standards provide minimal requirements concerning hydrographic surveys which, if met, may guarantee sufficient accuracy and layout of gathered data for an ensured safe navigation of particular users (merchant, naval, recreational and other vessels). The hydrographic data are also used to manage coastal zones, to perform research over natural environment, to exploit natural resources, for legal areas of concern, meteorological and oceanographic modelling, aquatic engineering and hydro-techniques as well as for many other fields of human activities. This is to conclude that users of the hydrographic data make a very diverse group. To raise usefulness, the users require the data to be updated, detailed, reliable and given in a digital form. Even though these standards do not meet needs of all the users, they shall make a basis for evaluation of the hydrographic data quality.

In order to assign various accuracy requirements to various surveyed water regions in a systematic way, four categories of the hydrographic surveys have been defined: special category and categories 1, 2 and 3 [IHO 1998].

The hydrographic works covered by the Special Category are nearly to meet engineering standards. Their performance is limited to specific critical water regions of minimal depth or in those places where characteristics of bed is potentially hazardous for vessels. Those areas must be clearly defined by an institution responsible for quality of the works. **Sea port areas and anchoring places for ships, including canals**, are examples of such water regions. All error sources should be minimized. The special category requires **usage** of more number of **profiles**, including **side scan sonars**, multi-processing echo-sounders or **multi-beam echo-sounders** of high resolution, in order to perform a full search of the bed.

The measuring devices must ensure detection of objects represented by spatial dimensions over 1 m. **Usage of side scan sonars may be necessary in water regions in which presence of thin and dangerous obstacles is expected** [IHO 1998].

The special category, considered as a category of highest requirements in respect to accuracy of the depth measurements and position definition, is subject to analysis in this elaboration. For this category, IHO has determined the following minimal accuracy requirements [IHO 1998]: horizontal accuracy (95% of confidence level) – 2 m, depth accuracy $\pm\sqrt{a^2+(bd)^2}$ for reduced depth (95% of confidence level), where: a = 0.25 m and stands for a constant depth error, i.e. it is a sum of all constant errors, bd stands for an error dependent on the depth, i.e. it is a sum of all errors dependent on the depth, b = 0.0075 and stands for an error coefficient dependent on the depth, while d stands for the depth itself.

In order to ensure the safe navigation, the full search of the bed is obligatory and it may be performed with a use of strictly determined mechanical sweep guarantying a minimal safe depth in the entire water region. Moreover, the spatial dimension > 1 m of the searched objects resting on the bed has been defined.

1.1.3 REQUIRED ACCURACIES OF DEPTH MEASUREMENTS FOR HYDROGRAPHY

Today, it is to be expected that many hydrographic measurements shall still be executed by means of single-beam sounders capable of performing point depth measurements along sounding profiles, while the techniques of full bed search shall be introduced only in water regions of special importance. Such an assumption has resulted in keeping the term “distance between profiles” still binding, even if the distance is not related to a scale of the survey elaboration.

Accuracy of the depth should be perceived as an accuracy of reduced depth. It is necessary to determine a number of particular errors’ sources in order to define the depth accuracy. All error sources should be related to each other in order to determine an overall error of propagation. The overall error is a result of all partial errors taken into consideration and, among the others, the following ones belong to them [Makar 2002]:

- errors of measuring system and those resulting from impact of sound speed,

- errors in modelling interference of sounding vessel's movement,
- errors of tides' measurements and modelling,
- impact of acoustic wave refraction on acoustic ray trajectory.

Creation of definition of sea bed's general topography, reduction of tides, detection, classification and description of underwater obstacles are fundamental tasks of the hydrographic survey. Depths above the obstacles should be determined with an accuracy meeting at least requirements of Category 1 for the hydrographic works.

The measured depths should be reduced to map values or to the measurements' reference level by correcting them in respect to a tide height or sea level. Depths higher than 200 m should not be reduced with the tide height, unless the tide has significant impact on the overall error of the depth measurement.

1.1.4 REQUIRED ACCURACIES IN DEFINING POSITION FOR HYDROGRAPHY

Basic activities of hydrographic services, requiring accurate determination of positions, are as the following:

- performance of surveys,
- placement and control of navigation marks,
- laying and marking out new water lanes and navigation water regions,
- inspection over correctness of dredging works' execution and others.

A required position error, depending on kind of works under execution, is 1-10 metres (95% of confidence level). Also, the requirements set by IHO should be met to protect the hydrographic works. Not so long ago, accuracy of the position determination with the 95%-confidence level was proportional to a chart scale or plotting board and should have not exceeded a value of 1.5 mm in a given scale. Requirements for accuracy in defining positions in hydrographic surveys, in respect to map scales (survey plotting boards) have been given in the Table 1.1.

Table 1.1 Position error in hydrographic surveys for typical scales of charts and plotting boards

Chart's scale	Position accuracy for 95%-confidence level	Position accuracy for 65%-confidence level
1 : 3 000	4.50 m	2.60 m
1 : 5 000	7.50 m	4.40 m
1 : 7 500	11.25 m	6.50 m
1 : 10 000	15.00 m	8.70 m
1 : 15 000	22.50 m	13.00 m
1 : 25 000	37.50 m	21.70 m
1 : 50 000	75.00 m	43.40 m
1 : 100 000	150.00 m	87.00 m

Last version of IHO standards for hydrographic surveys has been issued in 1998 [IHO 1998]. In the previous editions of SP-44, requirements for the position accuracy mostly were resulting from drawing limitations in given scales. Improvement of the hydrographic surveys' accuracy has been done up to date due to implementation of satellite systems for the position determination, in particular accompanied by introduction of differential techniques and GPS-RTK. Automation of the data processing allows presenting them in any scale these days. That is why the requirements for position accuracy are function of errors in the current SP-44 edition - delivered by positioning systems and then processed in an appropriate way.

1.2 CONTEMPORARY HYDROGRAPHIC SYSTEMS

In Poland, the hydrographic works are performed by Navigation Marking Divisions of Maritime Offices in the towns of Gdynia, Szczecin and Slupsk, and by Hydrographic Office of Polish Navy. Maritime Institute in Gdansk, possessing modern measuring devices, also executes the works. A block diagram of those systems is given in the Fig. 1.1.

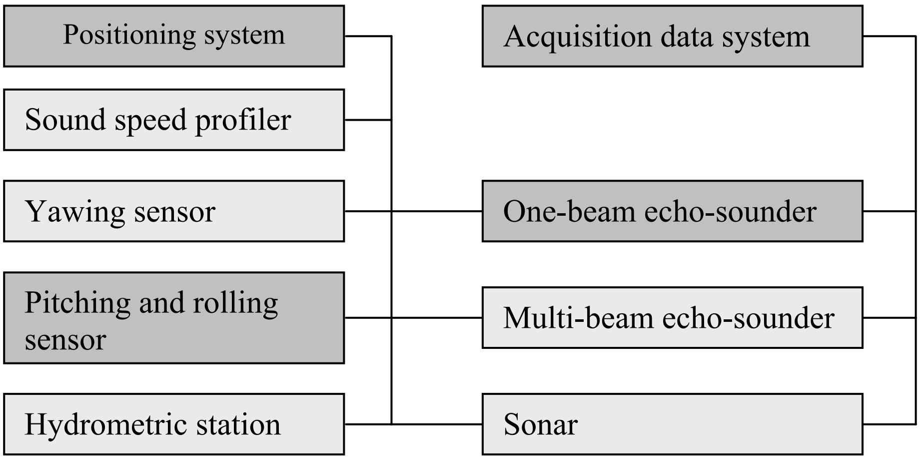

Fig. 1.1. Block diagram of hydrographic system

The hydrographic surveys are often performed against orders of owners of berths and harbour basins not equipped with own measuring devices but obliged to pass periodic inspections of the bed configurations. Then, the measurements are executed with single-beam echo-sounders and medium class DGPS or EGNOS system receivers by small private companies. Block diagram of the simplified hydrographic system is a modification (simplification) of the full system, and its elements have been marked in Fig. 1.1 with darker colour.

Yawing motion and pitching and rolling sensors, minimizing the measurements' errors, are not introduced in such a system using only the single-beam echo-sounder for the depth measurements. The sound speed in water is estimated based on a known water temperature in the surface layer, while the water level changes are taken from staff gauges.

1.3 REFERENCE SYSTEMS USED IN HYDROGRAPHY

On the Polish coast, various reference systems have been used in the hydrographic surveys for many years – among the others, the following ones:

- system 42, based on Krassowski ellipsoid, with the application point in the town of Pulkowo near Leningrad;

- Rauenberg system, based on Bessel ellipsoid, with the application point in Berlin;
- Potsdam system, based on Bessel ellipsoid, with the application point in Helmertturm;
- Borowa Gora system, based on Bessel ellipsoid, with the application point in Borowa Gora;
- system 1965 – system of rectangular plane coordinates;
- geocentric systems WGS-72 and WGS-84.

Convention on the law of the sea of 1982 forces information about the coordinate system of baselines to be given on sea charts (Article 16). Borders of economic zones and continental shelf must be determined in the same way (Articles 75, 76 and 84).

Current capabilities of computer techniques allow collecting the position data and bathymetric data on digital carriers. This makes direct usage of that information possible when elaborating sea charts, plans and plotting boards with computer programs. No consideration, or partial consideration, of corrections between the reference systems may result in additional errors. It is important because there are more and more mentions about replacement of the bathymetric data making a source of information for production of sea charts between national hydrographic offices.

In order to standardize the hydrographic surveys, IHO recommends that positions are referred to the geocentric reference system – to WGS 84 at best. In case the positions have been referred to a local reference system, they should be incorporated into the geocentric system. At the same time, it is recommended to document accuracy of the adaptation with appropriate documents.

1.4 DEFINITIONS OF REFERENCE COORDINATE SYSTEMS

1.4.1 SYSTEM OF NATURAL AND GEODESIC COORDINATES

System of natural and geodesic coordinates $\Im^E$ is commonly known in the hydrography as well as it is used in bathymetric measurements and in elaborating the measurements' results. This system is related to Earth in such a way that its versors e_x^E, e_y^E, e_z^E are respectively determined by its axes X, Y, Z [Czarnecki, 1997, chapter 1].

System of geodesic coordinates B, L, H is a coordinate system for which the geodesic longitude and geodesic latitude have been described in [Czarnecki, 1997, chapter 2]. H is an ortometric height represented by depth for bathymetric surveys.

1.4.2 COORDINATE SYSTEM RELATED TO EARTH, OF THE BEGINNING SET ON SOUNDING VESSEL

Coordinate system $\Im^V = \{O^V, (e_x^V, e_y^V, e_z^V)\}$ is connected with a position of sounding vessel or with a given position, and its beginning and base are determined in respect to coordinate system $\Im^E$. It is defined in the space in such a way that its beginning O^V is placed in point P (e.g. it may be centre of an object's mass) distant from ellipsoid surface by h; axis O_z^V runs along a normal versus the ellipsoid and towards Earth centre; axes O_x^V and O_y^V are parallel to the horizon surface.

1.4.3 COORDINATE SYSTEM RELATED TO SOUNDING VESSEL

Transfer from the coordinate system related to the sounding vessel onto the normal coordinate system related to Earth and of the beginning set on this unit may be performed by executing three rotations of the coordinate system around three axes laying in different planes, by the following angles of spatial orientation:

- angle of yaw Ψ,
- angle of inclination Θ,
- angle of roll Γ.

These angles refer respectively to:

- yawing – turn around axis e_z^V,
- pitching – turn around axis e_y^V,

- rolling – turn around axis e_x^V .

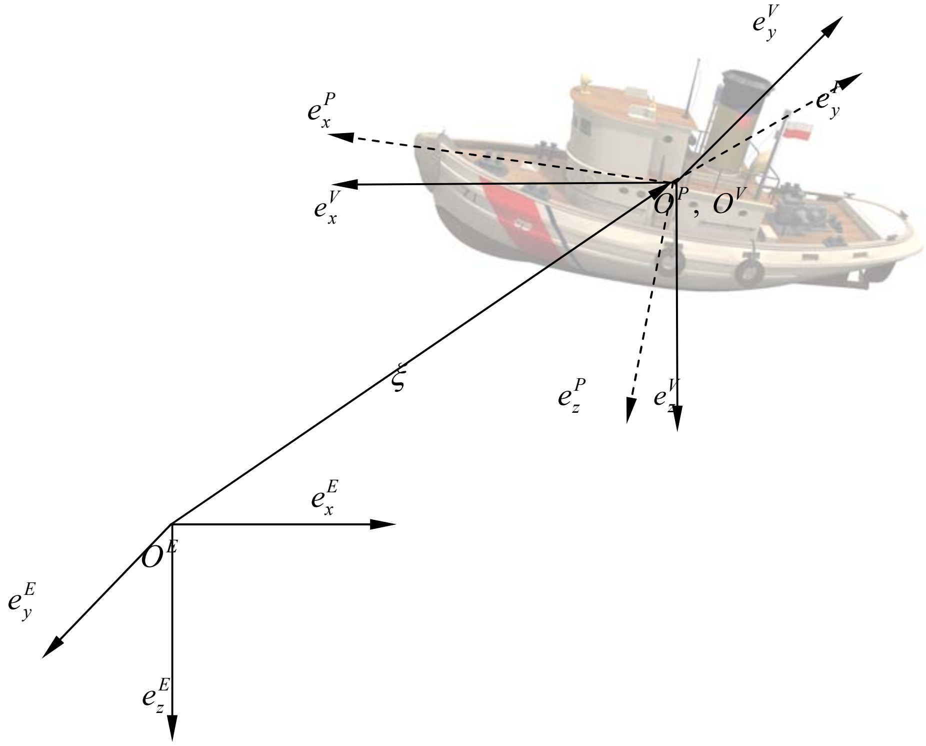

Fig. 1.2. Interpretation of position change in measuring and geocentric coordinate system

Other methods of transfer from one coordinate system to another are as the following:

- Euler angles,
- direction cosines,
- Rodrigez- Hamilton parameters,
- Cayley-Klein parameters,
- Hamilton quaternion,
- Pade approximation.

These methods are commonly used in interactive measuring systems [Ortyl 1994].

1.5 SOUND SPEED IN WATER AND ITS MEASUREMENT

Sound speed is one of the most important value defining conditions of acoustic waves' propagation in water. It is determined with direct methods and with an error up to Δc = ±0,03 m/s, or calculated based on empirical formulas [Wilson - 1962, Clay and Medwin - 1977, Chen and Millero - 1977, Klusek – 1990, Dera 2003] from measurements of other physical values, mostly from the so-called initial water parameters, i.e. temperature T, salinity S and hydrostatic pressure p_h. It is a non-linear function of these parameters and is expressed in a general as $c = c(T, S, p_h)$.

Speed of the acoustic wave propagation can be determined:

- on the basis of Medwin's formula:

$$\begin{aligned} c = 1449{,}2 - 4{,}6 \cdot T - 5{,}5 \cdot 10^{-2} \cdot T^2 - 2{,}9 \cdot 10^{-4} \cdot T^3 - 1{,}34 \cdot 10^{-2} \cdot T \cdot (S - 35) \\ - 1{,}6 \cdot 10^{-2} \cdot H \end{aligned} \tag{1.1}$$

- on the basis of Wilson's formula:

$$c = 1449{,}14 + \Delta c_T + \Delta c_p + \Delta c_S + \Delta c_{STp} \tag{1.2}$$

where c_T, c_P, c_S, $c_{ST\,P}$ are following components of vertical distributions of sound speed in water:

$$\begin{aligned}
\Delta c_T &= 4{,}5721\, T - 4{,}4532 \cdot 10^{-2}\, T^2 - 2{,}6045 \cdot 10^{-4}\, T^3 + 7{,}9851 \cdot 10^{-6}\, T^4 \\
\Delta c_p &= 1{,}60272 \cdot 10^{-1}\, p + 1{,}0268 \cdot 10^{-5}\, p^2 + 3{,}5216 \cdot 10^{-9}\, p^3 - 3{,}3603 \cdot 10^{-12}\, p^4 \\
\Delta c_S &= 1{,}39799(S - 35) + 1{,}69202 \cdot 10^{-3}(S - 35)^2 \\
\Delta c_{STp} &= (S - 35)(-1{,}1244 \cdot 10^{-2} T + 7{,}7711 \cdot 10^{-7}\, T^2 + 7{,}7016 \cdot 10^{-5} p - 1{,}2943 \cdot 10^{-7} p^2 + 3{,}1580 \cdot 10^{-8}\, pT + 1{,}5790 \cdot 10^{-9} pT^2) + p\,(-1{,}8607 \cdot 10^{-4} T + \\
&\quad 7{,}4812 \cdot 10^{-6} T^2 + \\
&\quad + 4{,}5283 \cdot 10^{-8} T^3) + p^2(-2{,}5294 \cdot 10^{-7} T + 1{,}8563 \cdot 10^{-9} T^2) - p^3 \cdot 1{,}9646 \cdot 10^{-10} T
\end{aligned}$$

- on the basis of Kinsler-Frey's formula:

$$\begin{aligned} c = {} & 1449{,}05 - 45{,}7 \cdot T - 5{,}21 \cdot T^2 - 0{,}23 \cdot T^3 - \\ & - (1{,}333 - 0{,}126 \cdot T - 0{,}009 \cdot T^2)(S - 35) - \\ & - 16{,}3 \cdot H \cdot 0{,}001 \cdot (1 - 0{,}0026 \cos 45) - \\ & - 0{,}18 \cdot (H \cdot 0{,}001 \cdot (1 - 0{,}0026 \cos 45))^2 \end{aligned} \tag{1.3}$$

Usage of CTD measuring instruments (measuring conductivity, temperature and depth) is not practical from this elaboration point of view. Determination of vertical sound speed distribution in water is singular for one measurement and the device must be connected to a computer before performance of the next measurement. Devices which operation is based on measurement of time of the acoustic wave pass along a strictly defined and known route is an alternative for this kind of the measuring instruments.

These measuring devices are often sunk down the bed and the data are stored in their memories by the time they have been opened.

Incorporation of individual calculation algorithms may result in the vertical sound speed distributions varying from each other – a given algorithm may bring, more or less, the real profile closer. Vertical sound speed distributions in water have been presented for the following formulas in Fig. 1.3: Medwin, Wilson and Kinsler-Frey, out of which the last one is the most different from the others. This formula is assigned for arctic waters.

