

ANNUAL of NAVIGATION

MAREK GRZEGORZEWSKI

**NAVIGATING AN AIRCRAFT
BY MEANS OF A POSITION POTENTIAL
IN THREE DIMENSIONAL SPACE**

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Published in affiliation
with the Gdańsk Branch of the Polish Academy of Sciences

Citation

When citing papers from this volume the following reference should be used:
Author(s), Title, Annual of Navigation, No.9, 2005, Pages

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INTRODUCTION

The end of the 20th century marked an era of extraordinary technological developments. Owing to telecommunication capabilities, intercontinental television transmissions and the Internet, the world has become smaller and we have gained a possibility of visiting our friends at their homes at any time. Thanks to satellite navigation systems we have become more precise. A miniature receiver of satellite navigation systems allow us to find our precise position on the Earth's surface and in the close space surrounding our planet. While carrying out everyday routines, everybody constantly solves navigation problems. On the way to work, on a trip to the country or while travelling abroad for holidays, we try to find solutions of navigation problems. Therefore we ask ourselves the following questions: "What is our current position and what are our co-ordinates in a reference system?" "What is the final point of our trip, and what direction we should follow to get there in a fixed time?"

Nobody is surprised any longer by the use of satellite navigation in aviation. Combat, search and rescue (SAR) capabilities have been considerably enhanced due to the more and more widespread use of satellite receivers. In carrying out combat missions such as attacking ground targets there has been a considerable technological progress due to the use of satellite navigation systems and precision guidance missiles. When the co-ordinates of a particular segment of an attacked target are known, a strike can be performed to destroy only this particular element of the target taken as a main point of impact. Using satellite navigation system is of the utmost importance in maritime SAR operations and in search and rescue of pilots shot down over the enemy territory. In the latter situation it is vital to have communications (e.g. via modem) between the pilot and the rescue team.

The problem of en-route aircraft satellite positioning regarding the position error assessment has been solved. Achieved accuracy meets the standards accepted all over the world and facilitates the proper flight safety. The main aspects of current studies in air navigation concern: the credibility, availability, reliability and continuity during the approach phase of flight¹[European Radionavigation Plan 1996]. Continuity criterion is the most important, regarding the safety of carrying out such a navigation task as landing. It concerns a given time interval t in which the process of landing has to commence, and when the navigation system has to be available. In case of other navigation tasks their commencement can be delayed until the navigation system is available. Landing process has a defined initial time of manoeuvre and missing that time necessitates aborting and repeating the whole approach. The problem of continuity is no less important when the aircraft descends on the glide path. Any breaks in system availability during the descent are also vital to the safety of landing. Attempts have been made to solve that problem by means of combining several navigation systems or by developing special navigation filters. In the context of air navigation such interpretation of *continuity* is obvious. Literature IALA (2001)² [Recommendation R-121 2001] defines continuity as reliability within a short time interval. Such a definition is totally different from the previous one. In aviation the main point is the navigation system status, i.e. its availability or the lack thereof, when the manoeuvre is commenced and executed.

How can, therefore, full safety of the landing be ensured?

In order to solve the problem of making the information concerning the position of the aircraft and the accuracy thereof on the glide path continuously available to onboard navigation instruments, the following research problems had to be solved:

1. Preparing and conducting research experiments, aiming at the assessment of position co-ordinate errors in en-route flights, as well as during the approach and landing phase.
2. Estimation of time intervals at which no GPS information was available and pointing out probable causes of such situations en-route and during the landing phase..
3. Developing a mathematical model (a new navigation filter), “Navigating by means of a position potential in three-dimensional space” in order to secure the continuity of the aircraft position data in space.

The main thesis: We assume that the unavailability of GPS information in certain time intervals will be compensated by the implementation of the mathematical model, which will process the aircraft co-ordinates on the basis of the last received position.

The present study has been divided into four chapters aiming at and presenting the solutions of the above mentioned research problems:

Chapter 1 – “Review of the Literature” – deals with the works which have been published since the beginning of 1990s and which concern flight analyses of aircraft and error analyses – standard deviations of basic parameters obtained in aviation experiments. The analysis also concerned mathematical models designed to supplement GPS satellite navigation system.

Chapter 2, “En-route Flights - Aircraft Position Accuracy Assessment”, comprises the analysis of the flight experiments, carried out between 1999 and 2003, whose aim was to analyse the data obtained during en-route flights and to assess the possibility of carrying out flights within flight corridors and of sustaining vertical separation. It also deals with the assessment of the time intervals, when the GPS information was unavailable and with the mean errors of B, L and h .

Chapter 3, “Approach and Landing Phase - Aircraft Position Accuracy Assessment”, tackles the problem of aircraft landing accuracy in flight experiments, which compared the accuracy of landings using the Air Traffic Control Radar Landing System (RALS) with GPS-assisted landings.

Chapter 4, “Navigating By Means of a Navigation Potential In Space”, proposes a new solution being a development of Vanicek’s theory³ [Berlin Xu, Vanicek P. 2000] by expanding it into the third dimension. Using a given density function of a particle, it was assumed that the time-related position potential field can be established for a sequence of position fixes. Subsequently, a model of motion for an individual particle was set up so as to find the solution of the equation of its motion. In order to reflect changing navigational environment, the potential function contains certain variable parameters, α and G . When these parameters and the initial conditions are not known, the number of possible positions of the particle is infinite, and, what it involves, further data is required to select a proper trajectory. Therefore, a „self-learning” procedure for the filter was set up so that parameters and conditions from earlier observations could be taken into consideration.

The presented mathematical model fills the gap in determining aircraft position in the instance of the lack of navigation information. Knowing the position is of vital importance on the glide path, when the accident risk is very high in adverse weather conditions.

When discussing the use of satellite navigation systems in aviation, not only single satellite navigation receivers are taken into consideration, but also whole inter-operating navigation complexes, working in order to optimise the flight route in such a way as to minimise the flight time and fuel consumption, and thus to minimise the costs, at maximum flight safety. Flight safety problems and the credibility and reliability of the obtained data accuracy will be the primary condition taken into consideration in the certification process of any basic navigation instrument.

Mathematical fundamentals of position determination and of its accuracy assessment will be decisive in qualifying certain receivers for use in performing certain tasks in aviation. Safe altitude during the approach as well as touchdown precision are crucial. The level of touchdown precision will be a decisive factor in determining whether GPS (GLONASS) receivers will enter into common use to assist aircraft landings.

1. REVIEW OF THE LITERATURE

The following aspects of the research works dealing with the implementation of satellite techniques were analysed:

- En-route and landing accuracy of military aircraft;
- Results of attempts at mathematical modelling of the final flight phase, i.e. landing.

The first results of aviation experiments were published in the early 1990s. Professor Schänzer's team⁴ [Schänzer 1991] from the Technical University of Braunschweig conducted experiments on using GPS instead of currently used systems INS, VOR/DME, ILS/MLS. The study contains a number of detailed comparative analyses of flight trajectories, errors and differences, which emerge in the determination of the aircraft position by means of traditional methods and using GPS/DGPS. In 1994 Clarc E. Cohen⁵ [Clarc 1994] and his team prepared a work which concerned obtaining RNP (Required Navigation Performance). In order to define the procedures of Cat III precision landing with the assistance of GNSS the study gives definitions of numerous institutional and technical problems, which, when solved shall facilitate the accurate reliability of the newly-implemented system. The study was based on the experimental data obtained in test flights using GNSS. It contains the analysis of the application of combined GNSS simulators as well as considerations of reaching the norms for RNP. In 1995 the same team published⁶ [Clarc 1995] a synthetic description of basic terms related to experimental data and mathematical methods used in the analysis of the phenomena and problems of ensuring the proper accuracy level of GPS. The work places special emphasis on creating a maximum accuracy mathematical model, supplementing the navigation system. Three years later, i.e. in 1998 Takey Asu Sakai and Kazumobu Koremura⁷ [Takey 1998] published an article containing algorithms for assessing availability and continuity of data processing being two fundamental indicators of GPS reliability during precision approach and landing.

In the same year R.Pasquali and S.Viviano⁸ [Pasquali 1998] published the results of the research showing the advantages of GPS/GNSS system supplemented with new modules. Their experiments were conducted in the Mediterranean Test Bed laboratories. Their objectives were as following:

- presenting the possibilities of GNSS application in the Mediterranean as a supplementary navigation system or the basic system for executing of navigation procedures.
- providing limited integrated corrections for the system in order to elaborate the data in local area.
- verification of technical capabilities of GNSS in conducting the approach procedures in accordance to Cat I requirements.

The article which inspired me to continue the research studies and search for new solutions was „Navigating by Means of a Position Potential”⁹ [Berlin Xu,1998]. The authors presented a new navigation algorithm which was worked out on the basis of measuring velocities and positions of moving vehicles and statistical errors thereof. In order to make the algorithm more flexible, so that it may be used in the event of

volatile navigation environment, the authors leave some parameters undetermined so as to calculate and compare their values with the data from other experiments. The work concerns the two-dimensional co-ordinate system(x, y). The authors left the problem of the third dimension unresolved.

In 1999, a new navigation system (New Integrated Navigation System)/New Integrated Landing System) was presented in Navigation News¹⁰ [Navigation News 03/04 1999]. System was tested that time. The components of that system comprise instruments which facilitate the proper level of navigation and of supervising the landing process. Onboard sensors supply the system with complete information without the need of external assistance. The system has an interface to communicate with other systems in order to secure the reliability.

Problems with the implementation of the new system were presented in the article by G.Sasi Bhushana Rao.¹¹ [Sasi 2001]. Authors present a detailed analysis of GPS being a supplementary navigation system in the final flight phase, i.e. landing. The following parameters are analysed: NSE (Navigation Sensor Error), FTE (Flight Technical Error) and TSE (Total Sensor Error), which is the total error of the navigation system in determination of the aircraft position. The study concerns the implementation of the new system, recommended by ICAO – not complete, without the additional WASS elements – in most Indian military and civil airfields

In 2001 a research project was presented in The Journal of Navigation Vol. 54¹² [Leighton 2001]. The work describes various aspects of implementing of advanced GPS system in air navigation. It also points out to a considerable improvement of all parameters of the integrated navigation system in all flight phases and especially in landing. The project contains error analyses – standard deviation of basic parameters used and obtained during laboratory experiments at Boscombe Down airfield The basic objective of the experiment was to perform technical assessment of GNSS system capabilities in assisting fully automated procedures of take-off, en-route flight, circling approach and landing.

In the same year, *Aviation Week and Space Technology*¹³ [Bruce 2001] published information concerning Joint Precision Approach and Landing System (JPALS) and its use by US Air Force. The article presented the segment responsible for tactical mission applications and for special operations missions from fixed-base conditions.

2. EN-ROUTE FLIGHTS - AIRCRAFT POSITION ACCURACY ASSESSMENT

2.1. Estimation of Su-22M-4 aircraft positioning accuracy during en-route flights in WGS'84 (ETRF'89) using RTK techniques (Powidz 99 experiment).

Accomplishing an air combat mission requires carrying out a flight via the planned route in a fixed time and following a planned mission profile. Special performance assessment norms have been established in order to assess the performance of the crew, regarding piloting and navigating techniques¹⁴ [Program Szkolenia 2002]. Assessment of the the quality of a mission accomplishment is one of the most important parameters used in assessing the overall competence of a pilot. Performance measuring tools used so far in measuring the navigation accuracy have high error margin. Modern destruction assets, which are used on contemporary battlefield, facilitate the execution of very high precision strikes. While accomplishing the combat mission requiring the use of such assets, the pilot has to carry out the flight according to a set of parameters (heading, speed, flight altitude) in order to find the *initial point* (IP), or *push point*, where the target attack phase begins. Flight precision guarantees mission safety.

Because of the above factors, a series of experiments in using satellite navigation techniques were planned and carried out with the aim to determine the precision of the flight via the planned flight path. The experiments included various aircraft types used by the Polish Air Force, including SU-22M4 fighter-bomber, TS 11 Iskra jet trainer and, in one case, the AN2 transport aircraft.

The first experiment was conducted in Powidz Air Base and its objective was to compute the WGS'84 (ETRF'89) co-ordinates for subsequent in-flight position fixes at one-second interval for the SU22M4 aircraft using the RTK "post-processing" technique. At the same time asesment of accuracy of those position fixes was performed. Because of the mission character, the main problem was the accomplishment of the mission in a fixed time, which was dependent on executing the flight via the planned route at fixed speed and constant or variable altitude. Air mission success was, therefore, dependent on precision in mission accomplishment. The flight was executed at 750-800 metres.

The following list of activities during the experiment was defined:

- designing the arrangement of satellite receivers and antennas and mounting them onboard of the SU 22 M4 aircraft cockpit;
- designing the reference point network and its incorporation in ETRF'89¹⁵ [Europen 1989];
- computing the co-ordinates of the reference points using Ashtech receivers;
- carrying out in-flight measurement sessions and computer post-processing of the data obtained during kinematic observations;
- comparing the obtained results.

Mounting the antennas and the receivers was carried out following the authors' own project, which was authorised by the Chief Engineer of the Polish Air and Air

Defence Forces. The antennas were placed in the cockpit under the windscreen (Fig.1).



Fig. 1. Mounting the satellite receiver antennas in Su-22M4 aircraft cockpit (25th April.1999.)

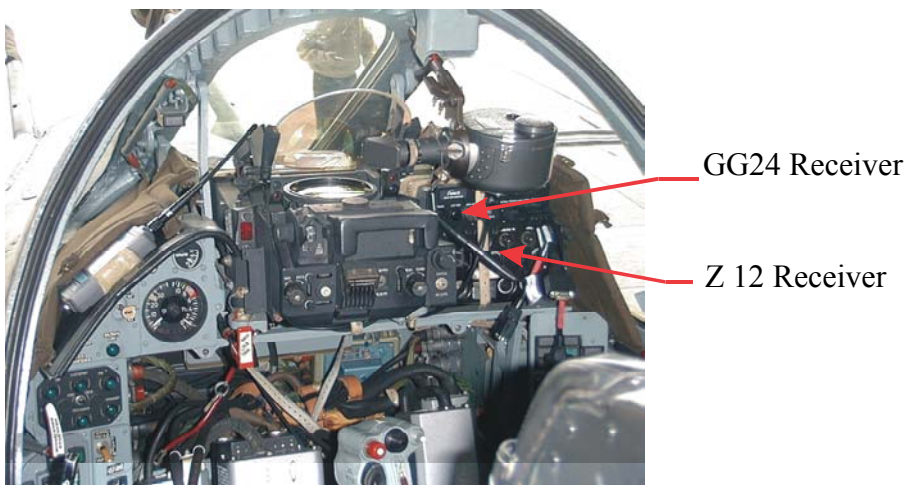


Fig. 2. Mounting GG24 and Z 12 satellite receivers in Su- 22M4 aircraft cockpit (25th April.1999)

The distance between the receivers was 10 cm. For safety reasons the receivers were fitted to the special mounting plate, screwed down to the flight control panel.

GG24 and Z 12 receivers were mounted on the starboard side of the cockpit, after removing the BI-91 plate¹⁶ [Instrukcja 1997] (Fig. 2.)¹⁷ [Instrukcja 1997]. This way the pilot had an easy access to control switches of both satellite receivers.

2.1.1. Task description

The main task was to compute the WGS'84 (ETRF'89) co-ordinates for subsequent in-flight position fixes at one-second interval for the SU22M4 aircraft using the RTK "post-processing" technique and to assess the accuracy of data gathered during the experiment.

2.1.2. Selection of the reference points

Four reference points, designated 0002, 0003, 0004, 0005, were established (Fig. 3.). The network was arranged in such way that two of the reference points were situated on the extension of the opposite ends of the runway axis and the other two at approximately half of its length¹⁸ [Grzegorzewski 1999]. Reference points were established only for the duration of the measurement sessions. In the same sessions, it was also necessary to compute co-ordinates of the GG24 receiver antenna, which served as a reference antenna for DGPS/DGLONASS measurements in the subsequent days. The antenna was mounted in REF0 point on the roof of the Air Traffic Control Tower at Powidz Air Base.

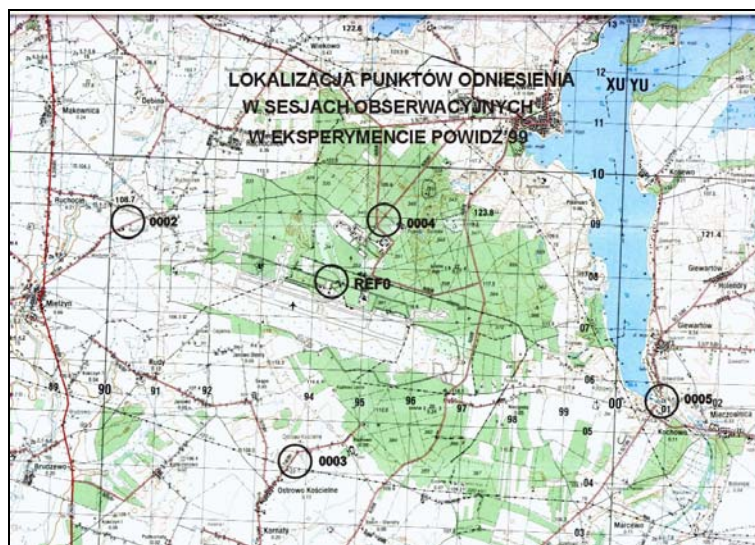


Fig. 3. Location of reference points in Powidz '99 experiment.

2.1.3. Computing the co-ordinates of the reference points in ETRF '89

Static satellite measurements were conducted on 25th April, 1999 using Ashtech dual-frequency satellite receiver. Prior to the commencement of measurements, observation conditions (constellation geometry and the total number of satellites)

were analysed using MPSM¹⁹ [Ashtech Inc]. Optimum time of arrival at particular points as well as the network geometry was also taken into consideration when the sequence of measurements was planned. On the basis of detailed observation conditions and analysis of reference points allocation, plan of GPS measurements was worked out.

ETRF'89 (European Terrestrial Reference Frame) is a practical geodetic realisation of WGS'84 in Europe. Three POLREF points with the numbers:

- 3304
- 3306
- 3405

situated in the vicinity of the experiment's location, were selected to be connectors to ETRF'89 reference system.

The location of the POLREF points facilitated proper geometry of the measured network. Geodesic and Cartographic Documentation Centre of Polish Main Geodesy and Cartography Authority supplied the co-ordinates of those points in ETRF'89.

Heights of antennas above the reference points were measured with millimetre precision. Measurement sessions lasted from 90 to 180 minutes and depended on the length of the measured vector. The measurements also comprised the location of the Ashtech GG24 receiver antenna (REF0), which served as a reference antenna for DGPS/DGLONASS measurements. Two measurement sessions were conducted and the results were interlinked with ETRF '89.

2.1.4. Post - processing of static session results

Approximate vector components of GPS points were calculated using GPSS (v. 5.2) software²⁰ [Ashtech Inc]. Process control calculations were performed in the field on a laptop computer with a Pentium processor. Final results were computed using an IBM Pentium II computer. Tropospheric correction for standard meteorological conditions and minimum elevation angle of 15 degrees were taken into account in the computations

Precise grid alignment was carried out using GEOLAB BitWise Ideas (v.1.9) software. Three constant points of the POLREF network were taken into consideration in the alignment and their co-ordinates were taken from ETRF'89. The aligned network consisted of eight points:

- three POLREF points,
- five reference points (four for post-processing RTK measurements - 0002, 0003, 0004, 0005, and one - REF0 - for DGPS/DGLONASS measurements).

Grid alignment results were obtained from the satellite system. Parameters of error ellipses were calculated with a 95% credibility rate. The mean semi-major axis length of an error ellipse was approximately 2.5 mm.

2.1.5. *Su-22M4 aircraft position determination in the post-processing mode*

Experiments carried out on 26th and 27th April, 1999 aimed to compute the coordinates of the Z-12 receiver antenna, mounted in the SU22M4 aircraft, for subsequent in-flight position fixes at one-second interval. Four previously designed reference points were used as reference stations. Two measurement sessions were carried out, one per day respectively. During each session four receivers set at one-second interval were recording raw observation data at reference points, while the fifth receiver - Ashtech Z-12 fitted with an aircraft antenna which was borrowed from the INS Company in Kraków – was mounted onboard the aircraft. That receiver was also recording raw observation data at one-second interval. At the same time, real time DGPS/DGLONASS measurements were conducted with two Ashtech GG24 receivers, one of which was situated on the roof of the control tower at Powidz Air Base and served as a reference station, sending DGPS/DGLONASS corrections to the other receiver mounted onboard the aircraft via radio comms line.