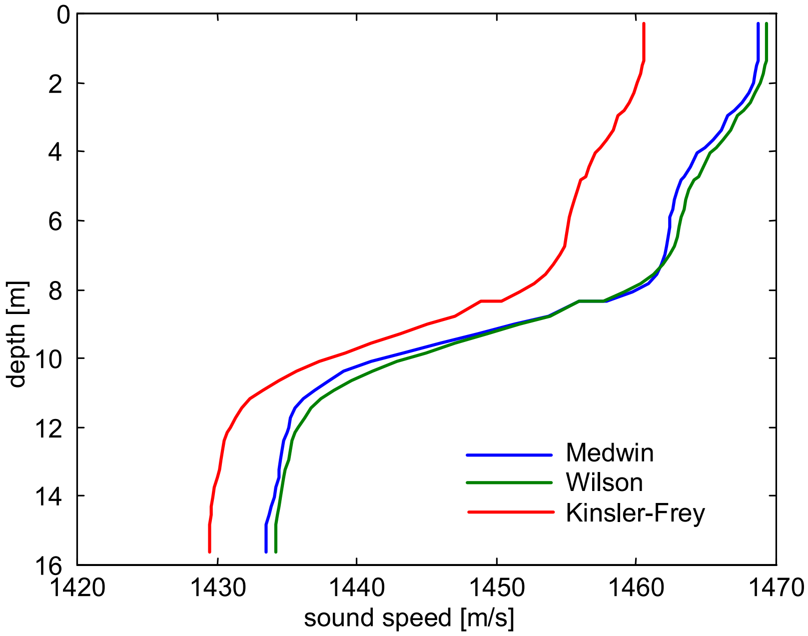

Fig. 1.3. Vertical sound speed distributions in water for formulas: Medwin, Wilson and Kinsler-Frey

1.6 ACOUSTIC CLIMATE OF SOUTH BALTIC SEA AND OF INLAND WATER REGIONS, AND ITS IMPACT ON ACCURACY OF DEPTH MEASUREMENTS

1.6.1 DYNAMIC PHENOMENA IN WATER ENVIRONMENT

Spacial-time classifications of dynamic phenomena in water, and related description of the sound speed field changes $c(x,y,z,t)$ acc. to DeSanto [Klusek, 1990], seem to be the most convenient system applied in tasks of the contemporary hydro-acoustics. Following this kind of classification, the observed local sound speed values are a sum of three components:

$$c(x,y,z,t) = c(x,y,z) + \delta c_1(x,y,z,t) + \delta c_2(x,y,z,t)$$

Component $c(x,y,z,t)$ determines average vertical sound speed profiles which are result of averaging for a period of at least one year and they define the so-called water region climate. Component $\delta c_1(x,y,z,t)$ stands for mezzo-scale processes occurring accidentally during acoustic tests lasting usually several days and it defines weather of the water region. Component $\delta c_2(x,y,z,t)$ refers to small-scale changes of the environment parameters caused by internal waves, fine structure, micro-structure and turbulence – also of accidental character.

1.6.2 SEASONAL CHANGES OF VERTICAL SOUND SPEED DISTRIBUTION IN WATER

Water temperature is a main factor having a long-term impact on changes of the sound speed distributions in water. Cold surface water in winter time and increase of its temperature down the bed result in higher and higher sound speed as water is deeper.

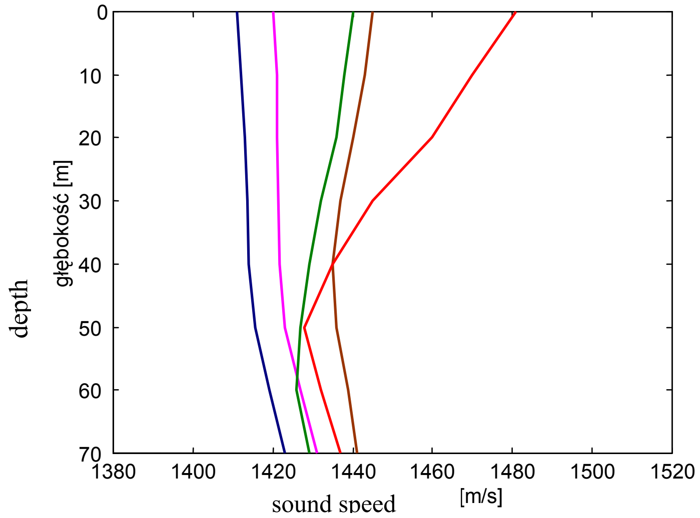

Fig. 1.4. Seasonal sound speed distributions in the Bay of Gdansk

🟦	Winter	1411 - 1423	m/s
🟪	Early Spring	1420 - 1431	m/s
🟫	Spring	1445 – 1441 (1435)	m/s
🟥	Summer	1481 – 1437 (1428)	m/s
🟩	Autumn	1440 – 1429 (1426)	m/s

The sound speed drops in the remaining seasons (spring, summer, autumn) due to temperature decrease of water which is the warmest on the surface. Minimal sound speed values have been given in parentheses. The sound speed drops in the remaining seasons (spring, summer, autumn) due to temperature decrease of water which is the warmest on the surface. Minimal sound speed values have been given in parentheses.Sound speed distributions for individual seasons have been presented on the Fig. 1.4 [Klusek, 1990, chapter 1].

Spatial distributions of sound speed field on water surface in the Bay of Gdansk are presented in Fig. 1.5, while Fig. 1.6 shows vertical section of the speed field from West to East in summer [Klusek, 1990, chapter 1]. Isopleths of the constant sound speed are distributed uniformly, hence there are no spatial changes of the vertical sound speed distribution in water for this direction.

Area of bathymetric soundings in an inland water region are presented in Fig. 1.7 and 1.8 – together with measuring points and vertical section of the sound speed field in water.

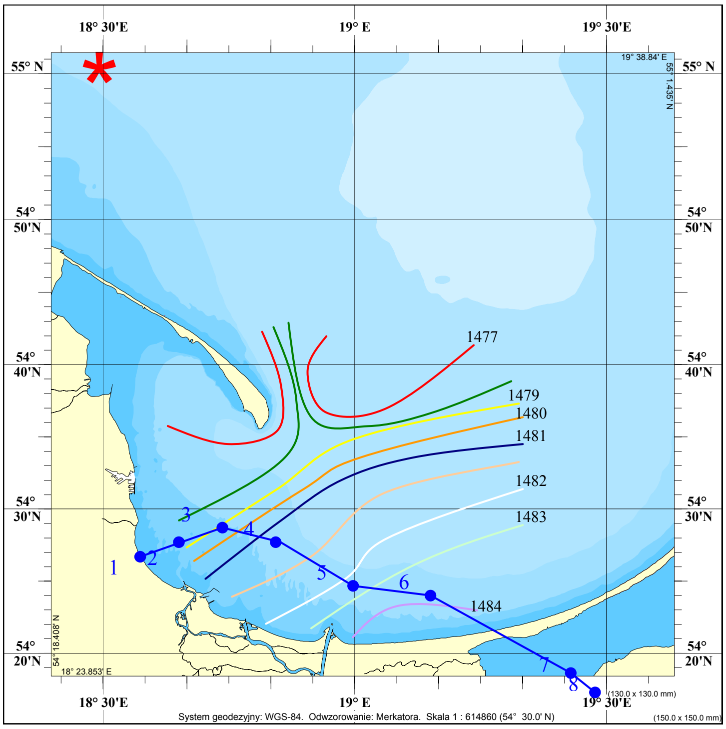

Fig. 1.5. Spatial distribution of sound speed field on water surface in the Bay of Gdansk in summer

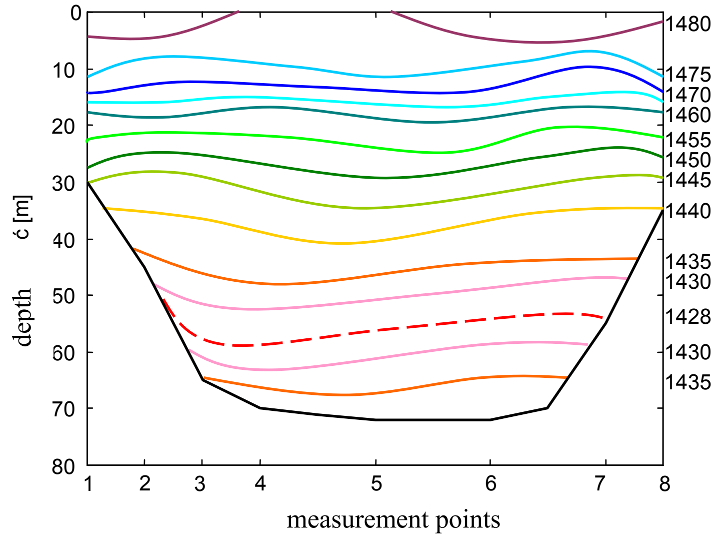

Fig. 1.6. Vertical section of sound speed field in the Bay of Gdansk in summer – from West to East

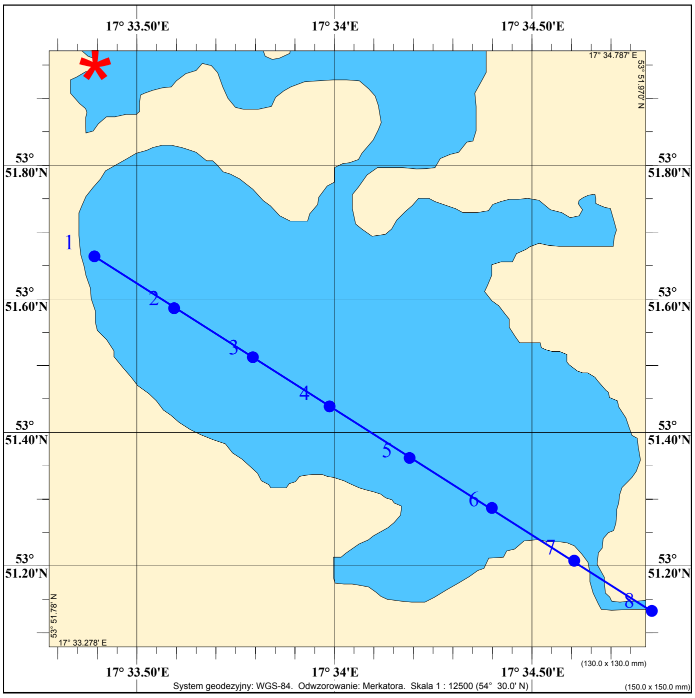

Fig. 1.7. Measuring points of sound speed in waters of the Lake Lackie

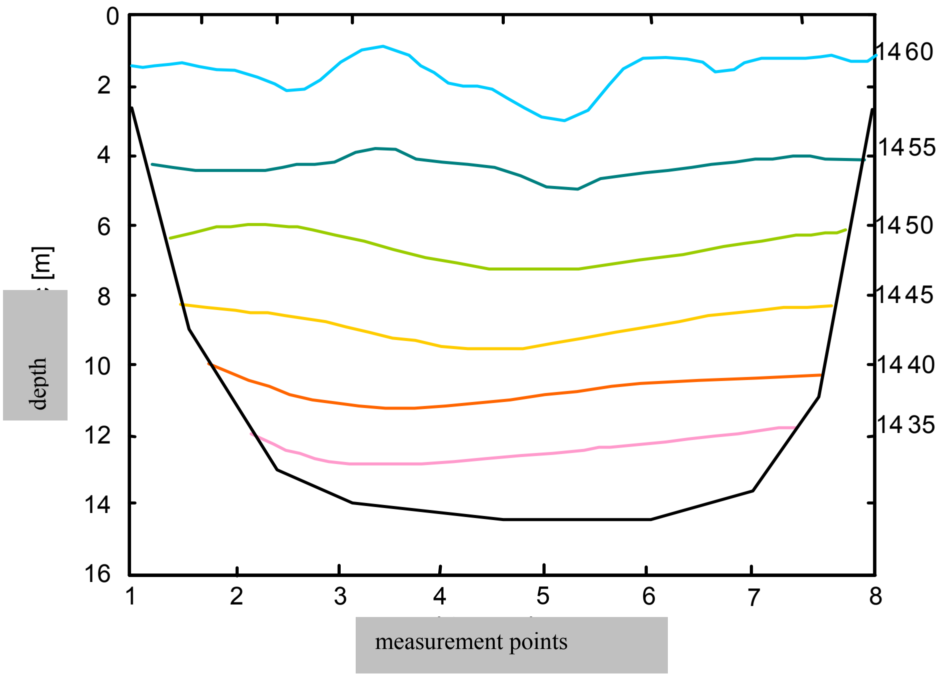

Fig. 1.8. Vertical section of sound speed field in the Lake Lackie in spring 24-hour changes of vertical sound speed distribution in water

1.6.3 24-HOUR CHANGES OF VERTICAL SOUND SPEED DISTRIBUTION IN WATER

Similar to seasonal changes, temperature difference of various water layer, caused by up warming of the surface layer by sun (afternoon effect), is the main factor influencing the 24-hour changes.

Ranges of the sound speed 24-hour changes and average 24-hour sound speed profile in spring have been presented on the Fig. 1.9. It is 20 m/s in the surface layer where the difference in water temperatures is the biggest.

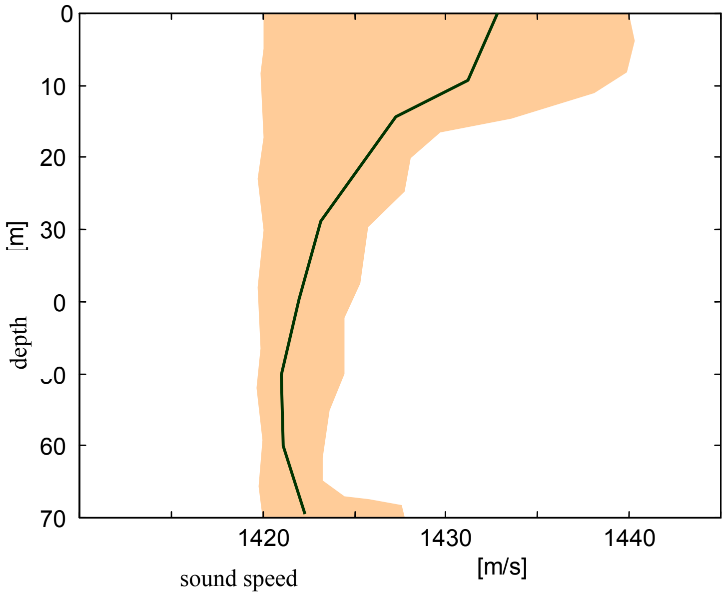

Fig. 1.9. Ranges of sound speed 24-hour changes and average 24-hour sound speed profile in the Bay of Gdansk in summer

The range of changes in the sound speed is even wider in summer, when there are bigger differences in water temperatures there, especially in the surface layer, and it may reach 40 m/s.

1.7 DETERMINATION OF SPATIAL ORIENTATION OF SOUNDING VESSEL

Contemporary professional hydrographic systems, equipped with multi-beam echo-sounders, use accelerometers (motion sensors) to compensate motion interferences. Their task is to determine angles of pitch, roll and heave. It is recommended to use heave sensors (in practice, they are of rare usage) for measurements executed with a single-beam echo-sounder; pitch and roll sensors are not used because of their big prices.

System of motion interference compensation, based on multi-sensor positioning system, is an alternative for accelerometers. There are two-antenna devices available, for determination of spatial direction with a phase method applied which in hydrography may be used for determination of one angle – of pitch or roll. Introduction of inexpensive GPS receivers for determination of tilts with the phase method applied are described in [Blake, 2008]. There were also tests on usage of three-antenna system for determination of spatial orientation performed there - in a stadiometric variant [Naus, Makar, 2002b, 2003].

Let's have the hydrographic system, consisting of echo-sounder containing transducer in point P_1 and of positioning system with antenna in point P_2, extra equipped with two receiving antennas enabling determination of points P_3, P_4 and making a reference measuring system of versors (p_1, p_2, p_3).

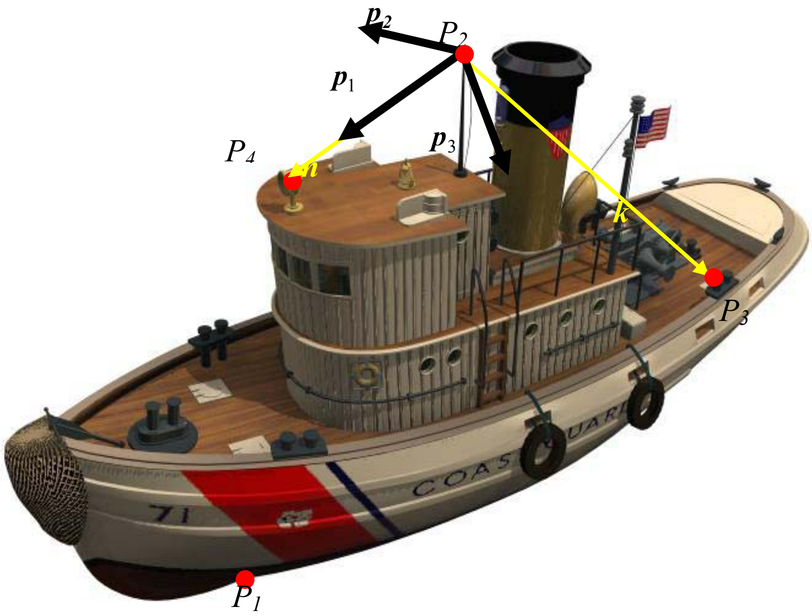

Fig. 1.10. Positions of non-collinear points P_2,P_3,P_4

Let's have coordinates of the points P_2,P_3,P_4, being positions of the positioning system's antennas, known:

Based on these points vectors were determined [Naus, Makar, 2002b, 2003]:

$$\boldsymbol{n} = \overrightarrow{\mathrm{P_2P_3}} = \begin{bmatrix} \boldsymbol{n}_X \\ \boldsymbol{n}_Y \\ \boldsymbol{n}_Z \end{bmatrix} = \begin{bmatrix} X_3 - X_2 \\ Y_3 - Y_2 \\ Z_3 - Z_2 \end{bmatrix}, \tag{1.4}$$

$$\boldsymbol{k} = \overrightarrow{\mathrm{P_2P_4}} = \begin{bmatrix} \boldsymbol{k}_X \\ \boldsymbol{k}_Y \\ \boldsymbol{k}_Z \end{bmatrix} = \begin{bmatrix} X_4 - X_2 \\ Y_4 - Y_2 \\ Z_4 - Z_2 \end{bmatrix}. \tag{1.5}$$

These vectors were used to define a base p_1, p_2, p_3 of system $\Im^P$, according to the below dependences:

$$\boldsymbol{p_1} = \begin{bmatrix} \boldsymbol{p}_1^X \\ \boldsymbol{p}_1^Y \\ \boldsymbol{p}_1^Z \end{bmatrix} = \frac{\boldsymbol{n}}{|\boldsymbol{n}|} = \begin{bmatrix} \dfrac{\boldsymbol{n}_X}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \\ \dfrac{\boldsymbol{n}_Y}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \\ \dfrac{\boldsymbol{n}_Z}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \end{bmatrix}, \tag{1.6}$$

$$\boldsymbol{p_2} = \begin{bmatrix} \boldsymbol{p}_2^X \\ \boldsymbol{p}_2^Y \\ \boldsymbol{p}_2^Z \end{bmatrix} = \frac{\boldsymbol{k} \times \boldsymbol{n}}{|\boldsymbol{k} \times \boldsymbol{n}|} = \begin{bmatrix} \dfrac{\boldsymbol{k}_Y\boldsymbol{n}_Z - \boldsymbol{k}_Z\boldsymbol{n}_Y}{\sqrt{\begin{vmatrix} \boldsymbol{k}_Y & \boldsymbol{k}_Z \\ \boldsymbol{n}_Y & \boldsymbol{n}_Z \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_Z & \boldsymbol{k}_X \\ \boldsymbol{n}_Z & \boldsymbol{n}_X \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_X & \boldsymbol{k}_Y \\ \boldsymbol{n}_X & \boldsymbol{n}_Y \end{vmatrix}^2}} \\ \dfrac{\boldsymbol{n}_Z\boldsymbol{k}_X - \boldsymbol{n}_X\boldsymbol{k}_Z}{\sqrt{\begin{vmatrix} \boldsymbol{k}_Y & \boldsymbol{k}_Z \\ \boldsymbol{n}_Y & \boldsymbol{n}_Z \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_Z & \boldsymbol{k}_X \\ \boldsymbol{n}_Z & \boldsymbol{n}_X \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_X & \boldsymbol{k}_Y \\ \boldsymbol{n}_X & \boldsymbol{n}_Y \end{vmatrix}^2}} \\ \dfrac{\boldsymbol{n}_X\boldsymbol{k}_Y - \boldsymbol{n}_Y\boldsymbol{k}_X}{\sqrt{\begin{vmatrix} \boldsymbol{k}_Y & \boldsymbol{k}_Z \\ \boldsymbol{n}_Y & \boldsymbol{n}_Z \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_Z & \boldsymbol{k}_X \\ \boldsymbol{n}_Z & \boldsymbol{n}_X \end{vmatrix}^2 + \begin{vmatrix} \boldsymbol{k}_X & \boldsymbol{k}_Y \\ \boldsymbol{n}_X & \boldsymbol{n}_Y \end{vmatrix}^2}} \end{bmatrix}, \tag{1.7}$$

$$\boldsymbol{p_3} = \begin{bmatrix} \boldsymbol{p}_3^X \\ \boldsymbol{p}_3^Y \\ \boldsymbol{p}_3^Z \end{bmatrix} = \boldsymbol{p_1} \times \boldsymbol{p_2} = \begin{bmatrix} \boldsymbol{p}_1^Y \cdot \boldsymbol{p}_2^Z - \boldsymbol{p}_1^Z \cdot \boldsymbol{p}_2^Y \\ \boldsymbol{p}_1^Z \cdot \boldsymbol{p}_2^X - \boldsymbol{p}_1^X \cdot \boldsymbol{p}_2^Z \\ \boldsymbol{p}_1^X \cdot \boldsymbol{p}_2^Y - \boldsymbol{p}_1^Y \cdot \boldsymbol{p}_2^X \end{bmatrix}. \tag{1.8}$$

Position of centre O_p of the measuring reference system $\Im^p$ in respect to a system $\Im$ is defined by a vector $\overrightarrow{OP_2}$ which beginning is situated in a centre of the geocentric, ortocartesian reference system $\Im$, and its end – in point P_2.