2.1.6. *Analysis of observation results – 26th April, 1999 session*

On 26th April, 1999, no real-time position fix of the SU22M4 aircraft could be recorded using the Z12 receiver. However, raw observation data were recorded by GG24 receivers at REF0 and onboard the aircraft. No file recorded on that day contained data from GLONASS satellites.

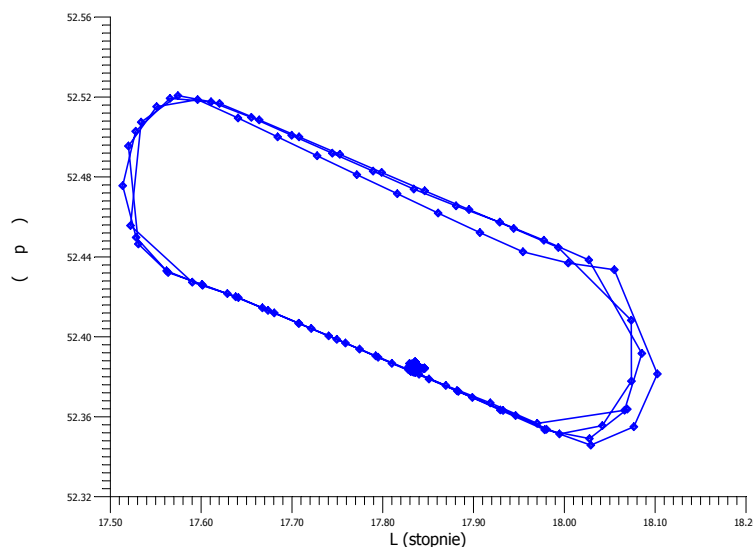


Fig. 4. Flight trajectory of Su-22M4 aircraft with onboard GG24 receiver.

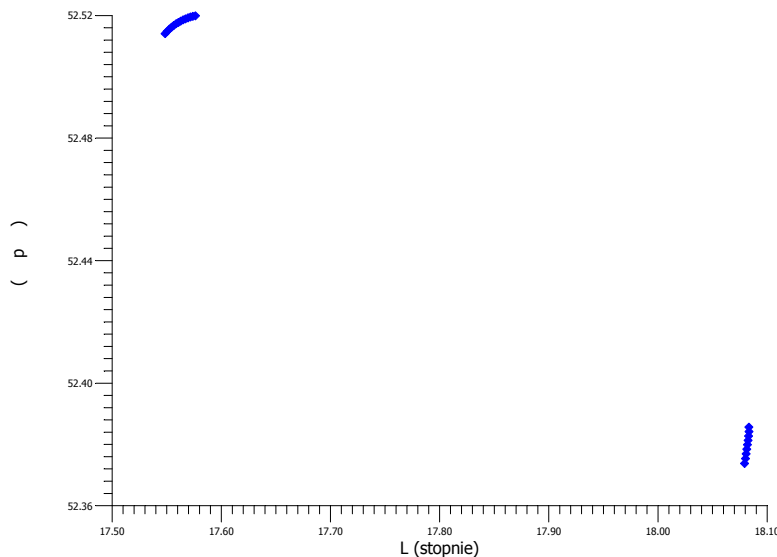


Fig. 5. Flight trajectory of Su-22M4 aircraft with onboard Z-12 receiver (26th April, 1999).

The Z12 receiver (mounted onboard the aircraft) was not working properly. Most of the time it was recording the observations from up to 3 GPS satellites, while the onboard GG24 receiver was using data from a larger constellation.

The obtained data was processed using Ashtech PPDIFF software, being a module of Geodetic Post-Processing Software v. 5.5., Ashtech Inc. In computations, the software uses solely the pseudoranges measured in the C/A code and, unlike, e.g. PNAV software, it does not require the availability of 5 satellites. Unfortunately no time intervals could be found when aircraft positioning data from both receiver types²¹ [Grzegorzewski 1999] were available.

The following computations were performed:

- generating files containing subsequent position fixes of the Z12 receiver antenna onboard the aircraft,
- generating files containing subsequent position fixes of the GG24 receiver antenna onboard the aircraft,
- calculating arithmetic means of the computed position fixes and subsequent standard deviations from arithmetic mean for both receivers,
- short analysis of the obtained results.

The Su22M4 trajectories (relation $B(L)$) calculated with Z12 and GG24 receivers are shown in Figs. 4 and 5. The plots in Figs. 6, 7 and 8 show the ellipsoidal height as a function of time for both receivers, where in case of the GG24 receiver the aircraft flight time was separated from the standstill time for the sake of clarity. Fig. 9 and 10 show the plots of the subsequent deviations from mean value of B, L and h parameters for position fixes, gathered by the onboard Z12 and GG24 receivers respectively, from reference points Ref0, 0002, 0003 and 0005.

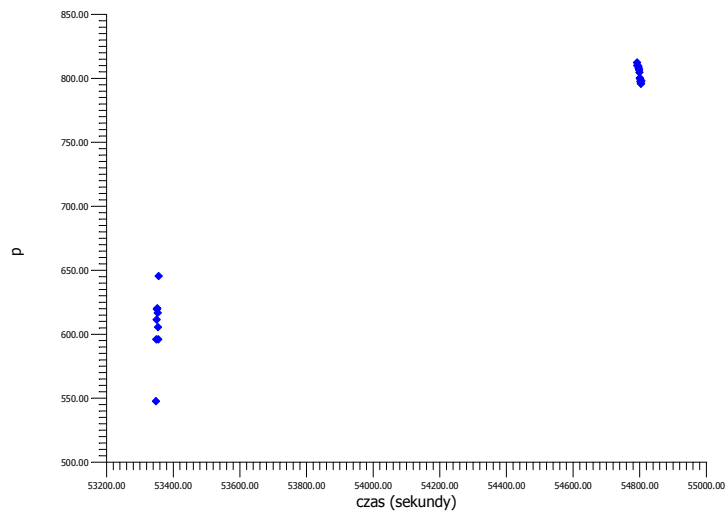


Fig. 6. Calculated Z12 receiver antenna ellipsoidal heights (26th April, 1999).

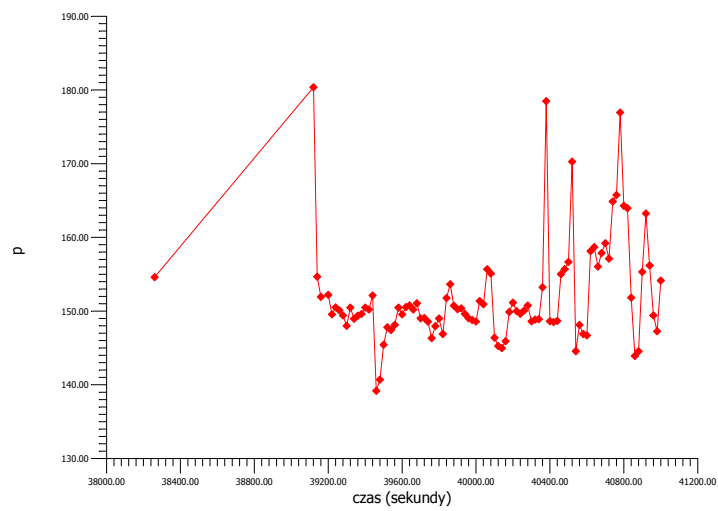


Fig. 7. Calculated GG24 receiver antenna ellipsoidal heights (26th April, 1999 – aircraft standstill time).

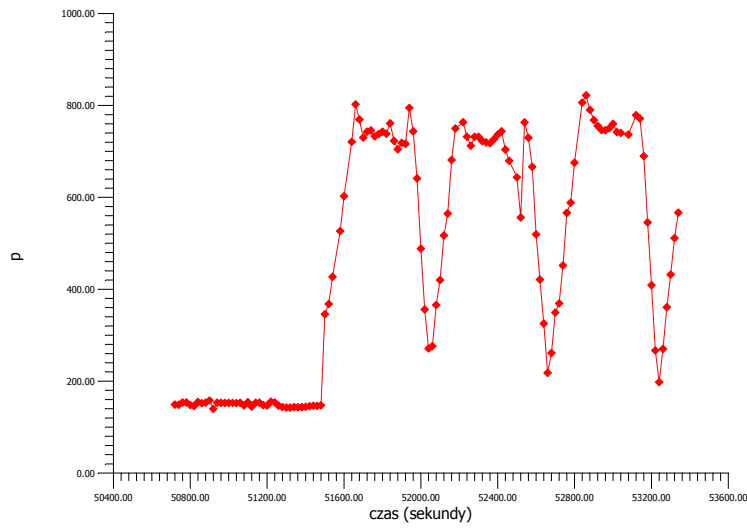


Fig. 8. Calculated GG24 receiver antenna ellipsoidal heights (26th April, 1999 – flight time).

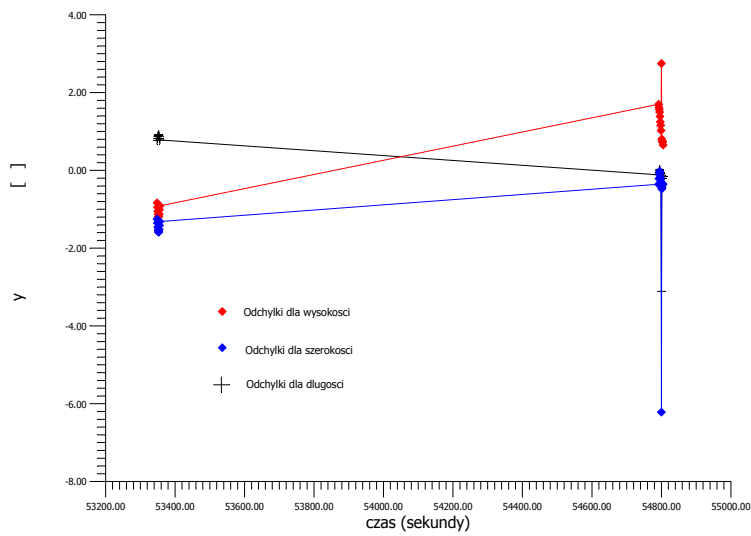


Fig. 9. Deviations from arithmetic means for Z12 receiver antenna (26th April, 1999).

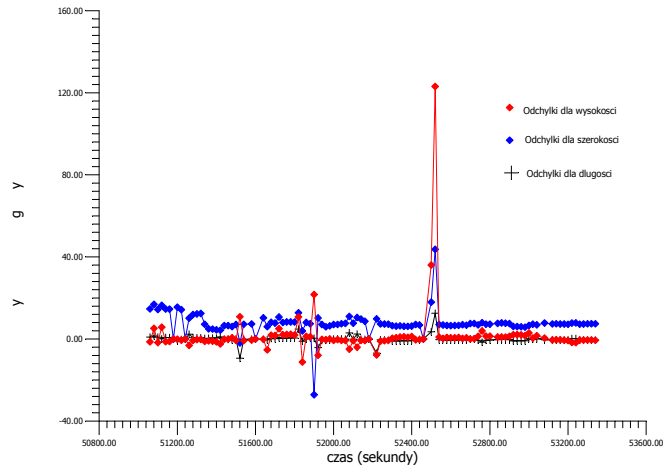


Fig. 10. Deviations from arithmetic means for GG24 receiver antenna (26th April, 1999).

The following conclusions can be drawn from the above graphs:

- there were only a few epochs in which aircraft position could be calculated especially in case of the onboard Z12 receiver;
- deviations from arithmetic mean are considerably lower for the Z12 than for the GG24; However, only a very few position fixes could be calculated with the Z12;
- in case of GG24 deviations, geodetic latitude (B) deviations are significantly greater (up to 15 metres) than for the remaining co-ordinates.
- Z12 deviations are in the range of 1-2 m, as they could be expected for DGPS positioning (solely on the basis of pseudo-range measurements).

2.1.7. Analysis of observation results – 27th April, 1999 session

On 27th April, 1999, no real-time position fixes of the SU22M4 aircraft were recorded from 10:19:09 to 12:02:41 GPS time. When analysing satellite availability recorded by the onboard Z12 receiver throughout that period of time, no common observation epochs were found - epochs in which GG24 in-flight position fixes (real time) could be compared with those of Z12 (post-processing). At the same day, the Z12 receiver mounted onboard the aircraft was not working properly. During most of the observation session receiver was recording data from only three satellites. Z12 (post-processing) and GG24 (real time) position fixes obtained at subsequent observation epochs were compared only for those intervals in which Z12 antenna position was available. The comparisons were made using the PPDIFF software. The following analyses were carried out only for the interval when real-time GG24 position fixes were available:

- comparison of post-processing GG24 position fixes with PPDIFF and AOSS software;
- analysis of the set of co-ordinates obtained in real time from the GG24 receiver.

a) *Comparison of real time GG24 position fixes with post-processing Z12 position fixes obtained in subsequent observation epochs.*

A data set containing averaged (arithmetic mean) B, L and h for the antenna of the onboard Z12 receiver was created in order to facilitate such a comparison²². Arithmetic mean was calculated for all position fixes available in a given observation epoch. If only one position fix was available, it was assumed as an average. Standard deviations were also calculated. They were assumed to be equal to zero whenever the position fix resulted from the positioning of only one reference station. Mean standard deviations for all co-ordinates are in the range of 1 m. Averaged co-ordinates of the Z12 antenna were compared with the real-time position fixes of the GG24 antenna. The results of the comparisons for the observation epochs prior to aircraft take-off are shown in Fig. 11. On the basis of analysis of the graph, we can assume that there is considerable regularity in differences – Z12 deviations are always greater than those of the GG24. Similar regularity can be observed with regard to geodetic longitude, where longitudes measured by Z12 are always smaller than those of GG24 and the differences range from -10 to -14 m. With regard to ellipsoidal height, the values obtained from Z12 are smaller than those of GG24 in real time. Differences range from -10 to -26 m.

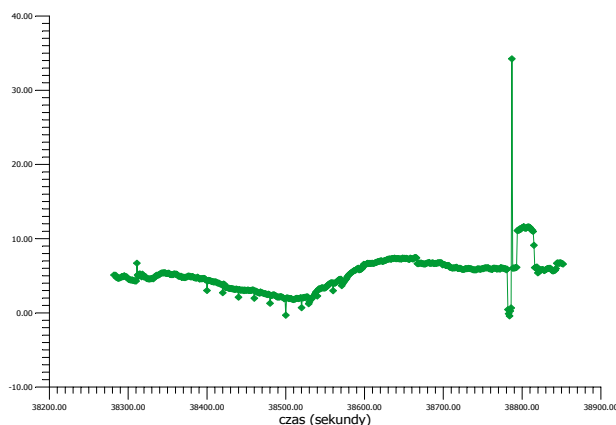


Fig. 11. Differences in geodesic latitude values [m]; GG24 – real time, Z12 – post-processing.

Moreover, we can find a limited number of epochs in which comparison of in-flight position fixes, obtained from both types of receivers, was possible. Unfortunately, the data set of real-time GG24 position fixes contains numerous epochs in which outdated co-ordinates were recorded. In the observation epochs in which no corrections are supplied, GG24 receiver records co-ordinates from the last epoch in which the corrections were available. Given the high speed of the aircraft, differences between the real position and that recorded in the data set of real-time co-ordinates are significant. Figs. 12 and 13 show the differences between the real-time position

fixes obtained by the GG24 receiver with those obtained with the Z12 in the post-processing mode. There are differences of up to as much as 2 km caused by the lack of updated real-time positions in case of corrections unavailability. Consequently, there is a need of creating a special module sustaining the work of the system until GPS signal is received again.

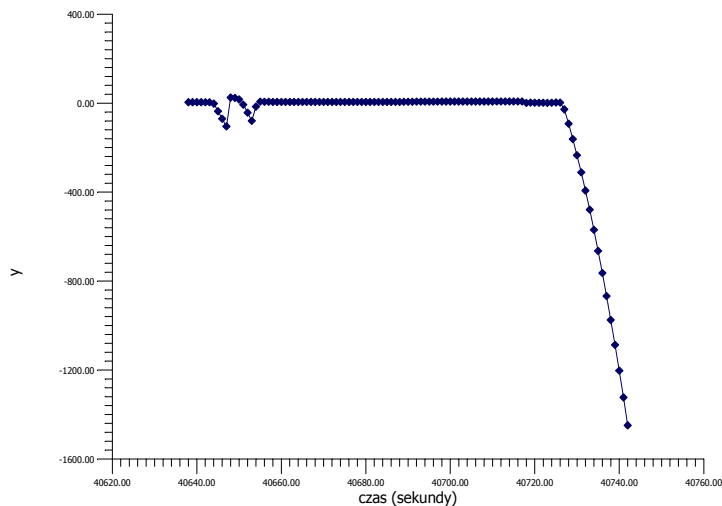


Fig. 12. Differences in real-time geodesic latitude values [m]; Z12 – “Post-processing” mode. Moving aircraft.

Nevertheless, the analysis of the graph in Fig. 12 reveals that there are about 50 observation epochs in which position fixes given by both types of receivers are within the range of 10 m from each other for B and L, and within 25 m in ellipsoidal height. Those epochs have been shown separately in Fig. 13. However, their number is too small to draw definite conclusions.

b) Comparison of post-processing GG24 position fixes obtained using PPDIFF and AOSS software.

Raw observation data sets recorded by GG24 receivers in REF0 and onboard the aircraft were available for the time interval from approximately 9:30:00 to approximately 10:02:00 GPS time. They could be processed using the available software (post-processing). Positioning obtained by PPDIFF software for the vector connecting 0004 reference point with the Z12 onboard antenna was compared with the positioning obtained by AOSS software²³ [Ashtech Office Suite for Survey] for the vector connecting REF0 with the GG24 onboard antenna. The results of the comparisons were collected in the file AOS PPD.DAT on a disk. Differences in B and L range from -1.5 to +1.5 m. and differences in ellipsoidal height from -2.8 to +0.8 m. The comparison shows the compatibility of both programs. There are no GLONASS satellites in the file containing the position fixes calculated by AOSS.

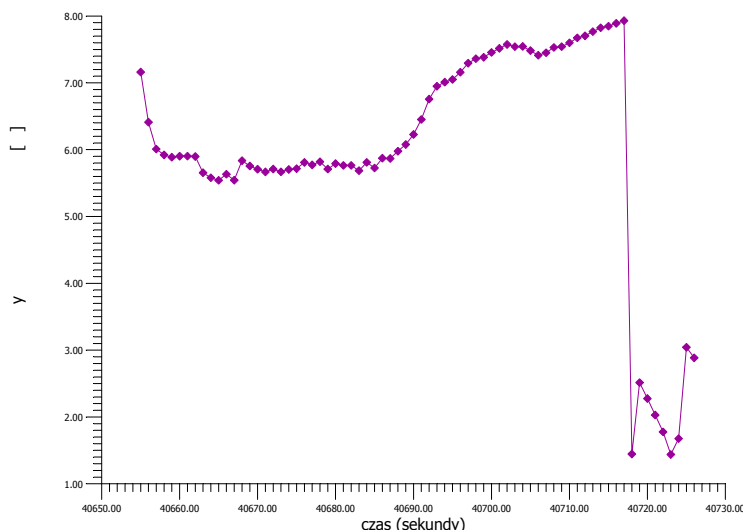


Fig. 13. Differences in real-time geodesic latitude values [m]; GG24 – “Post-processing” mode. Moving aircraft.

c) Analysis of the set of real-time co-ordinates obtained with GG24 receiver.

The set of co-ordinates calculated with GG24 receiver comprises of two segments: co-ordinates of a stationary aircraft and in-flight co-ordinates (moving aircraft). Positioning accuracy analysis could be carried out for the time interval, when the aircraft remained stationary (from 37149 to 39100 second after midnight). Average values of B, L and h were calculated for the stationary aircraft.

$$B_{AV} = 52.38416711217$$

$$L_{AV} = 17.84613311381$$

$$h_{AV} = 154.633$$

Having the arithmetic mean of B, L and h of all positionings, standard deviations for each co-ordinate were then calculated from formula (1):

$$S.D.B = \sqrt{\frac{\sum_i (B_{sr} - B_i)^2}{n-1}}, \quad S.D.L = \sqrt{\frac{\sum_i (L_{sr} - L_i)^2}{n-1}},$$

$$S.D.h = \sqrt{\frac{\sum_i (h_{sr} - h_i)^2}{n-1}} \quad (1)$$

The following values were calculated:

$$S.D.B. = 1.6 \text{ m.}, \quad S.D.L. = 2.8 \text{ m.}, \quad S.D.h. = 4.6 \text{ m.}$$

These results are acceptable on the condition that they were obtained in real time using only the pseudo-ranges measured to GPS and GLONASS satellites. When the aircraft was in motion, there were many epochs in which corrections were unavailable. Position fixes recorded by the receiver in that case were obsolete.