In a special case, when the echo-sounder's transducer is installed under the receiver's antenna of the positioning system and the acoustic wave propagates rectilinearly, position of the measuring point P_r from which the acoustic wave reflects, based on coordinates of the positioning system's antenna may be determined from the following dependence:

$$\overrightarrow{P_1P_r} = \begin{bmatrix} (l+r)\cdot\cos\beta_x \\ (l+r)\cdot\cos\beta_y \\ (l+r)\cdot\cos\beta_z \end{bmatrix}, \tag{1.9}$$

where: $P_r = \begin{bmatrix} X_r & Y_r & Z_r \end{bmatrix}^T$, while $\cos\beta_x, \cos\beta_y, \cos\beta_z$ are direction cosines of angles between axes of the system connected with the sounding vessel and antenna-transducer axis, l is a distance between the antenna and transducer, and r is a length of acoustic radius, that is – a depth measured by the echo-sounder (distance between the echo-sounder's transducer and sea bed).

1.8 CONCEPTION OF HYDROGRAPHIC SYSTEM BASED ON SINGLE-BEAM ECHO-SOUNDER AND TWO-ANTENNA POSITIONING SYSTEM

Proposed system of hydrographic surveys is based on application of the following devices:

- single-beam echo-sounder for measurements of depth,
- two-antenna positioning system, additionally paying a role of waving and sounding vessel's rolling compensator,
- instrument measuring sound speed in water – for accompanying measurements; with the sound speed in water known, based on a change of the acoustic wave's impulse speed on a route electro-acoustic transducer – sea bed - electro-acoustic transducer, not only an accurate depth shall be determined but also position of the acoustic wave's reflection point – as a result of non-rectilinear propagation of the acoustic wave, defined based on models of spacial-time sound speed distributions.

Such a system can be used on boards of small sea-going vessels, very often performing surveys in sea port water regions and on fairways (water regions of special class) where required accuracy of soundings is the highest.

In order to obtain high accuracy of hydrographic soundings, this area of concern has been solved in the following four stages:

- Spatial-time modelling of the vertical sound speed distributions in water has been done in the first stage (Chapter 2). Based on measurements of the sound speed done in various points of water region under survey, its spatial changes have been defined – being surfaces of constant sound speed if presented as a graphic. With the spatial model of the sound speed distribution known, it is possible to determine a local sound speed. Moreover, due to time changes of the sound speed (during performance of hydrographic survey), it is necessary to make models of the distributions. The spatial-time sound speed distributions have been used not only to determine a depth, but also - to define a trajectory of acoustic ray as a result of occurrence of refraction phenomenon (in the third stage).
- A positioning system, paying a role of motion sensor, has been designed in the second stage (Chapter 3). It has been assumed that having knowledge about coordinates of two points suitably situated, it is possible to determine spatial orientation. They have been used for compensation of interferences

of the sounding vessel movement – for determination of real depth and of acoustic ray trajectory.

- Trajectory of acoustic ray has been defined in the third stage (Chapter 3) – based on knowledge about the local vertical sound speed distribution in water (Chapter 2) and outlet angle of acoustic ray leaving the acoustic transducer. A two-antenna positioning system, paying a role of a system for spatial orientation determination, has been applied to determine the outlet angle of the acoustic ray. Coordinates of the acoustic wave's reflection points have been defined.
- Methodology of geodesic bathymetric surveys have been proposed in the fourth stage, together with measurements of the sound speed in water – to determine the spatial-time model and local vertical sound speed distribution, evaluation of compatibility of the regression function with the measured values and calibration of the measuring system.

2 MODELLING OF SPATIAL-TIME SOUND SPEED DISTRIBUTIONS IN WATER

2.1 ESTIMATION OF IMPACT OF SOUND SPEED VERTICAL DISTRIBUTIONS ON ACCURACY OF INDICATIONS OF SINGLE-BEAM ECHO-SOUNDER

Results of measurements executed in freshwater region (own survey) and in the Bay of Gdansk [Klusek, 1990] were applied to analyze impact of the sound speed distributions on indications of the echo-sounder. Those regions are characterized by various depths and ranges of sound speed changes.

Based on the measurements of the sound speed distributions, its average value was defined – the value being taken into consideration when determining the depth. Following an absolute runtime of the acoustic wave from the transducer to the sea bed, a real time of the acoustic wave runtime from the transducer to the sea bed and back was determined. Then, based on the acoustic wave runtime and various values of the sound average speed, value of depth indicated by the echo-sounder was defined.

The real and approximate sound speed distributions in the lake and in the Bay of Gdansk are presented in Fig. 2.1 and 2.2. Distributions of errors of the depth measurements for various values of the sound average speed are given in Fig. 2.3 and 2.4 – expressed as a difference between values of the depths, determined with an average value of the sound speed and an absolute distribution taken into consideration.

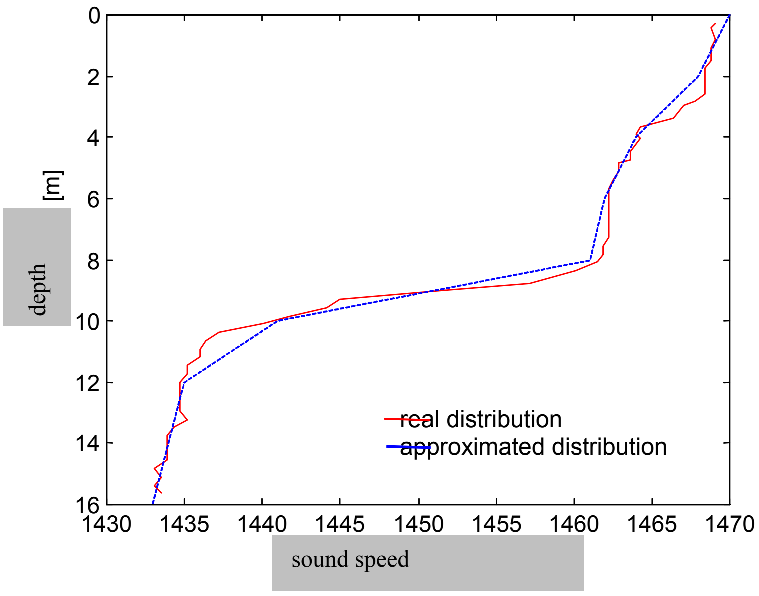

Fig. 2.1. Vertical sound speed distribution in freshwater region

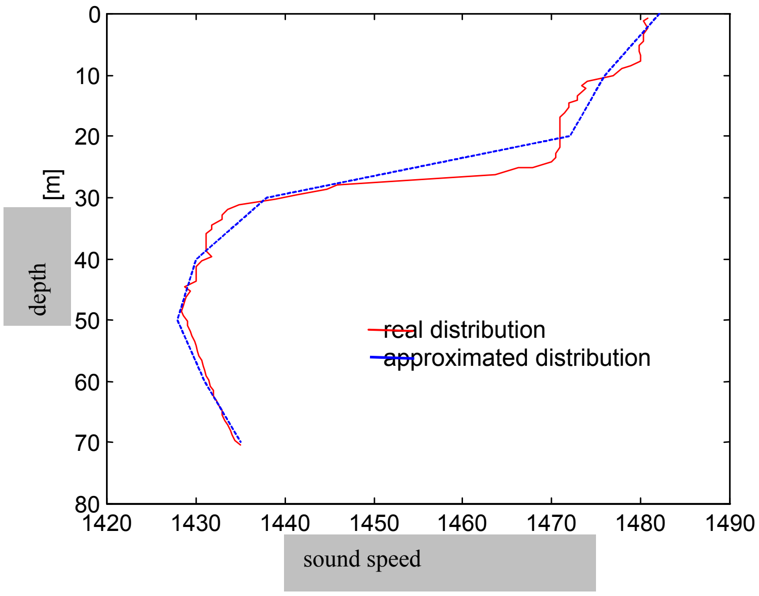

Fig. 2.2. Vertical sound speed distribution in the Bay of Gdansk

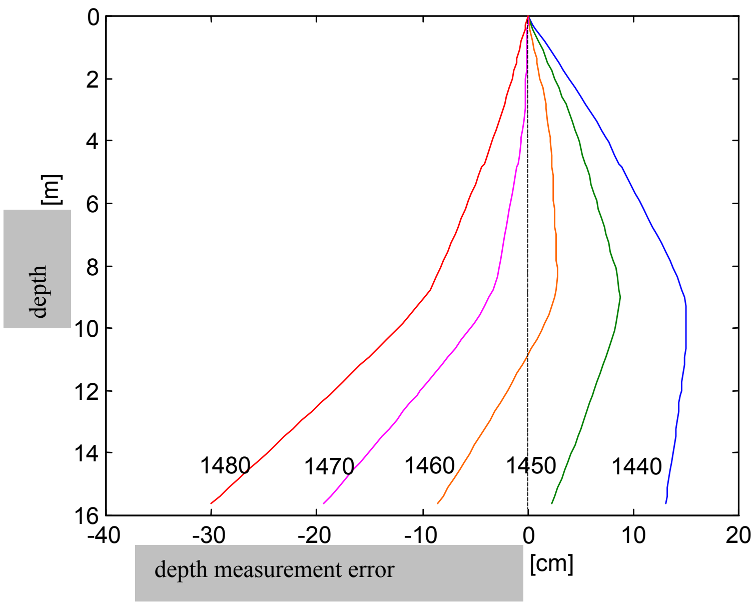

Fig. 2.3. Vertical distribution of errors of depth measurements in freshwater region

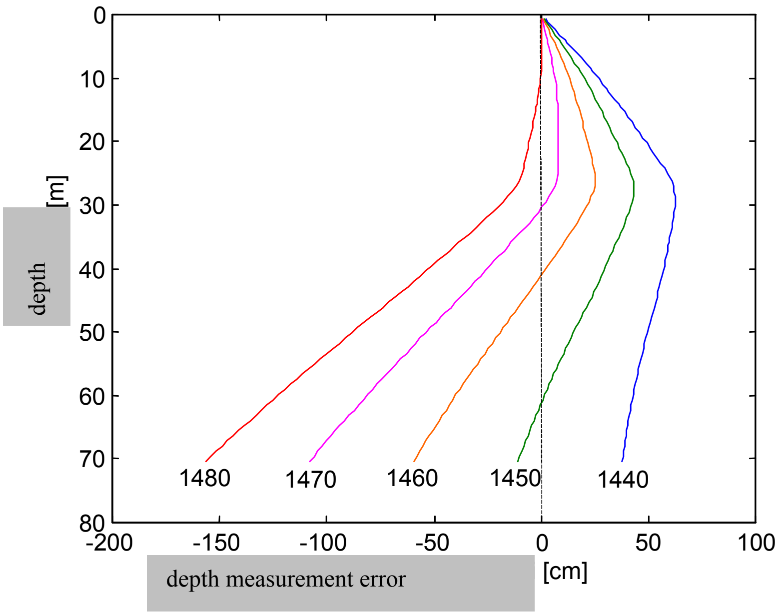

Fig. 2.4. Vertical distribution of errors of depth measurements in the Bay of Gdansk

2.2 MODELLING OF SOUND SPEED SPATIAL DISTRIBUTIONS IN WATER

Let's have - based on the measurement data - values of the sound speed in water $\tilde{c}_{x,y,t,k}$ - taken at the moment t, $t \in \overline{1,l}$ at the depths $H_k^{'}$, $k \in \overline{0,n}$, - known in the places of co-ordinates $P_k = [x_k \quad y_k]^T$:

$$\tilde{c}_{x,y,t,k} = \tilde{c}_{x,y,t}(H_k)\,. \tag{2.1}$$

Equation of the sound constant speed surface in water may be given as a linear combination

$$\mathbf{S}_{i,j}(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,j}\Phi_{i,p}(x)\Phi_{j,q}(y) \tag{2.2}$$

of basic B–spline functions, or in a form of NURBS (Non-Uniform Rational B-Spline) functions [Piegl and Tiller, 1997 – chapter 4]

$$\mathbf{S}_{i,j}(x,y) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\Phi_{i,p}(x)\Phi_{j,q}(y)w_{i,j}\mathbf{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}\Phi_{i,p}(x)\Phi_{j,q}(y)w_{i,j}}, \quad a \le x, y \le b\,, \tag{2.3}$$

where a set $\mathbf{P}_{i,j}$ makes a two-direction control network, $w_{i,j}$ stands for weighers, and $\Phi_{i,p}(x)$ and $\Phi_{j,q}(y)$ are rational basic B-spline functions, defined against a nodes range of co-ordinates:

$$\mathbf{X} = [\underbrace{0,\ldots,0}_{p+1}, x_{p+1},\ldots,x_{r-p-1},\underbrace{1,\ldots,1}_{p+1}]^T\,,$$

$$\mathbf{Y} = [\underbrace{0,\ldots,0}_{q+1}, y_{q+1},\ldots,y_{s-q-1},\underbrace{1,\ldots,1}_{q+1}]^T\,,$$

where $r = n + p + 1$ and $s = m + q + 1$.

$\Phi_{i,p}$ is an i-th basic B-spline function of p-degree, defined as [Piegl and Tiller, 1997 - chapter 4]:

$$\Phi_{i,0} = \begin{cases} 1 & \text{dla } H_i \le H < H_{i+1} \\ 0 & \text{for other} \end{cases},$$

$$\Phi_{i,p}(H) = \frac{H - H_i}{H_{i+p} - H_i}\Phi_{i,p-1}(H) + \frac{H_{i+p+1} - H}{H_{i+p+1} - H_{i+1}}\Phi_{i+1,p-1}(H)\,. \tag{2.4}$$

For example, B-spline function of 3-rd degree is given as:

$$\Phi_{i,3}(x) = \frac{1}{3!\,g_x^4}\begin{cases} 0 & x \in (-\infty, x_{i-2} \\ (x - x_{i-2})^3 & x \in [x_{i-2}, x_{i-1} \\ g_x^3 + 3g_x^2(x - x_{i-1}) + 3g_x(x - x_{i-1})^2 - 3(x - x_{i-1})^3 & x \in [x_{i-1}, x_i] \\ g_x^3 + 3g_x^2(x_{i+1} - x) + 3g_x(x_{i+1} - x)^2 - 3(x_{i+1} - x)^3 & x \in [x_i, x_{i+1}] \\ (x_{i+2} - x)^3 & x \in [x_{i+1}, x_{i+2} \\ 0 & x \in [x_{i+2}, \infty) \end{cases} \tag{2.5}$$

..

with equidistant nodes $x_i = x_0 + ig_x$, $g_x := \frac{x_N - x_0}{N}$, $i \in \overline{-1, N+1}$. Its graphical interpretation is presented in Fig. 2.5.