A sample graph ($h(t)$ relation) representing real-time GG24 positionings is shown in Fig.14. The most common scenario of correction availability is as following: when the aircraft is climbing, corrections are available, then, at the ceiling altitude, they are unavailable and remain unavailable in the descent phase till the aircraft reaches the lowest flight level, where they reappear.

The following conclusions may be drawn from the above analyses:

- a) The measurements ought to be repeated paying the utmost attention to the correctness of onboard antenna mounting and all connections within the equipment. Faulty operation of the Z12 receiver onboard the SU22M4 aircraft seems to be caused by loosing connection between the antenna cable and the receiver.
- b) „Post Processing” position fixes obtained with the Z12 onboard receiver are acceptable for those epochs, in which at least 4 GPS satellites were available
- c) „Post Processing” position fixes obtained with the GG24 onboard receiver on 26th April, 1999 are not accurate enough; regularity can not be found. For the position fixes from 27th April, 1999, there is no regularity neither for real-time nor for “Post Processing” position fixes.
- d) The lack of GLONASS satellite data in the raw observation data recorded by GG24 receivers seems strange, and their use in the real time positioning cannot be, therefore, determined.

The lack of continuity, resulting from various factors presented in the above conclusions, undermines the possibility of applying satellite navigation techniques in navigating aircraft flying at high velocities. However, since those experiments have been the first, the lack of experience may have been the primary reason for such poor results.

The last example sets the scope of the experiments to be carried out. Creating a mathematical model supplementing the aircraft navigation during the flight was a challenge, the author of the present work decided to face up to.

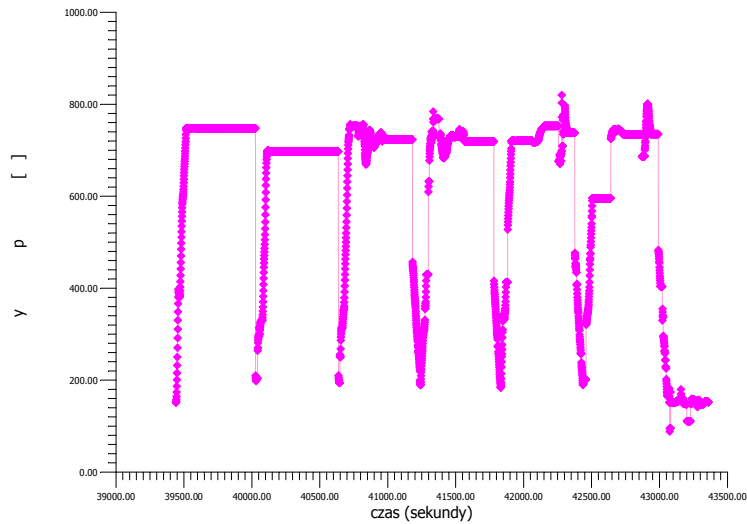


Fig. 14. Real-time ellipsoidal heights of the GG24 antenna

2.2. Studies on An-2 aircraft positioning

The experiment concerning determining the flight trajectory of the An-2 aircraft using GPS satellite system was carried out at Dęblin military airfield²⁴ [Grzegorzewski 2001] on 7th June, 2000. The main objective of the experiment was to use satellite DGPS and On-the-Fly (OTF) techniques during en-route, approach and landing phases of the flight. Precise aircraft trajectory was determined at one-second intervals with centimetre accuracy in WGS'84, using precision OTF technique in the "Post Processing" mode. OTF position fixes were subsequently compared with DGPS real-time, DGPS post processing as well as with autonomous position fixes.

2.2.1 Material and methods

Since the experiment involved determining of real-time DGPS positions of the aircraft, it was necessary to set up a reference station, with precisely determined position co-ordinates (REF1) to transmit pseudo range corrections. The reference station, REF1 was allocated on the roof of the Air Navigation Department of the Polish Air Force Academy, and its co-ordinates were calculated during a static observation session. During the static observation session, two reference points - 0002 and 0004 - were used. Their co-ordinates were linked to ETRF'89 at earlier research studies in 1999 (Grzegorzewski et al., 2000). Observation data were processed with *GPSS* (Ashtech, 1993) and *Fillnet* (Ashtech, 1992) software.

Upon completion of the static session, a satellite receiver - Ashtech Z-FX connected with the Satellite 2AsxE radio-modem – was placed in REF1. The reference station was recording the observation data (files B, E and S) and was transmitting real-time DGPS corrections in international RTCM SC-104 format (message 1 and 3). In order to conduct accurate analysis a set up of two additional base stations was necessary. Reference points 0003 and 0004, situated laterally and on the extension of the runway (Fig. 16), were used to that end. Both stations were working only in the "post-processing" mode. The WGS'84 co-ordinates of base stations are shown in Tab.1.

Tab. 1.

Co-ordinates of reference points in WGS'84 used in the experiment.

Point No.	Latitude (B)	Longitude (L)	Ellipsoidal Height (h)
REF1	51° 33' 16'',78093	21° 52' 03'',19967	167,875
0002	51° 34' 06'',14137	21° 54' 43'',31047	152,498
0004	51° 32' 08'',79107	21° 53' 06'',02778	150,634

The An-2 aircraft (Fig. 15), supplied for the experiment by the Polish Air Force Academy in Dęblin was furnished with the following instruments:

- dual-frequency satellite receiver Ashtech Z-Surveyor;
- GPS satellite antenna;
- Satellite 2AsxE radio-modem for data transmission in both directions.

(send/receive capability)

The satellite receiver and the radio-modem were installed in the passenger cabin to make their assembly and disassembly process easier and to facilitate easy switch on/off operation. The satellite antenna was attached to a special frame in the cockpit. The GPS receiver was connected with a radio-modem, which was receiving DGPS corrections from the base station in real time.



Fig. 15. An-2 aircraft used in the experiment.

Dual frequency satellite receiver used in the experiment is a high quality precision instrument. Satelline 2ASxE (Astor, 1997) radio-modems, transferring the data in the duplex mode, on frequency 438.7 MHz at 9600 bps, represent state-of-the-art technology. The only parameter limiting the efficiency of the system in real time was the relatively low output power of the radio-modem, 1.0 watt, which made impossible to perform RTK observations .

Observation conditions (constellation, its geometry, etc.) were analysed prior to the experimental flight using *Mission Planning* (Ashtech, 1998b) software. The flight commenced on 7th June, 2000 at 11:52:17GPS time and lasted 1542 seconds - until 12:18:00. Within that timeframe, the aircraft circled the airfield twice (the flight route and the arrangement of the three reference stations are shown in Fig. 16).

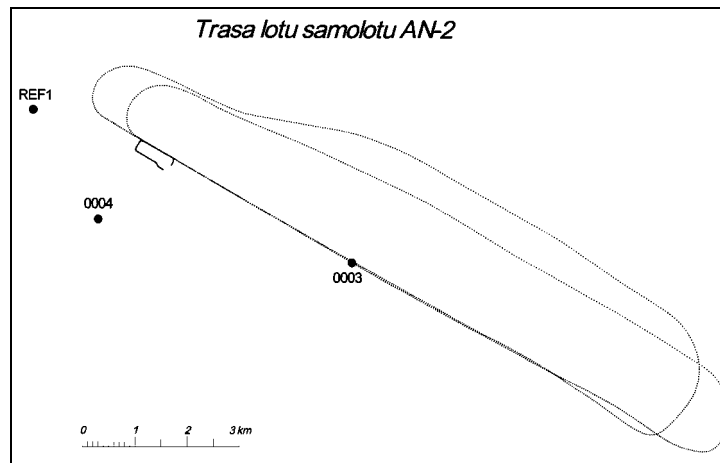


Fig. 16. An-2 flight route and the arrangement of three base stations.

During the flight, the number of available satellites and the PDOP values were relatively stable, which is illustrated in Fig. 17.

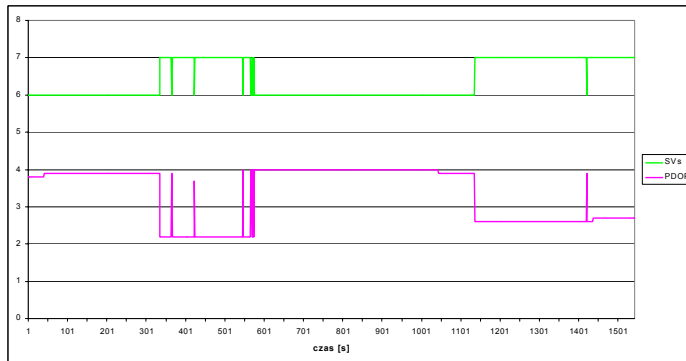


Fig. 17. Number of available satellites and PDOP values during the flight.

2.2.2 Assessing the parameters of the navigation system

In order to compare the accuracy of various GPS techniques it was necessary to have the knowledge of accurate and credible, “real” positions of the aircraft. Such position was calculated for each second of the flight as an arithmetic mean of three independent OTF position fixes in the “post-processing mode. The computations were conducted using *PNAV* (Ashtech, 1998c) software in order to determine the aircraft position for each epoch of the flight in relation to three independent base stations - REF1, 0003, 0004. Three autonomous OTF positions facilitated determining the mean error for each averaged position at each second of flight. Mean position errors were calculated separately for each co-ordinate, B, L, h (ellipsoidal height) from formula 1.