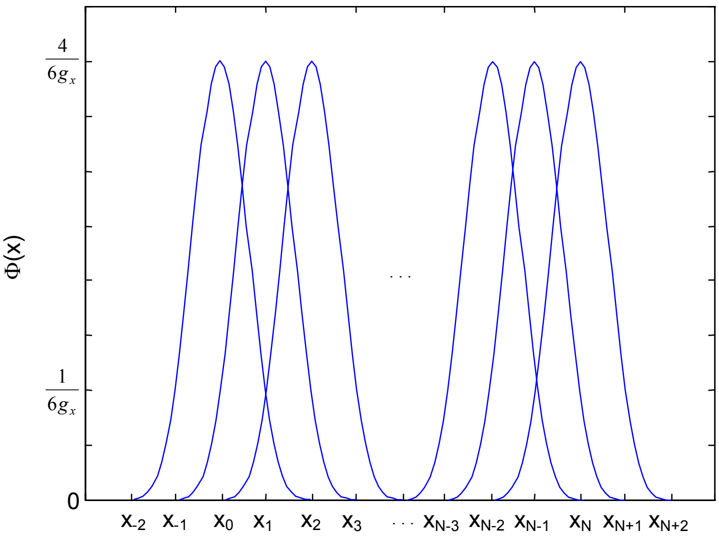

Fig. 2.5. Runs of basic B-spline functions of 3-rd degree for one variable

While the B-spline function of 5-th degree receives a form of:

$$\Phi_{i,5}(x)=\frac{1}{5!\,g_x^6}\begin{cases}0 & dla \quad x\le x_{i-3}\\ f_1(x) & dla \quad x_{i-3}\le x\le x_{i-2}\\ f_2(x) & dla \quad x_{i-2}\le x\le x_{i-1}\\ f_3(x) & dla \quad x_{i-1}\le x\le x_{i}\\ f_4(x) & dla \quad x_{i}\le x\le x_{i+1}\\ f_5(x) & dla \quad x_{i+1}\le x\le x_{i+2}\\ f_6(x) & dla \quad x_{i+2}\le x\le x_{i+3}\\ 0 & dla \quad x\ge x_{i+3}\,,\end{cases}, \tag{2.6}$$

$i\in\overline{-3,N+3}$,

where:

$$f_1(x)=\tau_1^5,\quad \tau_1=x-[x_0+(i-3)g_x]\,,$$

$$f_2(x)=g_x^{\,5}+5g_x^4\tau_2+10g_x^3\tau_2^2+10g_x^2\tau_2^3+5g_x\tau_2^4-5\tau_2^5\,,$$

$$\tau_2=x-[x_0+(i-2)g_x]\,,$$

$$f_3(x)=26g_x^5+50g_x^4\tau_3+20g_x^3\tau_3^2-20g_x^2\tau_3^3-20g_x\tau_3^4+10\tau_3^{\,5}\,,$$

$$\tau_3=x-[x_0+(i-1)g_x]\,,$$

$$f_4(x)=26g_x^5+50g_x^4\tau_4+20g_x^3\tau_4^2-20g_x^2\tau_4^3-20g_x\tau_4^4+10\tau_4^{\,5}\,,$$

$$\tau_4=x_0+(i+1)g_x-x\,,$$

$$f_5(x)=g_x^5+5g_x^4\tau_5+10g_x^3\tau_5^2+10g_x^2\tau_5^3+5g_x\tau_5^4-5\tau_5^5\,,$$

$$\tau_5=x_0+(i+2)g_x-x\,,$$

$$f_6(x)=\tau_6^5,\quad \tau_6=x_0+(i+3)g_x-x\,.$$

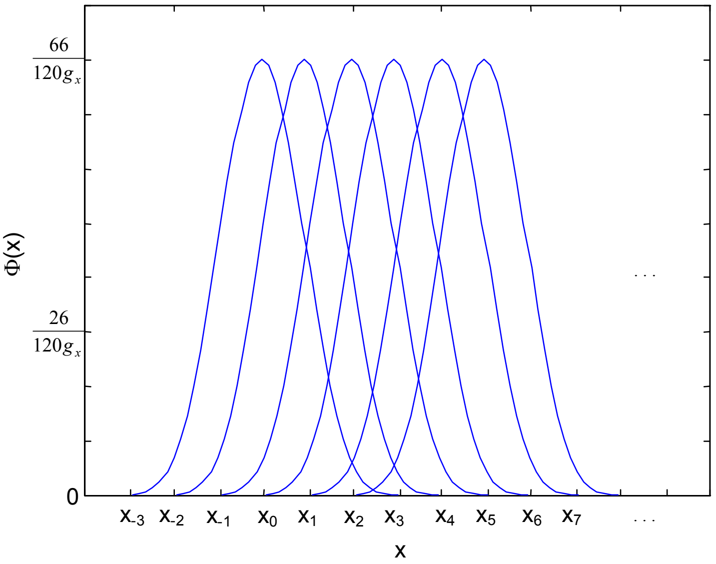

Fig. 2.6. Runs of basic B-spline functions of 5-th degree for one variable

Substituting

$$R_{i,j}(x,y)=\frac{\Phi_{i,p}(x)\Phi_{j,q}(y)w_{i,j}}{\sum_{k=0}^{n}\sum_{l=0}^{m}\Phi_{k,p}(x)\Phi_{l,q}(y)w_{k,l}}, \tag{2.7}$$

the equation (2.3) may be expressed as:

$$\mathbf{S}_{i,j}(x,y)=\sum_{i=0}^{n}\sum_{j=0}^{m}R_{i,j}(x,y)\mathbf{P}_{i,j}\ . \tag{2.8}$$

Coefficients $a_{i,j}$ are selected in such a way that the functional describing mean-square error of the method:

$$J(\hat{a}_{0,0},\cdots,\hat{a}n,m)=\sqrt{\sum_{k=0}^{r}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\hat{a}_{i,j}\Phi_{i,p}(x_k)\Phi_{j,q}(y_k)-\tilde{c}_k\right]^2} \tag{2.9}$$

reaches minimum [Musielak 1976]. Marking

$$\boldsymbol{v}_{i,j} := \begin{bmatrix} \Phi_i(x_0)\Phi_j(y_0) \\ \Phi_i(x_1)\Phi_j(y_1) \\ \vdots \\ B_i(x_n)B_j(y_n) \end{bmatrix}, \qquad \tilde{\boldsymbol{c}}_k := \begin{bmatrix} \tilde{c}_0 \\ \tilde{c}_1 \\ \vdots \\ \tilde{c}_n \end{bmatrix}, \tag{2.10}$$

the functional (2.9) may be given in a form

$$J(\hat{a}_{0,0},\cdots,\hat{a}_{n,m}) := \left\| \sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{a}_{i,j}\boldsymbol{v}_{i,j} - \tilde{\boldsymbol{c}}_k \right\|, \tag{2.11}$$

where $\| \;\|$ is a norm induced by a scalar product $(\,|\,)$ in Hilbert space l_n^2.

Assuming that the range $B := \{\boldsymbol{v}_{0,0}, \boldsymbol{v}_{0,1}, \cdots, \boldsymbol{v}_{nm}\}$ makes a set of vectors independent, in respect to linearity, in the space l_n^2,

$$LimB := \left\{ \boldsymbol{\omega} = \sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{a}_{i,j}\boldsymbol{v}_{i,j} \right\} \tag{2.12}$$

is a closed subspace of a normed linear space l_n^2.

Such a vector

$$\boldsymbol{c}_* = \tilde{a}_{0,0}^0\boldsymbol{v}_{0,0} + \cdots + \tilde{a}_{n,m}^0\boldsymbol{v}_{n,m} \tag{2.13}$$

of hyperplane $LimB$ should be determined which is of the nearest position (in a sense of norm $\| \;\|$) in respect to the given vector $\boldsymbol{c}$. Thus, such a range of real numbers $(\tilde{a}_{0,0},\cdots,\tilde{a}_{n,m})$ should be defined, that

$$J(\hat{a}_{0,0}^0,\cdots,\hat{a}_{n,m}^0) = \min\left\{ J(\tilde{a}_{0,0}^0,\cdots,\tilde{a}_{n,m}^0) \right\}. \tag{2.14}$$

2.3 MODELLING OF TIME-DOMAIN DISTRIBUTIONS OF SOUND SPEED IN WATER

Let's assume that the sound speed distribution in water c is expressed with a function $c(H)$. Based on the measurement data, $\tilde{c}_k, k \in \overline{0,n} := \{0,1,\ldots,n\}$ of this distribution, executed at depths H_k, i.e.

$$\tilde{c}_k = \tilde{c}(H_k), \tag{2.15}$$

the function $c^* = c^*(H)$ should be defined, while the function $c = c(H)$ should be approximated. Then, a function

$$\tilde{c} = a_0\Phi_0(H) + a_1\Phi_1(H) + \cdots + a_n\Phi_n(H) \tag{2.16}$$

should be determined and the function is the best least-square approximation of the function $c = c(H)$ in points H_k . Coefficients $a_i, i \in \overline{0,n}$ should be selected in such a way that the functional

$$J(\hat{a}_0,\cdots,\hat{a}_n) = \sqrt{\sum_{k=0}^{r}\left[\sum_{i=1}^{n}\hat{a}_i\Phi_i(H_k) - \tilde{c}_k\right]^2} \tag{2.17}$$

reaches a minimum.

Calculating derivatives against a_i, $i \in \overline{0,N}$, and then – comparing them with zero, the result is

$$\frac{\partial J}{\partial a_0} = 2\sum_{k=0}^{n}[a_0\Phi_0(H_k) + a_1\Phi_1(H_k) + \cdots + a_n\Phi_n(H_k) - \tilde{c}_k]\Phi_0(H_k) = 0$$

$$\frac{\partial J}{\partial a_1} = 2\sum_{k=0}^{n}[a_0\Phi_0(H_k) + a_1\Phi_1(H_k) + \cdots + a_n\Phi_n(H_k) - \tilde{c}_k]\Phi_1(H_k) = 0$$

....

$$\frac{\partial J}{\partial a_i} = 2\sum_{k=0}^{n}[a_0\Phi_0(H_k) + a_1\Phi_1(H_k) + \cdots + a_n\Phi_n(H_k) - \tilde{c}_k]\Phi_i(H_k) = 0 \tag{2.18}$$

....

$$\frac{\partial J}{\partial a_N} = 2\sum_{k=0}^{n}[a_0\Phi_0(H_k) + a_1\Phi_1(H_k) + \cdots + a_n\Phi_n(H_k) - \tilde{c}_k]\Phi_n(H_k) = 0$$

After conversions, the above set of equations may be given as the following

$$\begin{cases} a_0^0\sum\limits_{k=0}^{n}\Phi_0(H_k)\Phi_0(H_k) + \cdots + a_n^0\sum\limits_{k=0}^{n}\Phi_n(H_k)\Phi_0(H_k) = \sum\limits_{k=0}^{n}\tilde{c}_k\Phi_0(H_k) \\ a_0^0\sum\limits_{k=0}^{n}\Phi_0(H_k)\Phi_1(H_k) + \cdots + a_n^0\sum\limits_{k=0}^{n}\Phi_n(H_k)\Phi_1(H_k) = \sum\limits_{k=0}^{n}\tilde{c}_k\Phi_1(H_k) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_0^0\sum\limits_{k=0}^{n}\Phi_0(H_k)\Phi_{n-1}(H_k) + \cdots + a_n^0\sum\limits_{k=0}^{n}\Phi_n(H_k)\Phi_{n-1}(H_k) = \sum\limits_{k=0}^{n}\tilde{c}_k\Phi_{n-1}(H_k) \\ a_0^0\sum\limits_{k=0}^{n}\Phi_0(H_k)\Phi_n(H_k) + \cdots + a_n^0\sum\limits_{k=0}^{n}\Phi_n(H_k)\Phi_n(H_k) = \sum\limits_{k=0}^{n}\tilde{c}_k\Phi_n(H_k) \end{cases} \qquad (2.19)$$

This set is expressed as $\mathbf{D\,a} = \mathbf{G}$ in a matrix form, where:

$$\boldsymbol{D} = \begin{bmatrix} \sum\limits_{k=0}^{n_1}\Phi_0(H_k)\Phi_0(H_k) & \dots & \sum\limits_{k=0}^{n_1}\Phi_n(H_k)\Phi_0(H_k) \\ \sum\limits_{k=0}^{n_1}\Phi_0(H_k)\Phi_1(H_k) & \dots & \sum\limits_{k=0}^{n_1}\Phi_n(H_k)\Phi_1(H_k) \\ \dots\dots\dots\dots\dots & \dots & \dots\dots\dots\dots\dots \\ \sum\limits_{k=0}^{n_1}\Phi_0(H_k)\Phi_{n-1}(H_k) & \dots & \sum\limits_{k=0}^{n_1}\Phi_n(H_k)\Phi_{n-1}(H_k) \\ \sum\limits_{k=0}^{n_1}\Phi_0(H_k)\Phi_n(H_k) & \dots & \sum\limits_{k=0}^{n_1}\Phi_n(H_k)\Phi_n(H_k) \end{bmatrix}, \qquad (2.20)$$

$$\boldsymbol{a} = \begin{bmatrix} a_0 \\ a_1 \\ \cdots \\ a_{n-1} \\ a_n \end{bmatrix}, \qquad \boldsymbol{G} = \begin{bmatrix} \sum_{k=0}^{n_1} \tilde{c}_k \Phi_0(H_k) \\ \sum_{k=0}^{n_1} \tilde{c}_k \Phi_1(H_k) \\ \cdots\cdots\cdots\cdots \\ \sum_{k=0}^{n_1} \tilde{c}_k \Phi_{n-1}(H_k) \\ \sum_{k=0}^{n_1} \tilde{c}_k \Phi_n(H_k) \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \ldots \\ g_{n-1} \\ g_n \end{bmatrix}.$$

There are two various functions, different from zero, given in a form of (2.5) or (2.6) in the range $[H_i, H_{i+1}]$, depending on the degree of the B-spline function B: Φ_i, Φ_{i+1}, hence $\sum_{k=0}^{n_1} \Phi_i(H_k^{'}) \Phi_j(H_k^{'}) = 0$ for $|i-j| \geq 2$, i.e. **D** is a 3-diagonal band matrix.

Introducing the following designations:

$$\rho_i = \sum_{k=0}^{n} \Phi_i(H_k) \Phi_i(H_k), \qquad i \in \overline{0, n},$$

$$\vartheta_i = \sum_{k=0}^{n} \Phi_i(H_k) \Phi_{i+1}(H_k), \qquad i \in \overline{0, n-1},$$

$$g_i = \sum_{k=0}^{n} \tilde{c}(H_k) \Phi_i(H_k), \qquad i \in \overline{0, n}$$

we end up with the following matrixes of the set (2.23):

$$\mathbf{D}=\begin{bmatrix} \rho_0 & \vartheta_0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vartheta_0 & \rho_1 & \vartheta_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \vartheta_1 & \rho_2 & \vartheta_2 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \vartheta_2 & \rho_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \rho_{n-3} & \vartheta_{n-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \vartheta_{n-2} & \rho_{n-2} & \vartheta_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \vartheta_{n-1} & \rho_{n-1} & \vartheta_n \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \vartheta_n & \rho_n \end{bmatrix},$$

Matrix $\mathbf{D}$ is a symmetric matrix, hence there is a unique distribution

$$\mathbf{D}=\mathbf{U}\mathbf{W}\mathbf{U}^T,$$

where matrixes $\mathbf{U}$ and $\mathbf{W}$ receive respectively forms of

$$\mathbf{U}=\begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ p_0 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & p_1 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & p_2 & 1 & \ldots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & p_{n-2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & p_{n-1} & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & p_n & 1 \end{bmatrix},$$

$$\mathbf{W}=\begin{bmatrix} w_0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & w_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & w_2 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_3 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & w_{n-3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & w_{n-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & w_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & w_n \end{bmatrix}.$$

Distribution of matrix **D** may be obtained with Gaussian elimination performed, based on the dependence:

$$\begin{aligned} w_i &= \rho_i - w_{i-1} + p_{i-1}^2 - w_{i-2}q_{i-2}^2 - w_{i-3}s_{i-3}^2 \,, \\ p_i &= \frac{1}{w_i}(\vartheta_i - w_{i-1}p_{i-1}q_{i-1} - w_{i-2}q_{i-2}s_{i-2})\,, \\ q_i &= \frac{1}{w_i}(\eta_i - w_{i-1}p_{i-1}s_{i-1})\,, \\ s_i &= \frac{\varsigma_i}{w_i}\,, \end{aligned}$$

where: $i\in\overline{-1,n+1}$, $p_{i-2}=0$, $q_i=0$ for $i\in\overline{-3,-2}$, $s_i=0$ for $i\in\overline{-4,-2}$, $\varsigma_i=0$ for $i\in\overline{n-1,n+1}$, $\eta_i=0$ for $i\in\overline{n,n+1}$, $\vartheta_{n+1}=0$.

3 TRAJECTORY OF ACOUSTIC RAY

3.1 DETERMINATION OF LOCATION OF ACOUSTIC WAVE REFLECTION POINT AT NO INTERFERENCE OF MOTION

At no interference of sounding vessel's motions, when there is no pitching or rolling, location of reflection point of acoustic wave, radiating vertically towards the sea bed, is a location of antenna position projection shifted with offset. Two cases of related positions of the hydrographic system's sensors may be distinguished: when the sensors are positioned along symmetry line of the sounding vessel and when they are additionally shifted towards one of the vessel's side. Positions of the sensors and dependencies for determination of location of the acoustic wave reflection point have been presented in Fig. 3.1 – for the case of no interferences of the sounding vessel.

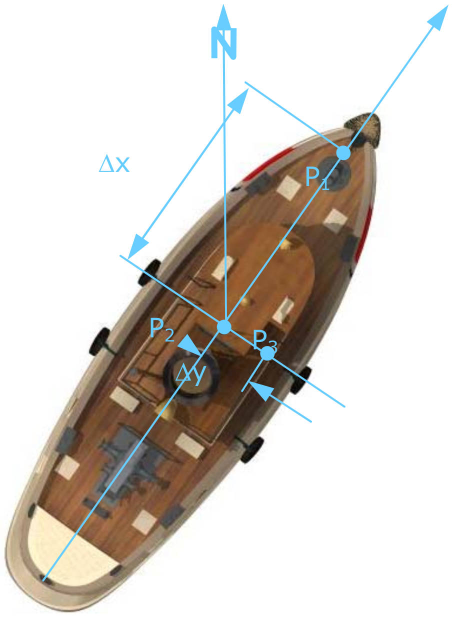

Fig. 3.1. Positions of hydrographic system's sensors

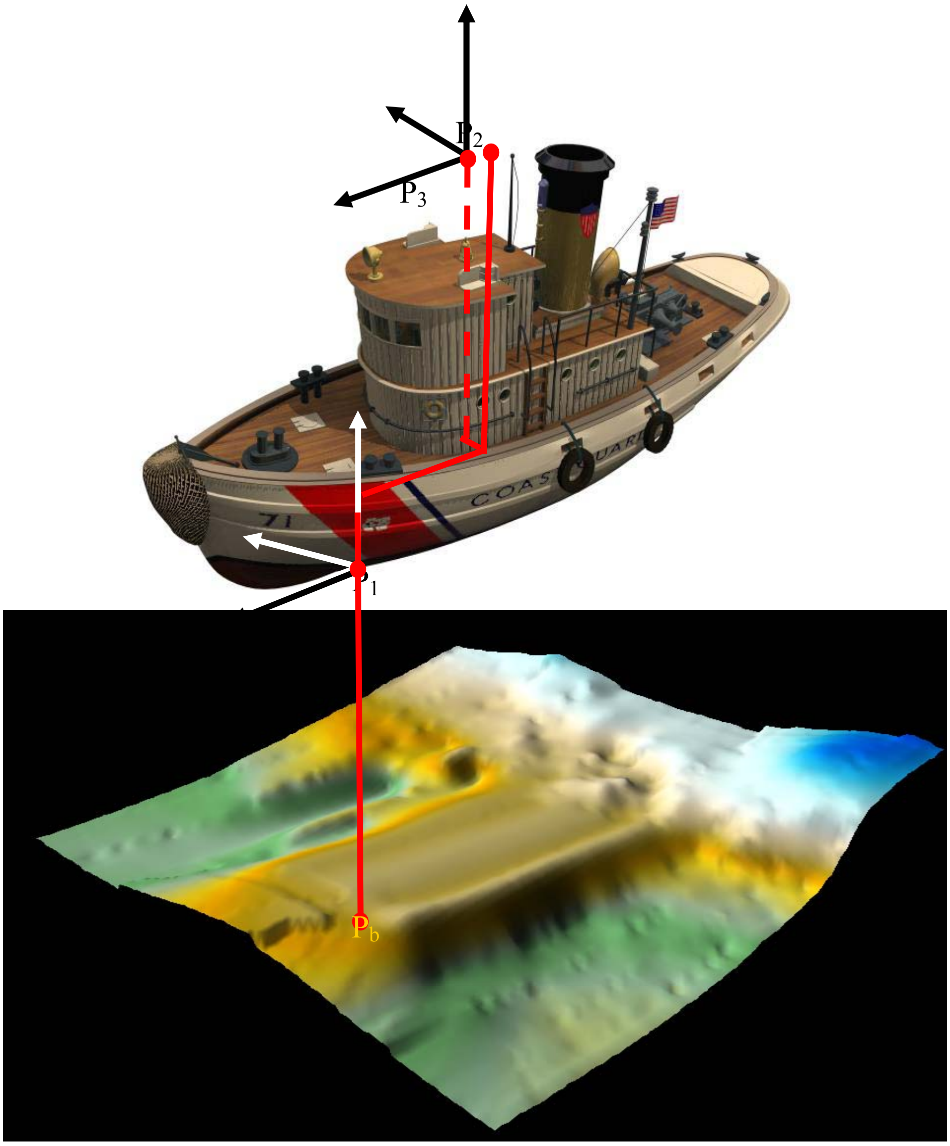

Fig. 3.2. Positions of hydrographic system's sensors

Position of the acoustic wave reflection point shall be determined for:

- sensors positioned along symmetry line of the sounding vessel,
- sensors not positioned along symmetry line of the sounding vessel.