Mean errors B, L, h for the remaining phase of the flight were as following: $m_B = 0,010$ m, $m_L = 0,005$ m, $m_h = 0,046$ m. The results gained for each second of the flight are presented in Fig. 18

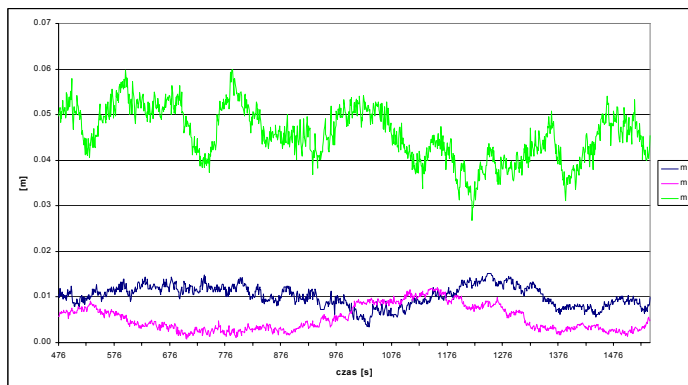


Fig. 18. Mean errors of geodesic co-ordinates B, L, h calculated at each second of the flight.

Having a precise aircraft position for each second of flight, it was possible to estimate the accuracy of the DGPS method and the autonomous positioning method. DGPS was used both in the “post-processing” and in the real-time mode.

The comparison was started from the assessment of the accuracy of determining of the aircraft trajectory using **DGPS in the “post-processing” mode**. DGPS „post-processing” positions were computed using *PPDIFF* program that was an element of the *GPSS* software (Ashtech, 1993). The aircraft trajectory was determined on the basis of arithmetic means from three independent positioning data. The error of geodetic co-ordinates B and L drawn with the method mentioned above was almost constant. Mean value was equal to 0.18 m for B and 0.22 m for L respectively, with maximum values of 0.65 m and 0.64 m. The observed errors of ellipsoidal height (h) had larger values. The mean error of h was equal to 0.64 m, with the maximum of -1.93 m. It should be underlined that DGPS “post-processing” position is very stable in safety-critical flight phases such as the approach and landing. Differences between the co-ordinates obtained by DGPS in the “post-processing” mode and the OTF aircraft position are shown in Fig. 19.

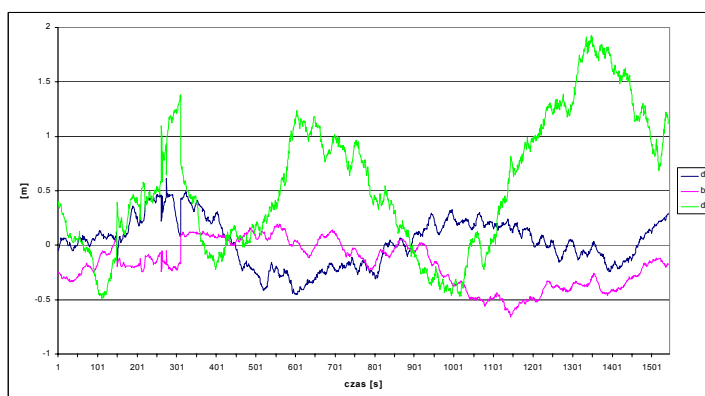


Fig. 19. Differences between „real” (OTF) aircraft co-ordinates and the co-ordinates obtained by DGPS in the “post-processing” mode.

The next stage was analysis of an accuracy of aircraft trajectory determination using **DGPS in the real-time mode**. Mean values of OTF co-ordinates were compared with the co-ordinates recorded by the rover type receiver in real time (C-file) The main problem encountered, was too low radio-modem output power (1.0 watt). Radio-modem was transmitting the DGPS corrections. For that reason the observation data file contains both the DGPS positioning data and the segments where the positioning accuracy was too low, due to the lack of differential corrections. This is particularly in evidence as far as the ellipsoidal height of the aircraft is concerned.

Mean B and L errors were 0.68 m and 0.25 m respectively, with the maximum values of 2.93 m and 0.69 m, when the corrections were unavailable. The fact that differential corrections were unavailable within some periods of time is much more strongly reflected on the h co-ordinate, for which the mean error was 4.71 m and the maximum value as much as 17.92 m. Comparison of co-ordinates obtained by DGPS in the real-time mode with the OTF aircraft position is shown in Fig. 20.

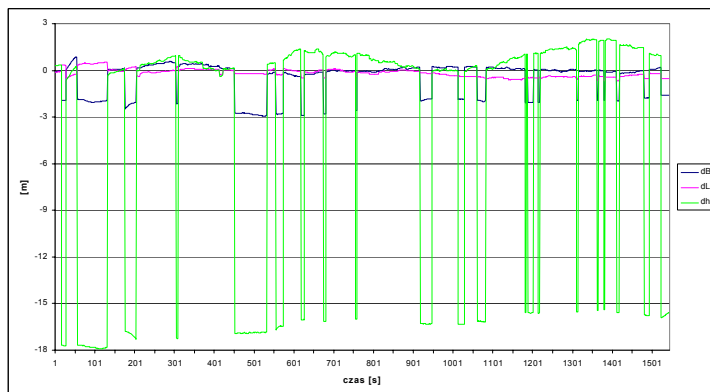


Fig. 20. Differences between „real” (OTF) aircraft co-ordinates and co-ordinates obtained by DGPS in a “real-time” mode.

The same case was analysed when all epochs in which differential corrections coming from the reference station were unavailable. All this epochs were rejected and not taken into consideration. Availability of DGPS method was estimated from the formula:

$$D_{\text{DGPS}} = \frac{\text{DGPS}}{\text{GPS}} \cdot 100\% \quad (2)$$

In our case parameter (2) was equal to 75%. Taking into consideration only those epochs in which the corrections were received, the results obtained were similar to those obtained in the “post-processing” mode. Mean B and L errors were 0.17 m and 0.22 m respectively, with the maximum values of 0.86 m and 0.61 m. The observed errors of ellipsoidal height (h) were a little bit larger. The mean error of h was 0.77 m with the maximum of -2.05 m. The comparison of DGPS real-time co-ordinates (only for these epochs in which the differential corrections were received), with the OTF positions are shown in Fig. 21.

The next stage was analysis of the accuracy of aircraft trajectory determination using **autonomous positioning**. Mean values of OTF co-ordinates were compared with the co-ordinates determined from the observation file (B-file) using BSHOW software from Ashtech.

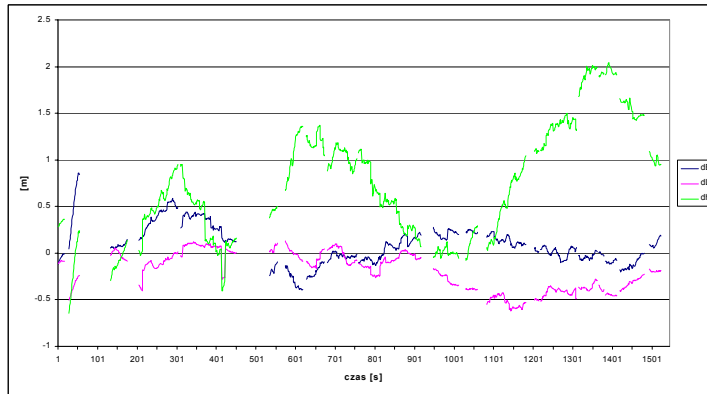


Fig. 21. Differences between „real” (OTF) aircraft co-ordinates and co-ordinates obtained by DGPS in a “real-time” mode (epochs with received corrections only).

Obtained results show very stable error values for each co-ordinate. The mean value of B error was 7.24 m with the maximum value of -9.36 m. The error of L was in this case considerably lower, having the mean value of 0.44 m and the maximum of -1.55 m. The mean error of h was 21.46 m and its maximum -25.80 m. The comparison of autonomously determined co-ordinates of the aircraft with its real position is shown in Fig. 22.

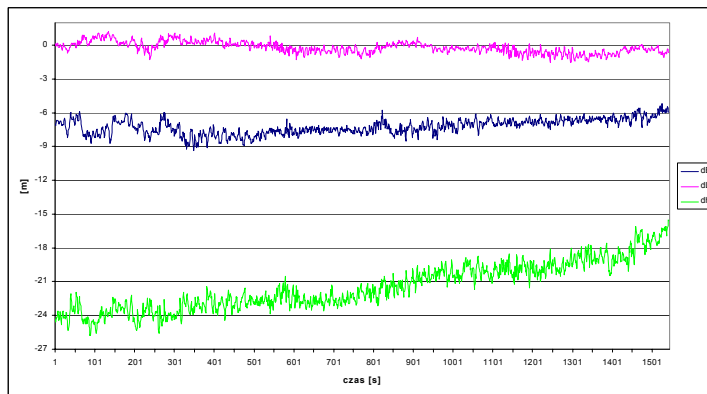


Fig. 22. Differences between „real” (OTF) aircraft co-ordinates and co-ordinates obtained in autonomous positioning.

Conclusions:

Summarizing, the research studies concerning the accuracy of position determination of An-2 aircraft we can assume that GPS techniques are very useful in air navigation. DGPS real-time method in particular can meet the accuracy requirements in aviation. (additional condition - an advanced radio modem with high output power has to be used).

2.3. ACCURACY OF TS-11 ISKRA AIRCRAFT TRAJECTORY DETERMINATION DURING EN-ROUTE FLIGHTS USING RADARS AND SATELLITE NAVIGATION SYSTEMS ACCORDING TO THE „DUNAJ” PROGRAM (EXPERIMENT DĘBLIN 2001)

A substantial change in views concerning the utilisation of satellite technique can be expected upon completion of the comparing process of satellite positioning results with the positioning data obtained by means of the radar technique. Therefore, the series of new experiments were planned, to use several satellite receivers, both onboard and on-the-ground, in a single trial and combine the results with radar observation data. A question whether the radar technique or GPS system is better (more accurate) in aircraft navigation was the main reason why a new quality of en-route flight studies were started.

The experiment conducted in co-operation with Industrial Telecommunication Institute in Warsaw and the Department of Satellite Geodesy and Navigation, University of Warmia and Mazury in Olsztyn was the next stage in the research studies concerning flight positioning accuracy. The Polish jet trainer TS-11 Iskra aircraft was used in the flights which were conducted at en-route altitude $H = 4,250$ m to $7,200$ m and speed $V = 420 - 480$ kph. During the flights the aircraft was fully tanked and had no external load. The following flight conditions were set: cloud coverage 10/10, cloud base: 300 m, wind force: 20 m/s, cross wind component 10 m/s.

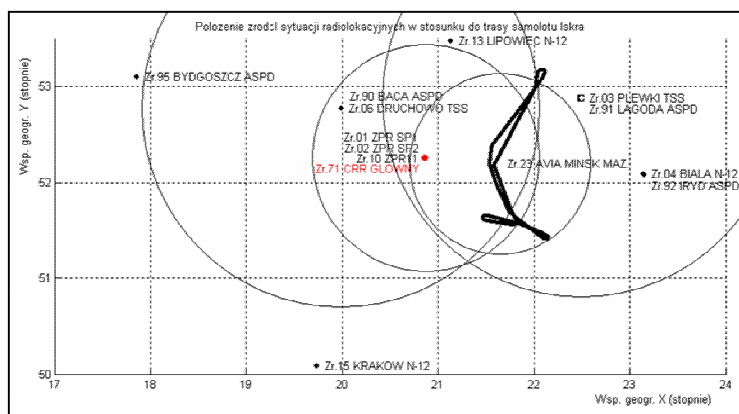


Fig. 23. Location of radar data sources during flight No. 1 of TS-11 Iskra aircraft.

The object of the study remained unchanged i.e. to determine the flight trajectory of the TS-11 Iskra aircraft using satellite navigation systems and assessing the time intervals in which the GPS information was unavailable.

Because of the requirements of the radar users, the distance between the extreme points of the flight route was 200 km.

Comparing the flight trajectory recorded by the Z-Surveyor satellite receiver with that recorded by a prototype of new radar opened new perspectives. It was an entirely new experience in the history of measurements carried out at the Polish Air Force Academy. This study presents the results of the comparisons carried out on the basis of flights No. 1 and 3. The location of radar data sources are shown in Figs. 23 and 24, which also present the plot of the flight No. 1 and 3 trajectories.

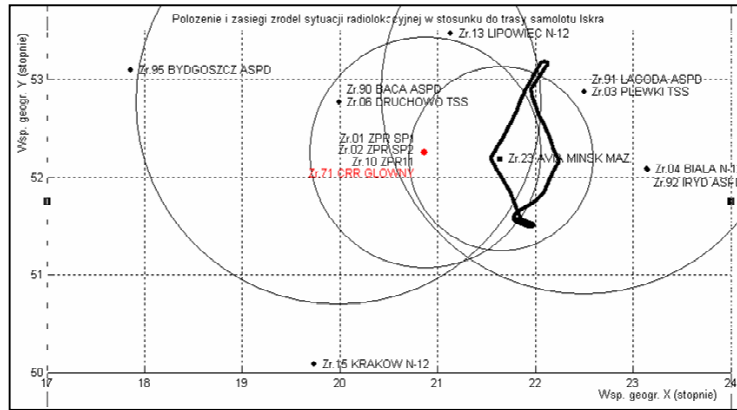


Fig. 24. Location of radar data sources during flight 3 of TS-11 Iskra aircraft.

2.3.1. Assembly of receivers onboard the aircraft

Prior to an experiment, working plan of onboard assembly and disassembly of the GPS receiver, antenna and radio modem was prepared in accordance to the appropriate procedures and formal requirements.



Fig. 25. Assembly of GPS antenna onboard the TS-11 Iskra aircraft

Documentation²⁵ [Grzegorzewski 2001] prepared, was submitted for and was officially authorised by the Chief Engineer of the Polish Air and Air Defence Forces. Following the plan, Z-Surveyor satellite receiver was mounted in the rear cockpit on the starboard side of TS11 Iskra aircraft (Fig. 26)²⁶ [Instrukcja pilotowania lot 1851/78]. The GPS antenna was mounted in the front cockpit, in place of the ASP-3NM-1 device, which was removed (Fig. 25).

Six reference ETRF'89 points along the flight route were designed in order to ensure the monitoring of the flight route. They were situated in the vicinity of Dęblin airfield, along the runway axis near the outer marker in both main and secondary landing directions, which resulted from the higher requirements of approach accuracy.



Fig. 26. Assembly of the GPS receiver and the radio modem onboard the TS-11 Iskra aircraft

The radio modem and the GPS antenna were installed between the ejection seat and the starboard side of the rear cockpit, opposite the 12th former (Fig. 26).

The experiment assumed RTK and DGPS real-time positioning of the aircraft, hence it was necessary to set up an “active” station transmitting RTK/DGPS corrections to one of the selected reference points. That point was REF1, stabilised on the roof of the Air Navigation Department building. During two days of flights, the base station, which was equipped with the Ashtech Z-12 satellite receiver connected with the Sateline 2Asx E radio modem and the 10 W SATELGAIN Plus Booster, was transmitting RTK corrections on the first day and DGPS corrections on the second day of flights.

2.3.2. Experiment description

The experiment was conducted on 25th-26th April, 2001. The reference stations (Figs. 28, 33) and the onboard GPS receiver recorded observations with one-second time increment. Six reference points were used in the experiment in order to determine the aircraft trajectory in WGS'84:

- 0002 – Dęblin;
- 0003 – Dęblin;
- REF 1 – antenna mounted on the roof of the Air Navigation Department;
- BOGO (Borowiec);
- IGIK – Warszawa;
- 4708 – Płoszyce.

Z12 receivers were mounted at REF1, 4708, BOGO and IGIK stations while at 0002 and 0003, Z-Surveyor receivers were used.

Ashtech Z-Surveyor receiver was also mounted onboard the TS-11 Iskra aircraft, following the plan authorised by the Headquarters of the Polish Air and Air Defence Forces. Co-ordinates of the aircraft were calculated independently at one-second time increment by each reference station using OTF and DGPS techniques. Computations were carried out using GeoGenius 2000 program. The subsequent phase of the experiment comprised the assessment of accuracy parameters using various GPS techniques. Because of the fact that the distance between the extreme points of the flight route was 200 km, the accuracy studies were divided into two stages.

The second stage of the experiment, concerning the approach and the landing (glide path) phase is discussed in Chapter 3. Unified system of co-ordinates – WGS'84/GPS time was applied throughout the experiment both for radar installations and the GPS receivers involved in the trial.

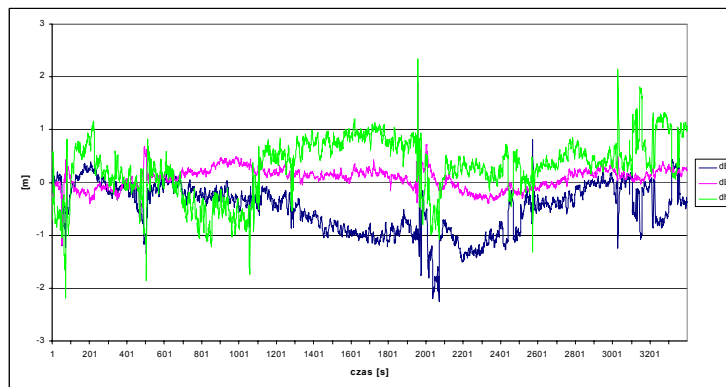


Fig. 27. Differences between the OTF aircraft co-ordinates and the DGPS „post-processing” co-ordinates in the en-route phase.

The experiment was carried out at the Dęblin military airfield and it comprised of four flights. The problem of radar-positioning accuracy is analysed in point 2.3.4.

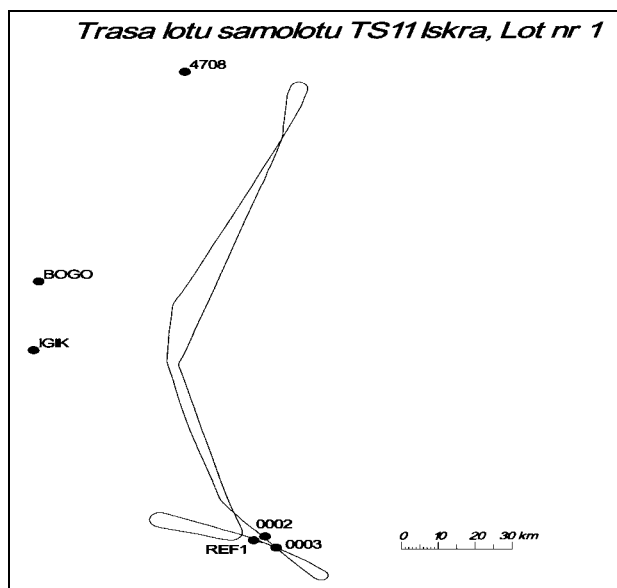


Fig. 28. Flight trajectory of TS-11 aircraft and the arrangement of six reference stations Flight No. 1.

2.3.3. Technical characteristics of the flight

In order to execute the flight in accordance with planned schedule, the following technical characteristics of the project were assumed:

- *Crew*: Class 1 test pilot (two test pilots).
- *Flight altitude*: 4250 – 7200 m.
- *Mission scenario*: stick to the assigned trajectory, speed $V = 420 - 480$ kph, and flight altitude in accordance to the commands of the air controller
- *Minimum weather conditions*: Cloud coverage 10/10, cloud base: 300 m, wind force: 20 m/s, cross wind component 10 m/s, visibility 4 km.
- *Execution guidelines*: Execute flights fully tanked without external load. Maintain radio contact with Air Traffic Control and the forward air controller in accordance with the radio procedures of the Polish Air Force. Monitor fuel consumption during all flight phases keeping in mind necessity of returning to Dęblin from the most distant point of the flight route. In the event of icing, follow the Flight Technique Manual.

Because of the much longer flight distance, the analyses of the en-route phase were based on the data received from all reference points situated along the flight route (REF1, 0002, 0003, IGIK, BOGO, 4708). It was noted that the position fixes calculated by the three Dęblin stations are significantly closer (due to the fact that ref. stations are situated much closer to one another) than the position fixes calculated by the remaining stations. The mean position fix was calculated from the data supplied by the Dęblin stations (REF1, 0002, 0003) and was treated as a position fix computed by a single reference station (DEBL) in order to avoid false weighing of the output data.

Finally, the aircraft position was computed as an arithmetic mean of four independent (DEBL, IGIK, BOGO, 4708) OTF positions in the “post-processing” mode. Position accuracy analysis during the take-off and en-route phases was performed using the surplus data.