3.1.1 POSITION OF ACOUSTIC WAVE REFLECTION POINT AT NO MOTION INTERFERENCES, FOR SENSORS POSITIONED ALONG SYMMETRY LINE OF SOUNDING VESSEL

Position of point P_b (point of acoustic wave reflection), in respect to geocentric ortocartesian reference system related to Earth and meant by vector $\overrightarrow{OP_b}$, is calculated with the following dependence:

$$\overrightarrow{O^E P_b} = \left(\mathbf{R}^{(L_2,\psi_2)}\right)^T \cdot \overrightarrow{P_2 P_b} + \overrightarrow{O^E P_2} =$$

$$= \begin{bmatrix} -\sin\psi_2\cos L_2 & -\sin L_2 & \cos\psi_2\cos L_2 \\ -\sin\psi_2\sin L_2 & \cos L_2 & \cos\psi_2\sin L_2 \\ \cos\psi_2 & 0 & \sin\psi_2 \end{bmatrix} \cdot \begin{bmatrix} \Delta x\cdot\cos KR \\ \Delta x\cdot\sin KR \\ l+r \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{a}{\sqrt{1-e^2\cdot\sin^2 B_2}} + H_2\right)\cdot\cos B_2\cdot\cos L_2 \\ \left(\frac{a}{\sqrt{1-e^2\cdot\sin^2 B_2}} + H_2\right)\cdot\cos B_2\cdot\sin L_2 \\ \left[\frac{a}{\sqrt{1-e^2\cdot\sin^2 B_2}}\cdot\left(1-e^2\right) + H_2\right]\cdot\sin B_2 \end{bmatrix}, \tag{3.1}$$

where:

$$\psi_2 = arc\,tg\left[\left(1-e^2\right)\cdot tg\,B_2\right] \tag{3.2}$$

is geocentric latitude of point P_2, r stands for a distance between transducer and sea bed measured by means of echo-sounder, l is a distance (altitude difference) between antenna of the positioning system and transducer of the echo-sounder, KR stands for a real course of the sounding vessel, $\mathbf{R}^{(L_2,\psi_2)}$ is a matrix of rotation of geocentric system's base in respect to base of horizontal topocentric system for which point P_2 is the beginning.

Matrix $\mathbf{R}^{(L_2,\psi_2)}$ describes rotation by the angles: L_2 of versors defining axes OX and OY (at the axis OZ not changing) and B_2 of versors defining axes OX and OZ (at the axis OY not changing) of the geocentric reference system's base and of the transformation following the rotation and consisting in mutual position exchange of the versors defining new positions of the axes OX and OZ.

3.1.2 POSITION OF ACOUSTIC WAVE REFLECTION POINT AT NO MOTION INTERFERENCES, FOR SENSORS NOT POSITIONED ALONG SYMMETRY LINE OF SOUNDING VESSEL

Position of point P_b (point of acoustic wave reflection), in respect to geocentric ortocartesian reference system related to Earth and meant by vector $\overrightarrow{O^E P_b}$, is calculated with the following dependence:

$$\overrightarrow{O^E P_b} = \left(\mathbf{R}^{(L_3,\psi_3)}\right)^T \cdot \overrightarrow{P_3 P_b} + \overrightarrow{O^E P_3}, \tag{3.3}$$

where:

- for $\Delta x \geq \Delta y$

$$\overrightarrow{P_3 P_b} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \cdot \cos\left(KR - arctg\dfrac{\Delta y}{\Delta x}\right) \\ \sqrt{\Delta x^2 + \Delta y^2} \cdot \sin\left(KR - arctg\dfrac{\Delta y}{\Delta x}\right) \\ l + r \\ \end{bmatrix}, \tag{3.4}$$

- for $\Delta y > \Delta x$

$$\overrightarrow{P_3 P_b} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \cdot \cos\left(KR - arcctg\dfrac{\Delta y}{\Delta x}\right) \\ \sqrt{\Delta x^2 + \Delta y^2} \cdot \sin\left(KR - arcctg\dfrac{\Delta y}{\Delta x}\right) \\ l + r \\ \end{bmatrix}. \tag{3.5}$$

3.2 DETERMINATION OF POSITION OF ACOUSTIC WAVE REFLECTION POINT WITH A TWO-POINT METHOD APPLIED

Let's have the positioning system, making - together with an echo-sounder – the hydrographic system, extra equipped with one receiving antenna enabling determination of positions of two points P_2, P_3 in respect to the geocentric reference system $\Im^E = \{O^E, (e_x^E, e_y^E, e_z^E)\}$ connected with Earth and these points – determining a vector with a point of origin $\boldsymbol{n}$ described by the dependence (1.4), and then, in the next step – a vessel free vector $\boldsymbol{p}_1$ of coordinates

$$\begin{bmatrix} \boldsymbol{p}_1^X \\ \boldsymbol{p}_1^Y \\ \boldsymbol{p}_1^Z \end{bmatrix} = \frac{\boldsymbol{n}}{|\boldsymbol{n}|} = \begin{bmatrix} \dfrac{\boldsymbol{n}_X}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \\ \dfrac{\boldsymbol{n}_Y}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \\ \dfrac{\boldsymbol{n}_Z}{\sqrt{\boldsymbol{n}_X^2 + \boldsymbol{n}_Y^2 + \boldsymbol{n}_Z^2}} \end{bmatrix} \tag{3.6}$$

in the base (e_x^E, e_y^E, e_z^E) of this system.

This vector is a direction vector of a straight line crossing point P_1 which – in special case - shall be a mounting place of the echo-sounder's transducer.

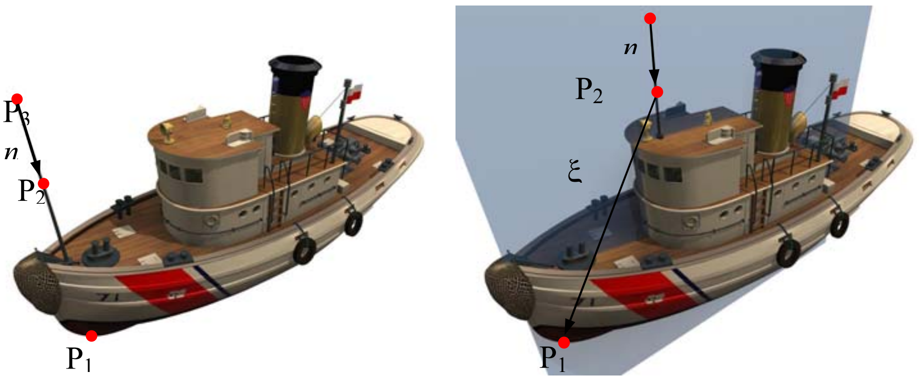

Fig. 3.3. Positions of points P_2, P_3 *of a two-antenna system*

That is why determination of position of the measuring point P_b (defined with the originated vector $\overrightarrow{O^E P_b}$ in respect to $\Im^E = \{O^E, (e_x^E, e_y^E, e_z^E)\}$), from which the acoustic wave reflects, can be executed based on known coordinates of direction vector of the straight line crossing the points P_1 and P_b, and position coordinates of one of the antennas, e.g. P_2, according to the dependence:

$$\overrightarrow{P_2 P_b} = \begin{bmatrix} (l+r) \cdot \boldsymbol{p}_1^X \\ (l+r) \cdot \boldsymbol{p}_1^Y \\ (l+r) \cdot \boldsymbol{p}_1^Z \end{bmatrix} \tag{3.7}$$

$$\overrightarrow{O^E P_b} = \overrightarrow{P_2 P_b} + \overrightarrow{O^E P_2}\,, \tag{3.8}$$

where: $P_b = \begin{bmatrix} X^b & Y^b & Z^b \end{bmatrix}^T$ and $\overrightarrow{O^E P_2}$ are described with the dependence (1.5) for $i = 2$.

In case the two-antenna system is located in another place (the right sided Fig. 3.3) but direction of the straight lines crossing the two antennas and of the acoustic ray are parallel, offset vector ξ between antenna and transducer should be defined.

3.3 ACOUSTIC RAY

Field of acoustic wave emitted in water environment by the echo-sounder's transducer and its changes in time-domain may be described mathematically with solving wave equation [Klusek 1990, Brekhovskikh and Lysanov 2001], which in hydro-acoustics is a basis for descriptions of small amplitude waves. This solution does exist, however – only for simple or idealized cases. In general, a point source of the acoustic wave and horizontal stratification of water – in which the sound speed $c(x, y, H) = c(H)$ - are assumed.

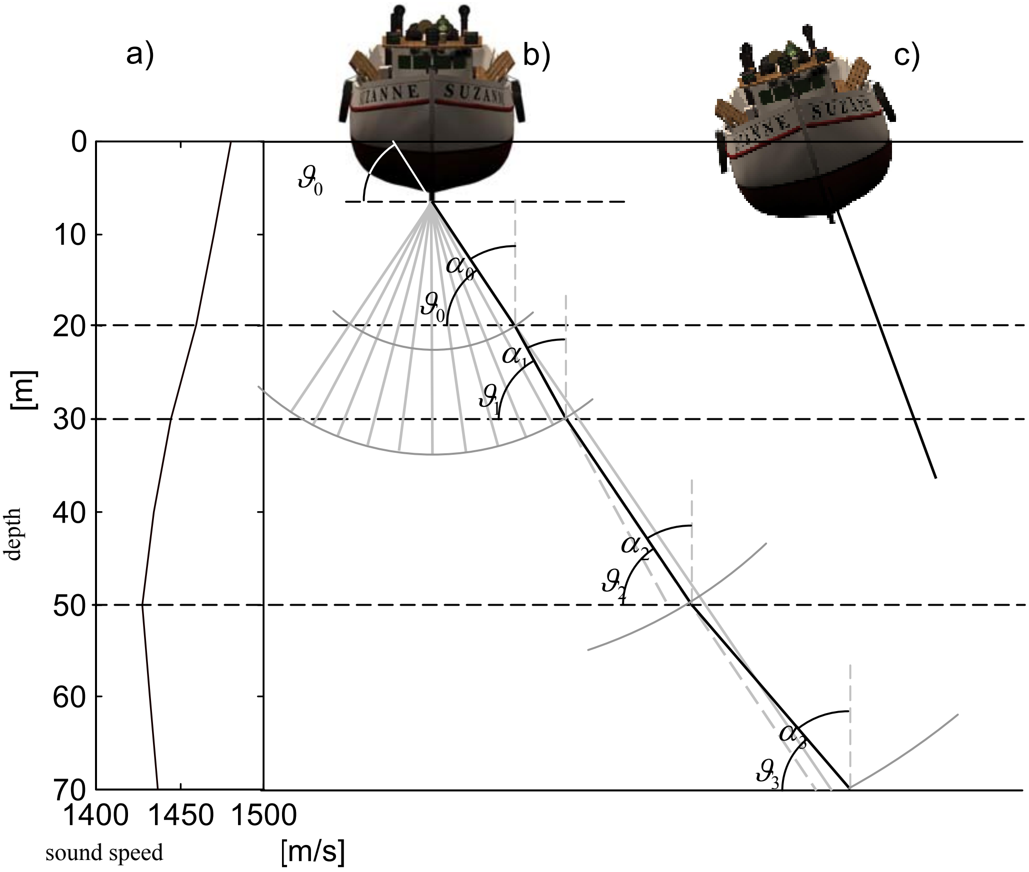

Fig. 3.4. Geometric sketch for ray theory of sound propagation in water

Eiconal equation, describing trajectory of the acoustic ray, is a partial linear differential equation which solutions provide tri-dimensional surfaces in a space

$W(x,y,H) = const.$ In respect to a field of acoustic interferences, these are waves' surfaces, i.e. surfaces of identical phase of the wave vibrations, and the wave front surface among them. Vector *dr,* perpendicular to the wave front, defines direction of the propagating wave front limited to the section it is surrounded by, and its next positions in time-domain determine a path of this section – the trajectory of the acoustic wave ray (Fig. 3.4).

Let's assume that c_0 is a constant sound speed in the environment of its source at the depth of H_0, and the ray under consideration is sent from this source with an angle α in respect to horizontal plane.

On the ray path, the sound speed may be represented by the values $c(H_1), c(H_2), c(H_3), \ldots, c(H)$ if more distant from the source, and the angle by which this ray crosses the next horizontal planes is of values $\vartheta_1, \vartheta_2, \vartheta_3, \ldots, \vartheta$.

Relation between directions of rays of incident and refracted waves depends on the sound speed values in the neighbouring environments, according to the refraction law [Klusek, 1990 - chapter 1, Brekhovskikh and Lysanov, 2001 - chapter 2].

Fig. 3.4 is a geometric illustration of the above-said relations – it presents vertical distribution of the sound speed $c(H)$ (Fig. 3.4.a) and its corresponding trajectory of the sound ray emitted from the source by an angle α in the plane xH (for one of rays of a multi-beam echo-sounder – Fig. 3.4.b, for a single-beam echo-sounder's ray in case of heel – Fig. 3.4.c). Following the Figure, reflected waves are not present in the environment horizontally stratified with constant sound distribution $c(H)$ of gentle run, and the refracted ray becomes a constant prolongation of the incident ray, making together one ray of changeable direction.

3.4 DISTRIBUTION OF ACOUSTIC RAYS AT REFRACTION OCCURRENCE

Refraction phenomenon (diffraction of a wave) occurs for acoustic rays emitted non-vertically. Deviation from rectilinearity of trajectory occurs for:

- single-beam echo-sounder at pitching and rolling,
- multi-beam echo-sounder in which slant emission of acoustic rays is a heart of operation,
- dipping sonar, emitting horizontal acoustic waves.

Paths of rectilinear and diffracted acoustic rays in the function of outlet angle, in case of occurrence of positive and negative refraction, have been presented in Fig. 3.5 and 3.7.

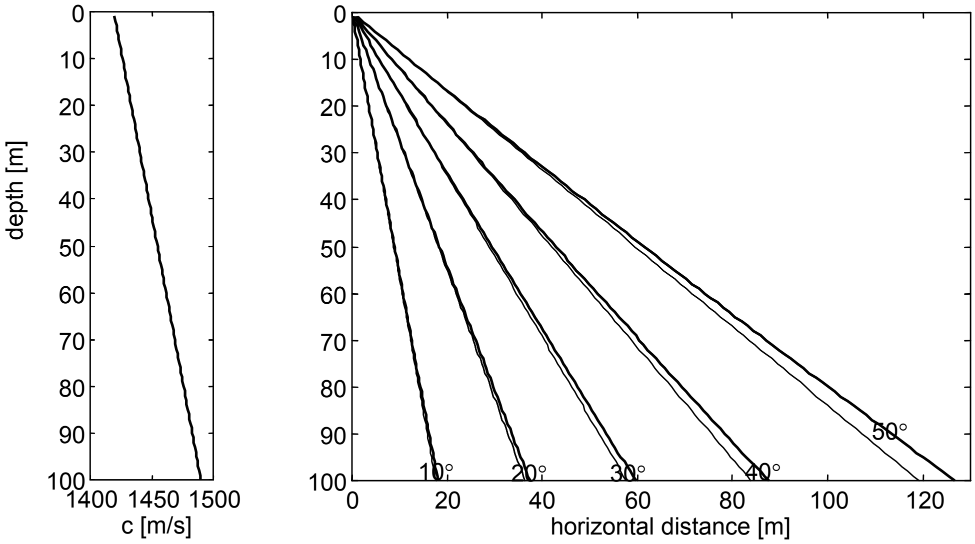

Fig. 3.5. Propagation of acoustic waves at positive refraction

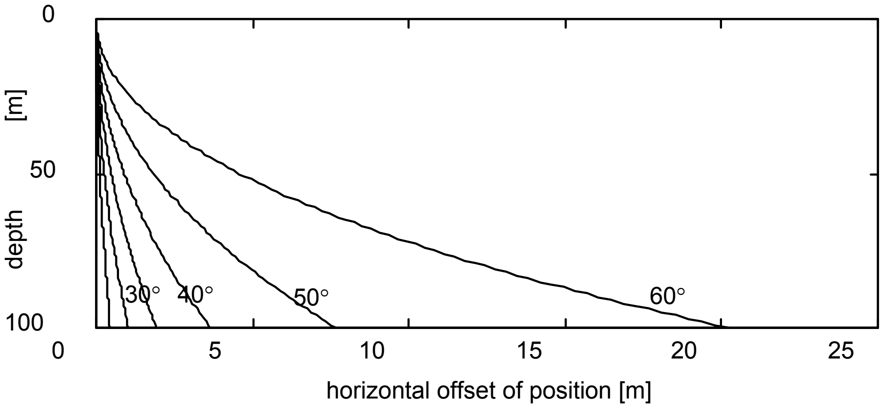

Fig. 3.6. Horizontal offset of acoustic wave reflection point at positive refraction

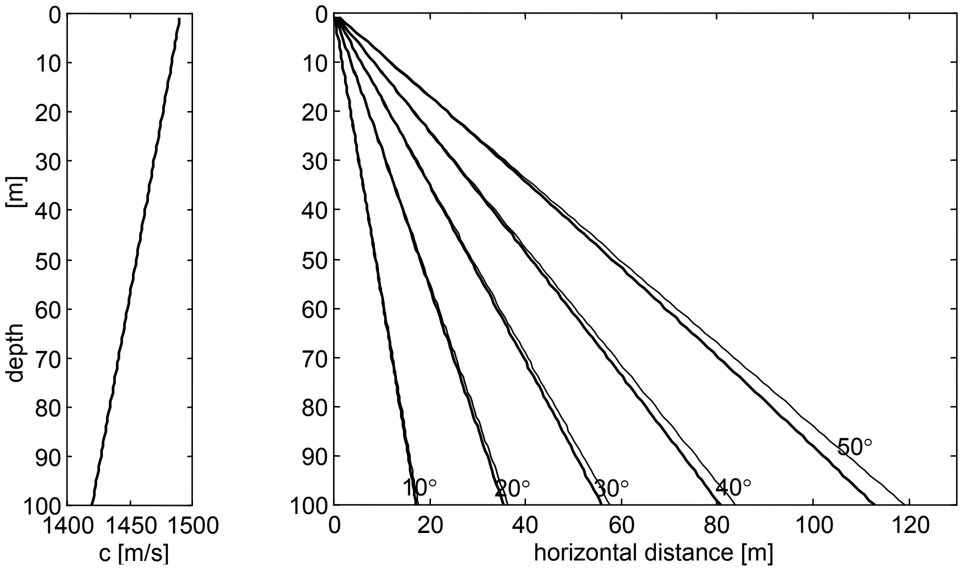

Fig. 3.7. Propagation of acoustic waves at negative refraction

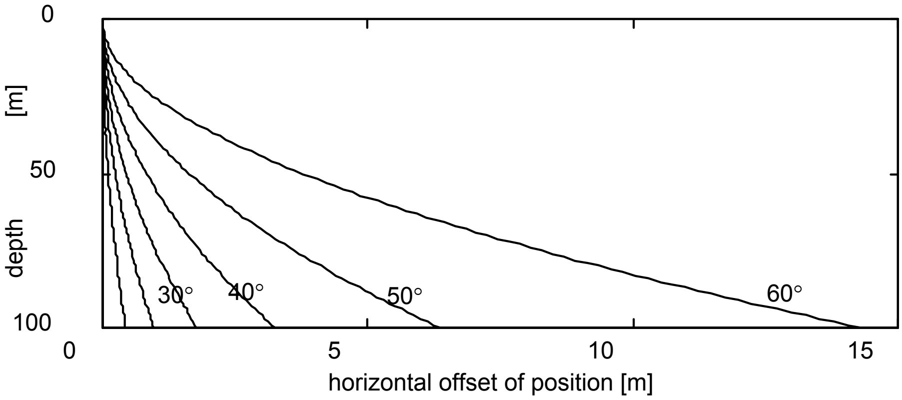

Fig. 3.8. Horizontal offset of acoustic wave reflection point at negative refraction

Thermal gradient is not constant in real conditions – it changes within quite a big range. It depends on air temperature and undulation in a surface water layer. This gradient rises along with the temperature growth. Sea undulation results in mixing of water surface layers and their temperature equalizes. This means that the temperature gradient equals zero. In such a case, a layer of constant water temperature forms near the surface known as an isothermal layer. The temperature drops along with rise of depth below this layer.

Path of rectilinear and diffracted acoustic rays in outlet angle function in case of occurrence of changeable refraction has been presented in Fig. 3.9.

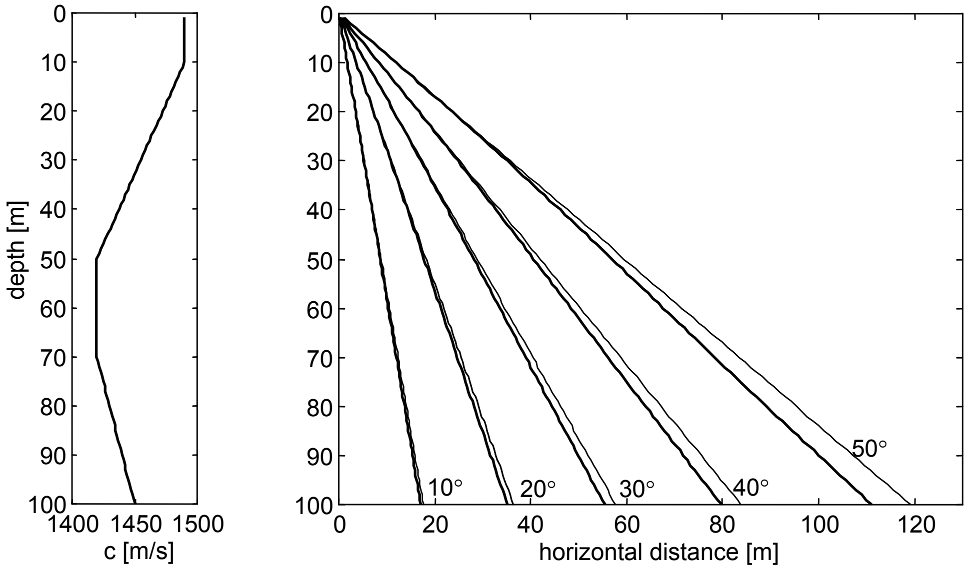

Fig. 3.9. Propagation of acoustic waves in conditions of changeable refraction

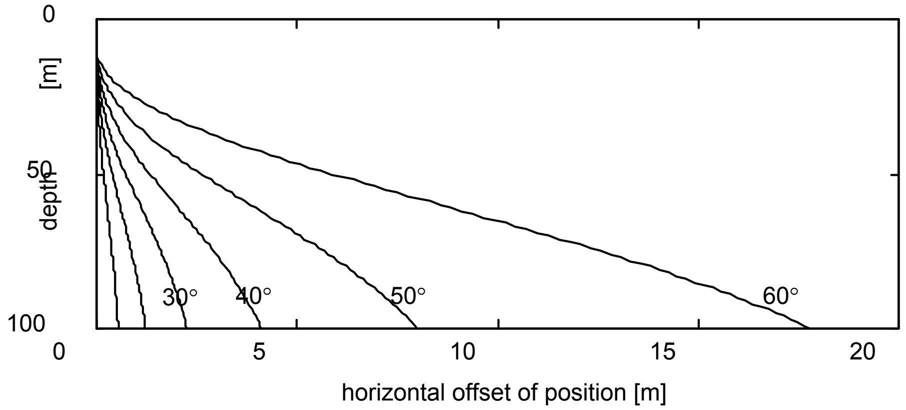

Fig. 3.10. Horizontal offset of acoustic wave reflection point in conditions of changeable refraction

3.5 HORIZONTAL DISTANCE OF REFLECTION POINT OF ACOUSTIC WAVE

Integration of acoustic ray's sections *dr* is required to determine its path – based on trigonometric interdependencies given in Fig. 3.4.

An assumption that propagation of acoustic wave is rectilinear enables to determine reflection points of the acoustic wave for every ray emitted in lower hemisphere of the sounding vessel. In fact, occurrence of refraction may result in diffraction of the acoustic wave and curving of its path towards a surface of no bed to reflect against. The ray takes horizontal direction in a dead point and this means that $\frac{dH}{dr}=0$; $\cos\vartheta=1$. Because of Snell law [Klusek, 1990 - chapter 1, Brekhovskikh and Lysanov, 2001 - chapter 2], this condition is met at the depth H, where $\frac{\cos\vartheta_0}{(H_0)}=\frac{1}{c(H)}$, that is – the ray's dead point at the depth H_z is described by the condition:

$$\frac{c(H_0)}{c(H)}=\cos\vartheta_0 . \tag{3.9}$$

Constant direction changes of the acoustic wave's rays in such a way that they always curve towards lower speed $c(H_1)$ is a characteristic feature of their trajectory.

A model in which the propagation environment is divided into layers of linear sound speed changes is a more accurate model. In this case, not the sound speed, but its gradient is subject to discontinuity at the limits. Acoustic ray is not rectilinear – it is a circular arc. Curvature of the ray is not continuous when it crosses the layers' limits.

Let's assume that $H_i - H_{i-1}$ $(i = 1,2,3...)$ stands for a layer depth of constant gradient of the sound speed in water, c_i and ϑ_i stand respectively for the sound speed and outlet angle of the acoustic ray in a lower limit of every layer. They are bound over with the Snell law [Klusek, 1990 - chapter 1, Brekhovskikh and Lysanov, 2001 - chapter 2], according to which a constant depends on the outlet angle ϑ_1 of the acoustic ray leaving a source (transducer of echo-sounder). Gradient of the sound speed in water, in this i-layer, is:

$$\frac{c_i - c_{i-1}}{H_i - H_{i-1}}. \tag{3.10}$$

With $\vartheta_1 = \vartheta_{i-1}$, application of the dependence $\vartheta = \frac{\cos\vartheta_1}{n(H)}$ where $n(H) = \frac{c_{i-1}}{c(H)}$, and performance of integration in respect to ϑ, a result is as the following:

$$s = \sum_i \frac{1}{\cos\vartheta_{i-1}} \frac{1}{a_i} \int_{\vartheta_{i-1}}^{\vartheta_i} \cos\vartheta d\vartheta = \sum_i \frac{1}{\cos\vartheta_{i-1}} \frac{1}{a_i} \left(\sin\vartheta_i - \sin\vartheta_{i-1}\right), \tag{3.11}$$

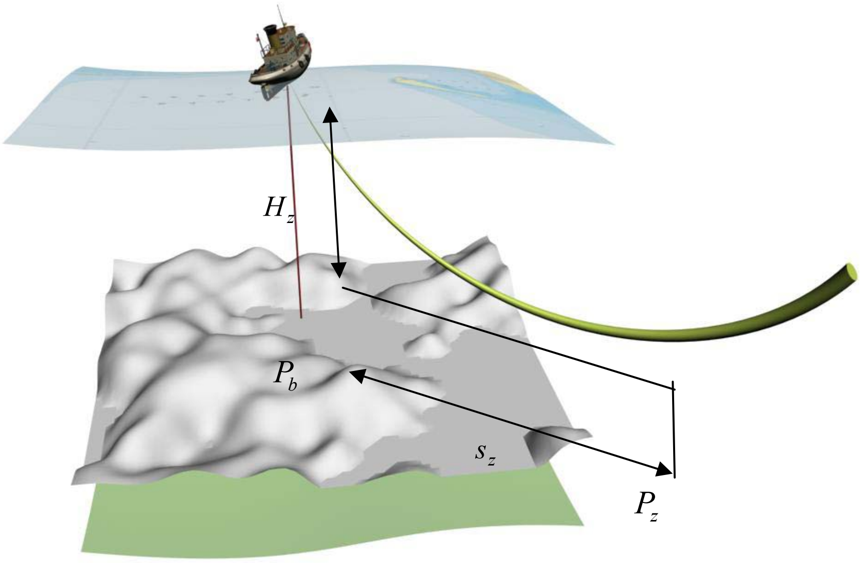

Fig. 3.11. Nonlinear trajectory of acoustic ray

3.6 RUN TIME OF ACOUSTIC WAVE

Run time of disturbance (of wave front, of impulse) equals:

$$t - t_0 = \int_{t_0}^{t} dt = \int_{h_0}^{h} \frac{dH}{c(H)\sqrt{1 - a_r^2 c^2(H)}} . \tag{3.12}$$

It comes out from the definition of sound speed $\frac{dr}{dt} = c$ that time element dt, in which the ray (wave front) makes a path section dr between the depths H_0 and H_1 can be expressed as the following:

$$dt = \frac{dr}{c(H)} = \frac{dH}{c(H) \cdot \sin \vartheta} . \tag{3.13}$$

Applying the dependence $c(H) = c_0(1 + aH)$ and expressing dH with the dependence of $d\vartheta$, run time of the acoustic wave impulse is:

$$t = \frac{1}{ac_0} \int_{\vartheta_0}^{\vartheta_1} \frac{d\vartheta}{\cos\vartheta}, \tag{3.14}$$

where $\vartheta_0 \equiv \vartheta(H_0)$ and $\vartheta_1 \equiv \vartheta(H_1)$. Solution of the above integral is as the following:

$$t = \frac{1}{2ac_0}\left(\ln\frac{1+\sin\vartheta_0}{1-\sin\vartheta_0} - \ln\frac{1+\sin\vartheta_1}{1-\sin\vartheta_1} \right). \tag{3.15}$$

With $\vartheta_1 = 0$, we result in the run time of the acoustic wave impulse on a path from the echo-sounder's transducer to the dead point when the acoustic wave returns towards water surface (given as $\frac{\Delta t}{2}$):

$$\frac{\Delta t}{2} = \frac{1}{2ac_0} \ln\frac{1+\sin\vartheta_0}{1-\sin\vartheta_0}, \tag{3.16}$$

i.e.

$$\Delta t \approx \frac{2\vartheta_0}{ac_0}\left(1 + \frac{1}{6}\vartheta_0^2 + \frac{1}{24}\vartheta_0^4 + \ldots \right). \tag{3.17}$$

3.7 RAY CURVATURE OF ACOUSTIC RAY

Alternatively, the Snell law [Klusek, 1990 - chapter 1, Brekhovskikh and Lysanov, 2001 - chapter 2] may be given in the following form:

$$c_0 \cos\vartheta = c\cos\vartheta_0 , \tag{3.18}$$

where ϑ_0 and $c_0 = c(H_0)$ are respectively outlet angle of the acoustic ray leaving transducer and acoustic wave speed at the depth of the transducer location, while ϑ and $c = c(H)$ are respectively outlet angle of the acoustic ray at the layers' limits, where change of value of the sound speed gradient in water takes place, and the acoustic wave speed at this dept. Differentiation of this formula in respect to the depth H , we result with:

$$-c_0 \sin\vartheta \frac{d\vartheta}{dH} = \cos\vartheta_0 \frac{dc}{dH} \text{ or } \frac{d\vartheta}{dr} = -q\frac{dc}{dH} , \tag{3.19}$$

where $dr = \dfrac{dH}{\sin\vartheta}$ is an elementary section of the acoustic ray path and

$$q = \frac{\cos\vartheta_0}{c_0} \tag{3.20}$$

is a constant parameter of every ray. It may be noticed that a derivative $\dfrac{d\vartheta}{dr}$ depends on the sound speed gradient at a given depth. Character "minus" means that the acoustic ray's outlet angle decreases along with growth of the sound speed in water, and vice versa.

Ray curvature of the acoustic ray $\Re$ equals

$$\Re^{-1} = \frac{d\vartheta}{dr} = q\frac{dc}{dH} . \tag{3.21}$$

When the gradient $\dfrac{dc}{dH} = const$, $\Re = const$, what means that the acoustic ray is an arc of circle of radius $\Re$ if the sound speed gradient is constant.

Applying (3.20) into (3.21), we receive

$$\Re = \frac{1}{a \cos \vartheta_0}\ , \tag{3.22}$$

where

$$a = \frac{1}{c_0} \frac{dc}{dH} \tag{3.23}$$

is a relative sound speed gradient. Bigger value a and smaller value ϑ_0 result in occurrence of stronger refraction phenomenon. When the outlet angle $\vartheta_0 = \frac{\pi}{2}$ $(\alpha = 0)$ – a case of vertical radiation of acoustic wave, $\Re = \infty$ and the refraction phenomenon does not occur.

3.8 USAGE OF GLOBAL MODEL OF EMG96 GEOID IN HYDROGRAPHIC SURVEYS AT NO MOVEMENT DISTURBANCES

Measurement of depth, as an element of hydrographic surveys, is executed according to a local co-ordinate system connected with sounding vessel. Submersion of echo-sounder's transducer taken into consideration allows determining distance from water surface to sea bed. Water level is changeable, especially in tide-water regions, even during performance of the measurements. This fact resulted in a necessity of elaborating many altitude reference systems. MSL (Mean Sea Level) system is commonly used in atidal water areas. Changes of water level ranges require registration and their incorporation into elaborations of the measurements' results.

Determination of depth on a reference ellipsoid, with a use of a global geoid model, may be an alternative for application of many altitude reference systems. Such a solution shall enable usage of positioning systems for depth measurements and compensation of undulation.

Concept on how to use the global EMG96 geoid model in hydrographic surveys may be introduced in the following stages:

- determination of the positioning system's ellipsoidal altitude and rigid connection between the positioning system and echo-sounder's transducer, allows defining co-ordinates, especially of the echo-sounder's transducer ellipsoidal altitude,

- knowledge on the local vertical sound speed distribution in water enables to determine trajectory of acoustic ray emitted by the echo-sounder's transducer and co-ordinates of its reflection from the sea bed, in particular of the ellipsoidal altitude,
- based on knowledge of local distance between the geoid and ellipsoid, there is a possibility of determining the depth in a given point.

Let's assume that we know co-ordinates of the point P_2, in a specific case – co-ordinates of point of a two-antenna system and base e_x^P, e_y^P, e_z^P of the system $\Im^P$.

Based on position of the echo-sounder's transducer in the two-antenna positioning system (Fig. 3.3):

$$\overrightarrow{P_2P_1} = \begin{bmatrix} l \cdot e_x^P \\ l \cdot e_y^P \\ l \cdot e_z^P \end{bmatrix} \tag{3.24}$$

and knowledge about distance between the geoid and ellipsoid in the place of bathymetric measurements' performance, given for water region of the Bay of Gdansk in Fig. 3.12, for the Lake Lackie – in Fig. 3.13 and about position of the positioning system's antenna, it is possible to compensate movement disturbances, especially due to yawing.

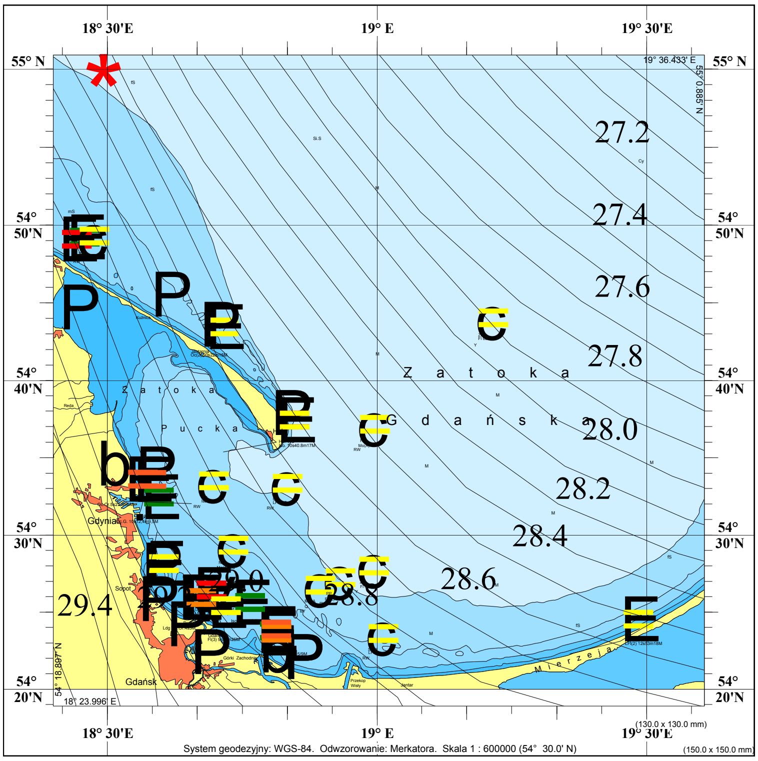

Fig. 3.12. Distribution of distance between geoid and ellipsoid for the Bay of Gdansk

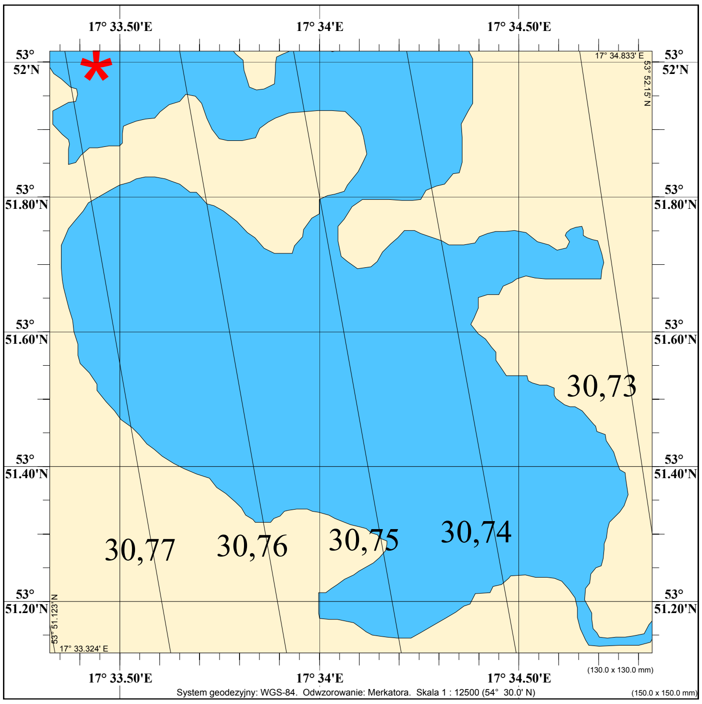

Fig. 3.13. Distribution of distance between geoid and ellipsoid for the Lake Lackie

Conventional point P_b (Fig. 3.14) is a point of acoustic wave reflection at no occurrence of movement disturbances. An ellipsoid of the WGS-84 reference system is positioned below the geoid in the Polish Sea Regions, hence ellipsoidal altitude of this point may result from the dependence $h_b = N - H_b$.

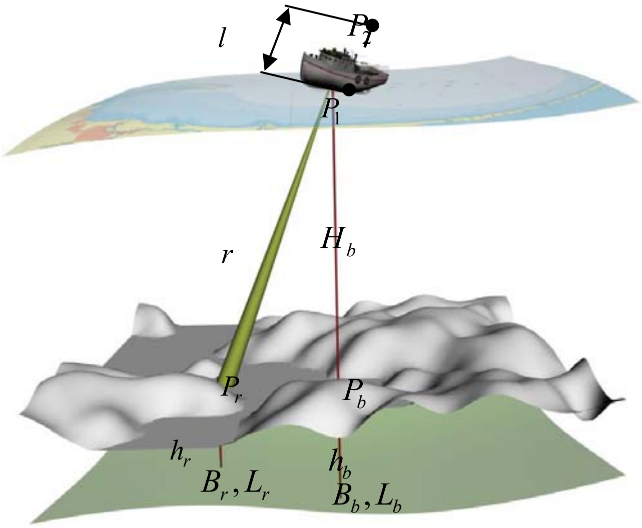

Fig. 3.14. Acoustic wave reflection point and its ellipsoidal altitude

3.9 DETERMINATION OF CO-ORDINATES OF ACOUSTIC WAVE REFLECTION POINT

The echo-sounder's transducer and positioning system's antenna are rigidly connected with a hull of the sounding vessel. Determination of position of the acoustic wave reflection point at no disturbances has been described in chapter 3.1. for sensors placed and not placed along the symmetry line of the sounding vessel. Co-ordinates of the acoustic wave reflection point, at no movement disturbance and with an assumption that it propagates linearly, is described with the dependence (3.8) if applying methods of determining spatial orientation of the sounding vessel.

In case of acoustic wave emitted diagonally, e.g. as a result of disturbances of the sounding vessel's movement, its trajectory shall be curvilinear (chapter 3). Determination of the acoustic wave reflection point shall be possible thanks to:

- knowledge about position of the transducer, as a result of performance of the measuring system's calibration prior to execution of bathymetric soundings,

- knowledge about outlet angle of the acoustic wave leaving the transducer, with a method of defining sounding vessel's spatial orientation applied, including the echo-sounder's transducer,
- trajectory of the acoustic wave, based on a spacial-time model of the sound speed in water and local vertical sound speed distribution determined based on the model.

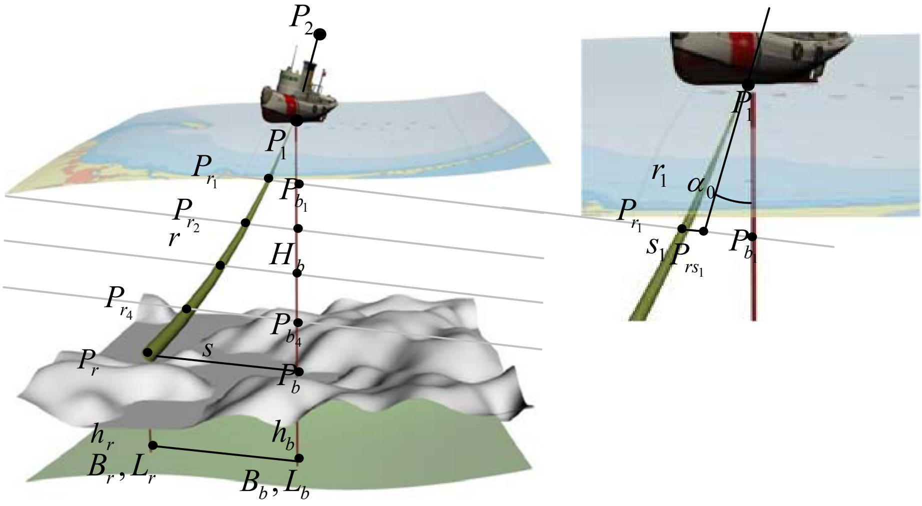

Fig. 3.15. Reflection point of acoustic wave propagating nonlinearly

Based on knowledge about spatial orientation of the sounding vessel, an estimated transformation of point's P_b position – by vector s described with the dependence (3.11) – should be performed in order to determine co-ordinates of reflection point P_r of the acoustic wave propagating nonlinearly at occurrence of movement disturbances:

$$\overrightarrow{O^P P_r} = \overrightarrow{P_2 P_r} + \overrightarrow{O^P P_2} = \overrightarrow{P_2 P_b} + s + \overrightarrow{O^P P_2} . \tag{3.25}$$

In case of no movement disturbances (linear propagation of the acoustic wave), co-ordinates of its reflection point $P_b = \begin{bmatrix} B_b & L_b & H_b \end{bmatrix}^T = \begin{bmatrix} B_2 & L_2 & l + r \end{bmatrix}^T$

are horizontal co-ordinates of point $P_2 = \left[B_2 \quad L_2 \quad H_2\right]^T$ represented by an ortometric altitude equal the measured depth.

Co-ordinates of point positioned at the limits of layers of the sound speed constant gradient in water P_{r_1} may be determined based on the dependence (3.7):

$$\overrightarrow{O^P P_{r_1}} = \overrightarrow{P_2 P_{r_1}} + \overrightarrow{O^P P_2} = \overrightarrow{P_2 P_{rs_1}} + s_1 + \overrightarrow{O^P P_2}\,, \tag{3.26}$$

that is

$$\overrightarrow{O^P P_{r_1}} = \begin{bmatrix} (l + r_1)\cdot \boldsymbol{p}_1^X \\ (l + r_1)\cdot \boldsymbol{p}_1^Y \\ (l + r_1)\cdot \boldsymbol{p}_1^Z \end{bmatrix}_{P_3 P_2} + \frac{1}{\cos\vartheta_0}\frac{1}{a_1}\left(\sin\vartheta_1 - \sin\vartheta_0\right) + \overrightarrow{O^P P_2}\,, \tag{3.27}$$

where $[\ ]_{P_3 P_2}$ stands for the point's co-ordinates determined based on knowledge about co-ordinates of direction vector of the straight line crossing the given point and points P_3 and P_2.

Analogically, co-ordinates of point positioned at the limits of next layers of the sound speed constant gradient in water P_{r_1} may be determined based on the above dependence, if co-ordinates of direction vector of the straight line crossing the points P_1 and P_{r_1} are known:

$$\overrightarrow{O^P P_{r_2}} = \overrightarrow{P_1 P_{r_2}} + \overrightarrow{O^P P_1} = \overrightarrow{P_1 P_{rs_2}} + s_2 + \overrightarrow{O^P P_1}\,, \tag{3.28}$$

that is

$$\overrightarrow{O^P P_{r_2}} = \begin{bmatrix} (r_1)\cdot \boldsymbol{p}_1^X \\ (r_1)\cdot \boldsymbol{p}_1^Y \\ (r_1)\cdot \boldsymbol{p}_1^Z \end{bmatrix}_{P_1 P_{r_1}} + \frac{1}{\cos\vartheta_1}\frac{1}{a_2}\left(\sin\vartheta_2 - \sin\vartheta_1\right) + \overrightarrow{O^P P_1}\,. \tag{3.29}$$

Assuming that the dependence (3.26) may be expressed with

$$\overrightarrow{O^P P_{r_1}} = \overrightarrow{P_1 P_{r_1}} + \overrightarrow{O^P P_1} = \overrightarrow{P_1 P_{rs_1}} + s_1 + \overrightarrow{O^P P_1}\,, \tag{3.30}$$

co-ordinates of reflection place of the acoustic wave, propagating nonlinearly at occurrence of the refraction phenomenon, may be calculated based on the dependence:

$$\overrightarrow{O^P P_r} = \overrightarrow{P_1 P_{r_1}} + \overrightarrow{P_{r_1} P_{r_2}} + \overrightarrow{P_{r_i} P_r} + \overrightarrow{O^P P_1}\,. \tag{3.31}$$

4 METHOD OF GEODESIC BATHYMETRIC SOUNDINGS

4.1 PREPARATION OF SOUNDING PERFORMANCE

Measurements of depth (sounding) result in creation of picture of the sea bed's height. However, some of its details differ from a typical aerial picture (levelling). First to say, all altitudes under measurements are of negative values; secondly, all points are concealed by a layer of water. And finally – thirdly – position of an active water surface constantly changing its placement should be taken into consideration during performance of the sounding. This special feature of the depth measurements requires all the works to be accompanied by an obligatory observation of the sea water level, leading to determination of the so-called zero depth, that is – the water level in a given region determined with a specified method applied, e.g. according to data taken from coastal water-level indicators from open sea mareographs.

The following aims are made prior to performance of the sounding:

1. Definition of general picture of the sea bed's profile;
2. Determination of positions of under-water hazards (obstacles);
3. Determination of positions of water lanes, manoeuvring regions, anchor grounds and roadsteads;
4. Possibility of determining positions according to characteristic features of the sea bed's profile and kind of ground;
5. Collections of materials indispensable for execution of dredging and underwater constructions.

These days, the soundings and elaboration of the sounding's materials may be executed in accordance with the below stages:

1. Elaboration of technical documentation for performance of the soundings;
2. Execution of the sounding and determination of the measured depths' positions;
3. Definition of the sea ground character;
4. Execution of indispensable observations and hydrologic surveys within region of the sounding performance;
5. Elaboration of the sounding and execution of reporting boards and sounding report.

Such stages of the works as:

1. technical preparation of the region of works;
2. placement of stations (antennas) of radio-navigation systems in order to protect performance of the sounding;
3. placement of water level indicators and observation of the water level in order to determine an average sea water level and the depth's zero in respect to which the depth is measured,

may be omitted and they result from new measuring technologies. In case a local positioning system (GPS-RTK) is used, preparation of the sounding may additionally require determination of the point's co-ordinates and placement of a reference station. In general, the stage of the bathymetric soundings' preparation shall include:

1. Installation of measurement devices on the sounding vessel for a mobile variant,
2. Determination of the echo-sounder's transducers positions with respect to the positioning system's antenna and to the respective positions of antennas for a tri-antenna (two-antenna) compensation system of movement disturbances,
3. Determination of reference altitude (ellipsoidal altitude of the positioning system's antenna referred to an average level of the Baltic Sea water in the Gulf of Finland, defined for a mareograph located in Kronstadt near Saint Petersburg).

4.2 CALIBRATION OF MEASURING SYSTEM

Calibration of the sounding system consists in determination of reciprocal positions of:

- the echo-sounder's transducers with respect to the beginning of the co-ordinate system connected with a multi-sensor positioning system, with respect to the point P_2 at best,
- the positioning system's antennas,
- orthometric altitude of the point P_2, making the beginning of the co-ordinate system connected with the multi-sensor positioning system.

Prototype of a mobile tri-antenna system and installation of a two-frequency system of transducers are presented in Fig. 4.1.

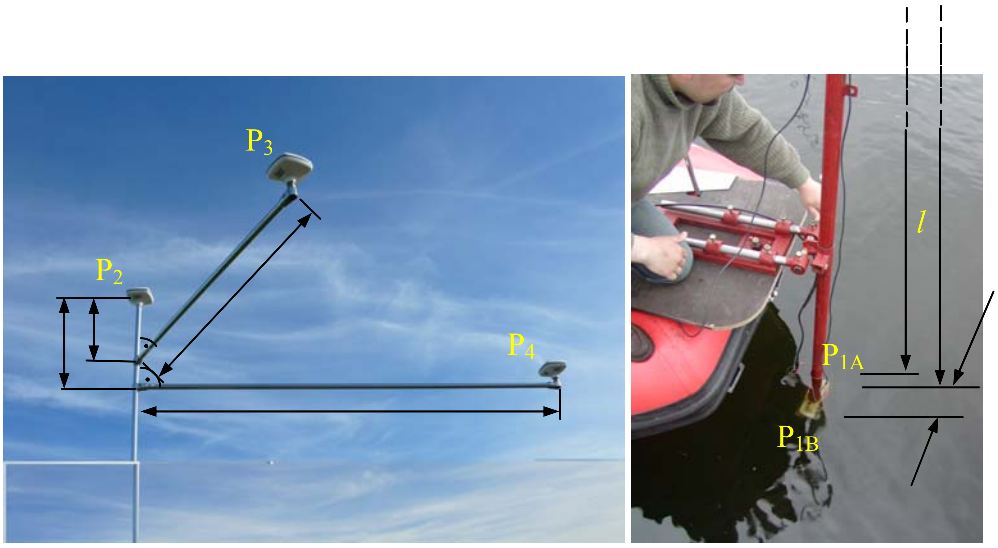

Fig. 4.1. Prototype of mobile tri-antenna system and installation of two-frequency system of transducers

4.3 REALISATION OF BATHYMETRIC SURVEYS

4.3.1 MEASUREMENT OF SOUND SPEED IN WATER

In order to perform modelling of spatial-time distributions of the sound speed in water, measurements of the speeds may be executed:

- by means of one device: CTD or ultrasound measuring instrument in many points and several times during performance of the survey,
- with a use of measuring buoys, automatically registering the vertical sound speed distribution, based on temperature measurement and assuming that salinity is constant in the buoy's anchoring point.

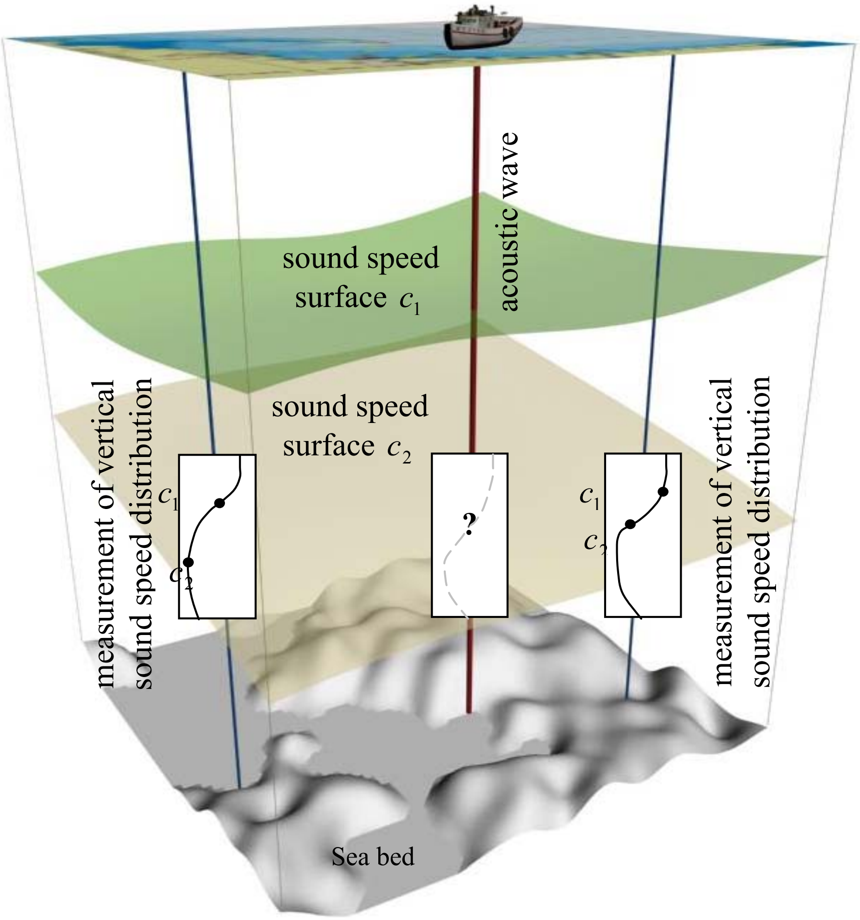

Fig. 4.2. Spatial model of constant surfaces of sound speeds in measurement point and distribution determined in depth measurement point

Surface of the constant sound speed in water shall be a plane fluctuating towards the water surface along with the temperature growth (from mornings to afternoons) in not large regions in which temperature and water salinity disturbances are not present (e.g. estuary of the Vistula River for the Bay of Gdansk, where the gulf's cold and salty water is supplied with warm and fresh water of the Vistula River) – only changes of time shall be registered. Spatial-time changes shall take place in water regions of disturbed distributions of temperature or salinity.

4.3.2 DETERMINATION OF SPATIAL MODEL OF SOUND SPEED DISTRIBUTION IN WATER – GLOBAL INTERPOLATION

With a given network $(n + 1) \times (m + 1)$ of points $\{Q_{k,l}\}$, $k = 0,..., n$ and $l = 0,..., m$, one may build a NURBS surface of (p,q) degree interpolating these points. The points $\{Q_{k,l}\}$ are measuring points of the sound speed in water.

$$Q_{k,l} = S(\bar{u}_k, \bar{v}_l) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(\bar{u}_k) N_{j,q}(\bar{v}_l) P_{i,j} . \tag{4.1}$$

Selection of appropriate values of the parameters $\bar{u}_k, \bar{v}_l$ is important. These values are contained in a range [0,1]. There are several methods of calculating the parameters existing there. The ones given hereby are used in surface modelling. It should be noted that the same methods are applied for calculating values of the parameters $\bar{u}_k$ and $\bar{v}_l$.

A method based on chord length [Piegl and Tiller, 1997] is the most common method:

$$d = \sum_{k=1}^{n} \left| Q_k - Q_{k-1} \right| , \tag{4.2}$$

where: d – chord length, $\bar{u}_0 = 0$, $\bar{u}_n = 1$ and

$$\bar{u}_k = \bar{u}_{k-1} + \frac{\left| Q_k - Q_{k-1} \right|}{d} , \ k = 1,..., n - 1. \tag{4.3}$$

Replacing the dependence (4.2) with dependence:

$$d = \sum_{k-1}^{n} \sqrt{\left| Q_k - Q_{k-1} \right|} \tag{4.4}$$

for $\bar{u}_0 = 0$ and $\bar{u}_n = 1$, surface of constant sound speed in water shall be determined. Moreover, similar to (4.3)

$$\bar{u}_k = \bar{u}_{k-1} + \frac{\sqrt{|Q_k - Q_{k-1}|}}{d}, \quad k = 1,\dots, n-1. \tag{4.5}$$

Averaging has been applied to calculate vectors of nodes U and V:

$$u_0 = \dots = u_p = 0, \quad u_{m-p} = \dots = u_m = 1,$$
$$u_{j+p} = \frac{1}{p}\sum_{i=j}^{j+p-1} \bar{u}_i, \quad j = 1,\dots,n-p. \tag{4.6}$$

In order to determine values of the parameters, first values $\bar{u}_0^l,\dots,\bar{u}_n^l$ are calculated for every l, and then all $\bar{u}_k^l$, $l = 0,\dots, m$ are averaged crosswise to obtain the values $\bar{u}_k$. The values $\bar{u}_k^l$ are calculated for every l

$$\bar{u}_k = \frac{1}{m+1}\sum_{l=0}^{m} \bar{u}_k^l, \quad k = 1,\dots, n. \tag{4.7}$$

Determination of co-ordinates of the control points $P_{i,j}$ may be performed as next curves' interpolations for the l data

$$Q_{k,l} = \sum_{i=0}^{n} N_{i,p}(\bar{u}_k)\left(\sum_{j=0}^{m} N_{j,q}(\bar{v}_l) P_{i,j}\right) = \sum_{i=0}^{n} N_{i,p}(\bar{u}_k) R_{i,j} \tag{4.8}$$

where

$$R_{i,l} = \sum_{j=0}^{m} N_{j,q}(\bar{v}_l) P_{i,j}. \tag{4.9}$$

The formula (4.8) is an interpolation of the points $Q_{k,l}$, $k = 0,\dots, n$. Points $R_{i,l}$ are control points of an isoparametric curve on a surface $S(u,v)$ for a given $v = \bar{v}_l$. Defining, in turn, an i, with the formula (6.10) applied, the points $R_{i,0},\dots, R_{i,m}$, get interpolated and obtained values $P_{i,j}$ make control points.

Calculation of all control points $P_{i,j}$ is executed according to the following sequence:

1. With a nodes' vector U and value $\bar{u}_k$ known, interpolations of the points $Q_{0,1},\ldots, Q_{n,1}$ for $l = 0,\ldots, m$ are performed $(m + 1)$, to obtain the values $R_{i,l}$;
2. With a nodes' vector V and value $\bar{v}_l$ known, interpolations of the points $R_{i,0},\ldots, R_{i,m}$ for $i = 0,\ldots, n$ are performed $(n + 1)$, to obtain the values $P_{i,j}$.

The above algorithm is symmetric what means that the same surface shall be obtained performing the calculations the opposite way:

1. First, $(n + 1)$ interpolations of the points $Q_{k,0},\ldots, Q_{k,m}$ shall be executed and the points $R_{k,j}$; shall be determined.
2. Then, $(m + 1)$ interpolations of the points $R_{0,j},\ldots, R_{n,j}$ shall be executed and the points $P_{i,j}$. shall be determined.

4.3.3 DETERMINATION OF SOUND SPEED LOCAL DISTRIBUTION IN WATER – LOCAL INTERPOLATION

Coefficients a_i of the B-spline functions, expressed with the dependence (2.19), for $N = 5, 10, 15$ (of vertical sound speed in water, described by 5, 10 and 15 B-spline functions) are given in the Table 4.1.

Table 4.1 Sheet of a_i coefficients of B-spline functions for N = 5, 10, 15

i	a_i		
	N=5	N=10	N=15
-2	238.48501248	246.58984259	244.54768342
-1	236.35200581	246.58984259	244.00741955
0	247.04803840	244.36749389	245.05894498
1	243.07336994	245.12669179	244.65521959
2	244.98425902	244.17917933	244.92490322
3	241.15223477	243.94167352	244.26697979
4	237.72675746	243.29436078	243.90176920
5	240.72295343	244.57848206	243.74464851
6	231.61964497	240.46248798	243.70617795
7	235.64957432	238.94143241	243.64241428
8		239.27803925	243.77546520

9		238.93451125	240.34356400
10		238.85590831	239.47793646
11		238.89167101	239.10885049
12		238.86342746	239.16419169
13			239.07020418
14			238.92114161
15			238.81075137
16			239.12124559
17			239.34229543

Sound speed distributions in water - real and approximated based on the dependence (2.19) - for the coefficients a_i listed in the Table 4.1 are presented in Fig. 4.3 and 4.4.

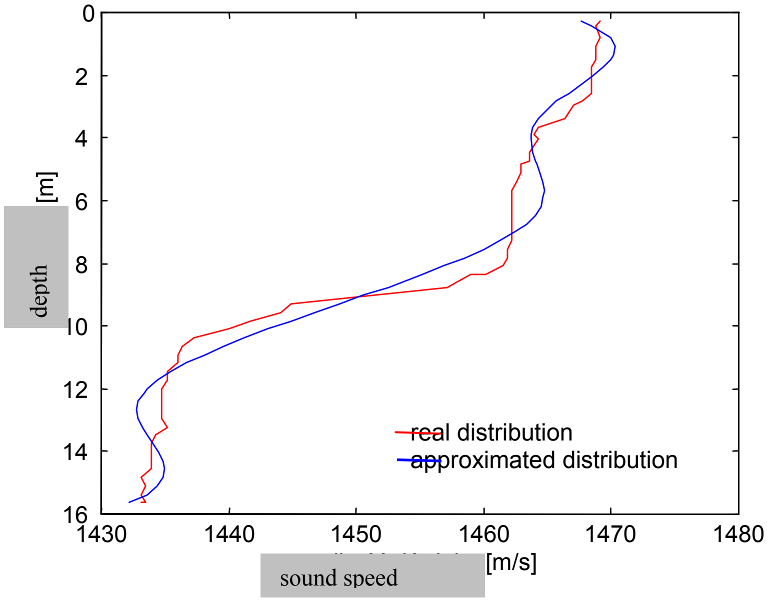

Fig. 4.3. Real and approximated (5 basic functions) sound speed distributions

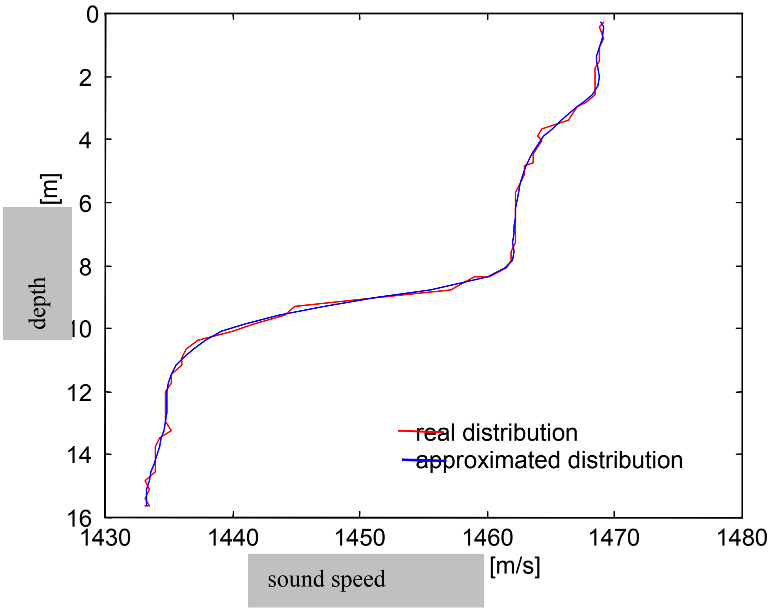

Fig. 4.4. Real and approximated (15 basic functions) sound speed distributions

4.3.4 ASSESSMENT OF REGRESSION FUNCTION'S COINCIDENCE WITH MEASUREMENT DATA

Command of regression function, characterizing a functional relation existing in statistical dependence, allows predicting an average behaviour of object. However, command of the regression function does not enable performance of assessment of discrepancy between values of the predictions and measurement data. Multidimensional correlation coefficient is used to determine intensity of relation between these two magnitudes.

An important feature, characteristic for a least square method, is demonstrated below, namely – an average measurement value

$$\bar{c} = \frac{1}{n}\sum_{k=1}^{n}\tilde{c}_k$$

equals average value from regression.

$$\bar{\hat{c}} = \frac{1}{n}\sum_{k=1}^{n}\hat{c}_k .$$

To this end, members of equations were respectively added up in the (2.23) system and the following equality was obtained:

$$\begin{aligned} &a_{-1}(\sum_{k=0}^{n}\Phi_{-1}(H_k)(\Phi_{-1}(H_k)+\ldots+\Phi_{N+1}(H_k)))+\ldots+ \\ &+a_{N+1}(\sum_{k=0}^{n}\Phi_{N+1}(H_k)(\Phi_{-1}(H_k)+\ldots+\Phi_{N+1}(H_k)))= \\ &=\sum_{k=0}^{n}\tilde{c}_k(\Phi_{-1}(H_k)+\ldots+\Phi_{N+1}(H_k)). \end{aligned} \tag{4.10}$$

With a use of the feature:

$$\sum_{i=-1}^{N+1}\Phi_i(H)=1 \quad \text{dla} \quad H\in[H_0,H_n], \tag{4.11}$$

(4.10) may be expressed with

$$a_{-1}\sum_{k=0}^{n}\Phi_{-1}(H_k)+\ldots+a_{N+1}\sum_{k=0}^{n}\Phi_{N+1}(H_k)=\sum_{k=0}^{n}\tilde{c}_k \tag{4.12}$$

and then

$$\sum_{k=0}^{n}(a_{-1}\Phi_{-1}(H_k)+\ldots+a_{N+1}\Phi_{N+1}(H_k))=\sum_{k=0}^{n}\tilde{c}_k\ . \tag{4.13}$$

Because

$$a_{-1}\Phi_{-1}(H_k)+\ldots+a_{N+1}\Phi_{N+1}(H_k)=\hat{c}_k\ , \tag{4.14}$$

hence

$$\sum_{k=0}^{n}\hat{c}_k=\sum_{k=0}^{n}\tilde{c}_k\ . \tag{4.15}$$

The required equality of the average values shall be obtained if both sides of this equality are divided by n

$$\bar{c} = \frac{1}{n}\sum_{k=0}^{n}\tilde{c}_k = \frac{1}{n}\sum_{k=0}^{n}\hat{c}_k \,. \tag{4.16}$$

The multidimensional coefficient of correlation between the magnitudes $\tilde{c}_k$ and $\hat{c}_k$, $k \in \overline{0,n}$,, expressing intensity of relation between values of signals defined during performance of measurements and values of the regression function is described with a dependence:

$$R = R_{c\hat{c}} = \frac{\sum_{i=1}^{n}(c_i - \bar{c})(\hat{c}_i - \bar{c})}{\sqrt{\sum_{i=1}^{n}(c_i - \bar{c})^2 \sum_{i=1}^{n}(\hat{c}_i - \bar{c})^2}} \,. \tag{4.17}$$

Considering equality of sums $\sum_{i=1}^{n}(c_i - \bar{c})(\hat{c}_i - \bar{c}) = \sum_{i=1}^{n}(\hat{c}_i - \bar{c})^2$, the formula (4.17) ends up with

$$R = \sqrt{\frac{\sum_{i=1}^{n}(\hat{c}_i - \bar{c})^2}{\sum_{i=1}^{n}(c_i - \bar{c})^2}} \,. \tag{4.18}$$

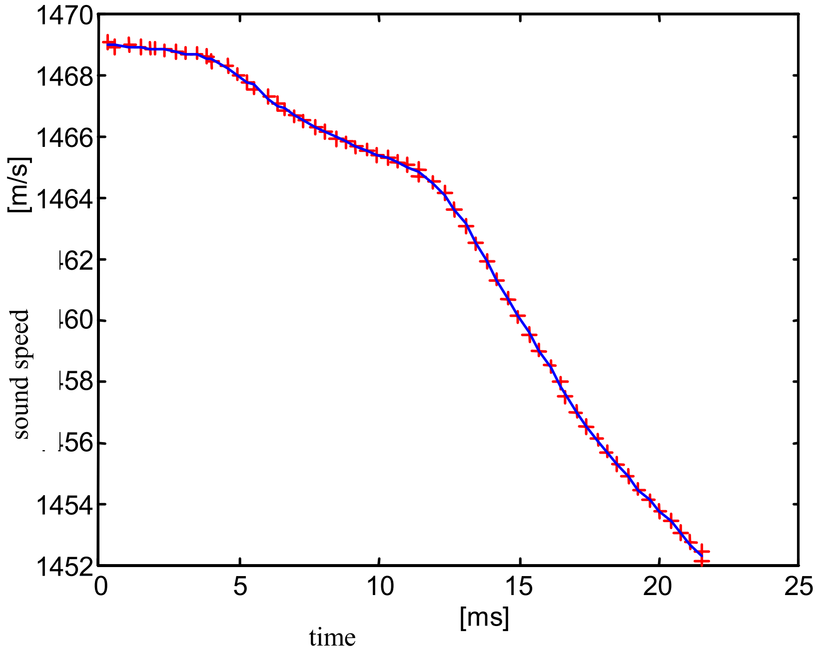

Fig. 4.5. Real and approximated time-domain run of sound speed in water

CONCLUSIONS

Determination of positions of the acoustic wave's reflection points is a basis for bathymetric soundings that are performed with a use of acoustic devices measuring the depth and radio-navigation positioning systems of various accuracy to allow defining the positions. Often, acceptance of assumption that the acoustic wave's propagation is rectilinear follows to achievement of lower accuracy of the performed measurements in comparison with application of the presented methodology by means of devices commonly used in the bathymetric soundings. The method presented in the study allows determining a real – versus approximate – positions of the acoustic wave's reflection points not only against usage of a complete hydrographic system, but also of its simplified variant containing a minimum number of measuring devices to execute the bathymetric soundings. The performed analysis of accuracy of determining reflection point of the acoustic wave radiating obliquely has proved that error in determining position of the acoustic wave's reflection point in water regions of minor depths (100 m) may exceed a value of 20 m at occurrence of positive refraction and some 15 m – at occurrence of negative refraction and of variable character. The two latter types of refraction happen in a period from spring to autumn, characterized by a broad range of the sound speed changes in water in summer time – this is when the bathymetric soundings are executed most often. It should be mentioned that these values refer to radiation of the acoustic wave with an angle of 30° against perpendicular, what for a multi-beam echo-sounder is a general, but not limiting, phenomenon. One should not expect that hydrographic surveys performed with a use of a single-beam echo-sounder shall be executed at heels reaching 30°. Moreover, non-linear trajectory of the acoustic wave lengthens its run time on a route transducer – sea bed – transducer, resulting in an average error of the depth measurement, with knowledge about the sound speed in water taken into consideration.

It is assumed in the study that the hydrographic system shall consist of the positioning system, single-beam echo-sounder and instrument measuring the sound speed in water, however theoretical considerations of the acoustic wave's trajectory and determination of its reflection point are valid for elementary beams radiated by the multi-beam echo-sounder. The area regarding determination of movement disturbances has been solved thanks to the multi-sensor positioning system (system of two or three non-collinear points). It is an alternative for accelerometers used in the hydrography for determination of pitching, rolling, and yawing. Usage of fluxgate type magnetic devices with floating cores or of GPS-Compass – a two-antenna GPS system capable of measuring direction - may be another possible solutions for determination of the movement disturbances. The latter solution requires development and an additional antenna of the GPS system due to capability of measuring only one angle: of pitching or rolling, depending on orientation of the devices in respect to symmetry line of the sounding vessel.

Orientation of the reference system connected with the sounding vessel has been used to determine output angle of the acoustic wave and – based on the developed spatial-time model of sound speed distributions in water – real trajectory of the acoustic ray. Findings on real positions of the acoustic wave's reflection points in the geodesic bathymetric measurements are the effect.

As far as the method of performing the hydrographic surveys, executed for decades by the time new devices were introduced such as satellite positioning systems, instruments measuring sound speed in water, hydro-metric stations and computers, execution of some stages of the soundings – included only in technical instructions and guidelines of hydrographic services - have been abandoned. It does not mean that these measurements are easier, calculations and processes are more complicated, but their execution may take less time and results may be more accurate thanks to the new hydrographic systems.

To conclude the performed study, it is possible to achieve high accuracy of the bathymetric soundings with by means of not complete (simplified) hydrographic system with the positioning system developed with one (or two) additional receiver.

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LIST OF ABBREVIATIONS AND SYMBOLS

a	-	length of the large semi-axis of the reference ellipsoid
a_i , $a_{i,j}$	-	coefficient of single-dimensional and double-dimensional B-spline functions
B	-	geodetic latitude
$\mathbf{C}$	-	NURBS curvate
c	-	sound speed in water
e	-	versor
e^2	-	square of the first eccentricity of the ellipsoid
H	-	orthometric height (for hydrographic surveys – depth)
h	-	ellipsoidal height
IHO	-	International Hydrography Organisation
IMO	-	International Maritime Organisation
$\Phi_{i,p}$	-	i -th B-spline function of p -th degree
KR	-	real course
L	-	geodesic longitude
n	-	sound refraction coefficient in medium
MSC	-	Maritime Safety Committee
MSL	-	Mean Sea Level
NURBS	-	Non-Uniform Rational B-Spline
$\mathbf{P}_\mathrm{i}$	-	i - th control point for NURBS curvate
$\mathbf{P}_{i,j}$	-	i, j - th control point for NURBS surface
p	- -	degree of the polynomial B-spline function base of the $\Im^P$ system
q	-	degree of the polynomial B-spline function
$\mathbf{R}$	-	matrix of rotation of geocentric system's base in respect to base of horizontal topocentric system
r	-	length of the acoustic ray
SOLAS	-	International Convention for the Safety of Life at Sea
$\mathbf{S}_{i,j}$	-	two-dimensional NURBS function

T	-	temperature
t	-	time
w	-	weight of the NURBS function
η	-	defect of the polynomial B-spline function
Ψ	-	deviation
Θ	-	inclination
Γ	-	tilt
ψ	-	geocentric latitude
α	-	incidence and outlet angle of waves between successive water layers
φ	-	latitude
λ	-	longitude
ϑ	-	planing angle of acoustic ray
$\Re$	-	radius of curvature of acoustic ray