The analyses were started from determining the accuracy of the aircraft position during flight No. 1. The flight trajectory and the arrangement of the six reference stations are presented in Fig. 28. The analysis started from the assessment of the observation conditions during the flight, which is shown in Fig. 29. As we can notice, PDOP value was never smaller than 2 and its maximum value reached 6. The number of available satellites during flight No.1 ranged from 7 to 10, which guaranteed accurate determination of the flight trajectory.

Precise flight trajectory was then determined together with the assessment of the main position errors of each co-ordinate, B, L and h during the take-off and en-route phases of flight. The aircraft position was computed as an arithmetic mean of four independent (DEBL, IGIK, BOGO, 4708) OTF positions in the “post-processing” mode. The computations were carried out with GeoGenius 2000 computer application.

Position accuracy analysis during the take-off and en-route phases was conducted using the surplus data. The analysis consists of determination of the mean position errors, which were calculated separately for each co-ordinate, B, L, h (ellipsoidal height) - formula (1).

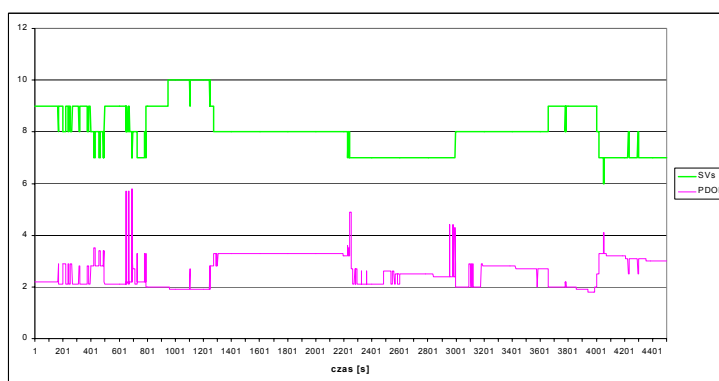


Fig. 29. Number of available satellites and the PDOP value during flight No.1

Mean errors of the B, L, h co-ordinates during the take-off and the en-route phases of the flight were finally equal to: $m_B = 0,34$ m, $m_L = 0,17$ m, $m_h = 0,32$ m. The results obtained for each second of the flight are presented in Fig. 30. The accuracy is worse by one order than accuracy obtained during other experiments. It is an outcome of large distances between the extreme points of the flight trajectory (close to 200 km).

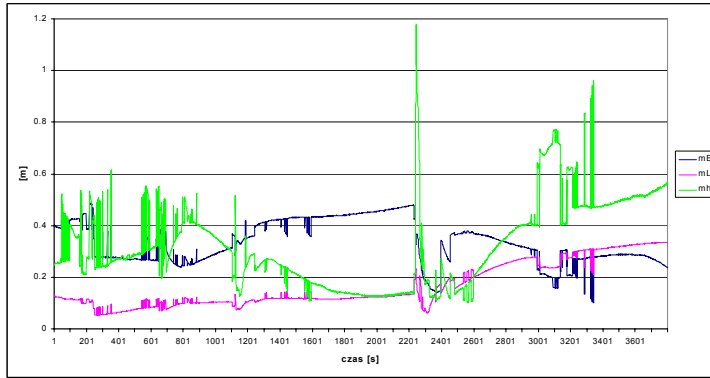


Fig. 30. Mean errors of geodetic co-ordinates B, L, h calculated at each second of en-route phase of the flight.

Having a precise aircraft position for each second of the flight, it was possible to estimate the accuracy of the DGPS method and the autonomous positioning method. The analysis was performed for en-route phase of the flight.

The comparison process was started from the assessment of the accuracy of determining the aircraft trajectory using DGPS in the “post-processing” mode. DGPS „post-processing” positions were calculated, using the GeoGenius 2000 software, as an arithmetic average value of four independent positionings. Position errors of B, L and h were for that method similar in both analysed flight phases. The mean errors for these flight phases were as follows: 0.36 m for B; 0.13 m for L and 0.56 m for h. The maximum errors were: 2.75 m; 1.55 m and 4.58 m, respectively. Taking into consideration the dynamics of the flight, accuracy of the order of 1-2 metres seems to be sufficient. DGPS positioning in the real-time mode, gives the same accuracy order, i.e. 1-2 m. But in that particular case, the problem of differential correction transmission by ground stations should be solved. There is also possibility to get corrections from the SABS (*Space Based Augmentation System*), such as EGNOS in Europe, WASS in America, or MSAS in Japan. The use of EGNOS system in determining the position of moving vehicles is the subject of the author’s future research. Differences between DGPS „post-processing” co-ordinates and the OTF position of the aircraft are shown in Figs. 31 and 32. The subsequent stage of the analysis concerns assessing the accuracy of aircraft trajectory determination using **autonomous positioning**. Mean values of OTF co-ordinates were compared with co-ordinates determined from the observation file (B-file) using the *BSHOW* software. The analysis dealt with the entire flight, since neither the number of the reference stations, nor their arrangement was relevant to the case taken into consideration.

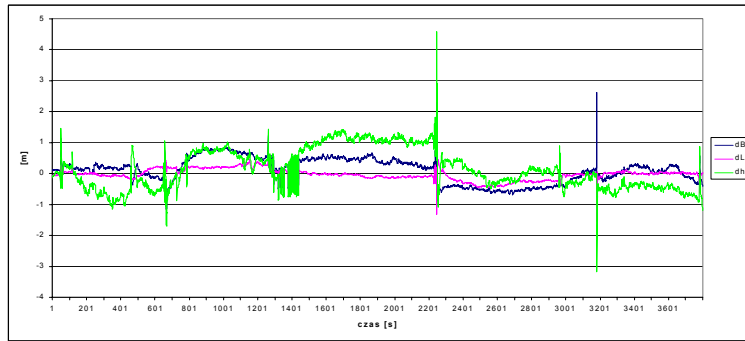


Fig. 31. Differences between the OTF aircraft co-ordinates and the DGPS „post-processing” co-ordinates in the en-route phase.

The obtained results show rather stable error values for the horizontal co-ordinates B, L, (latitude and longitude), whereas the observed errors of ellipsoidal height were considerably more varying. The mean value of B error was 7.56 m with the maximum value of - 15.53 m. The mean error of L was in this case 22.06 m and its maximum - 28.27 m. However, the mean error of h was 27.91 m and its maximum - 47.24 m.

The comparison of autonomously determined co-ordinates of the aircraft with its real position is shown in Fig. 32.

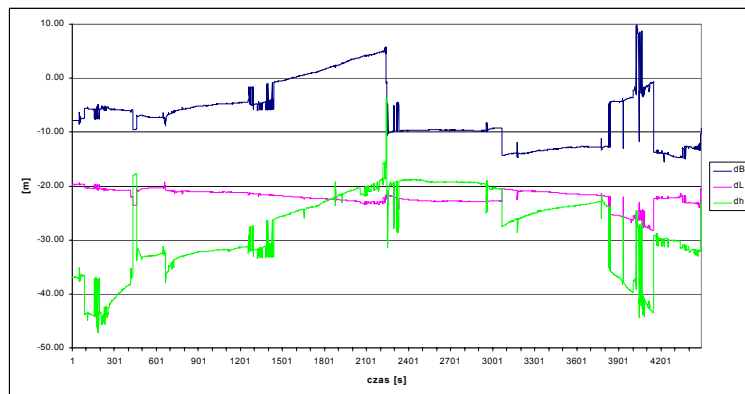


Fig. 32. Differences between OTF aircraft co-ordinates and its co-ordinates obtained in autonomous positioning.

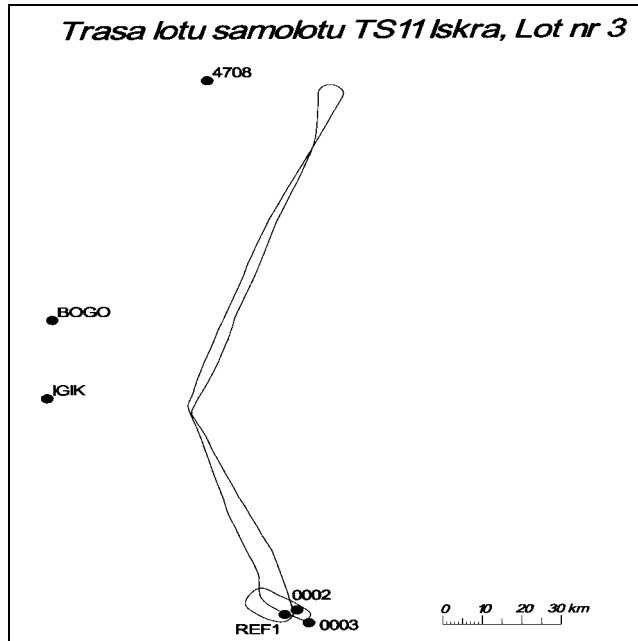


Fig. 33. Flight trajectory of TS-11 aircraft and the arrangement of six reference stations – flight No.3.

The trajectory of flight No. 3 was different from that of flight No.1. The number of reference stations and the radar network used for monitoring that flight were identical as in the case of flight No.1. The parameters of the TS-11 Iskra flight were also identical. The number of available satellites and the PDOP value during flight No.3 is shown in Fig. 34. The maximum value of PDOP was 6, and the number of available satellites ranged from 5 to 9.

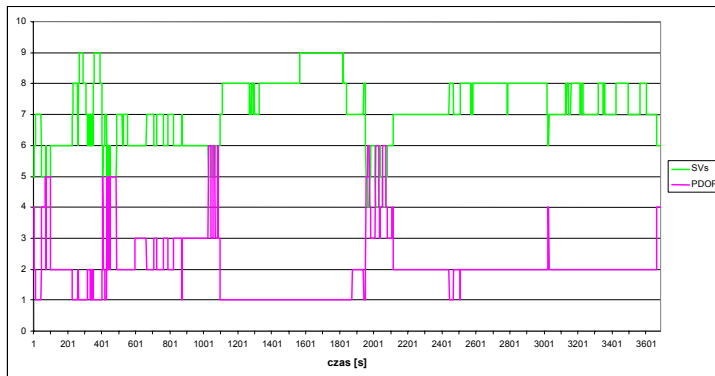


Fig. 34. Number of available satellites and the PDOP value - flight No.3.

Accuracy of the flight trajectory was assessed with DGPS in the “post-processing” mode (as it was performed for data from flight No.1). The computations were conducted with the same GeoGenius 2000 software.

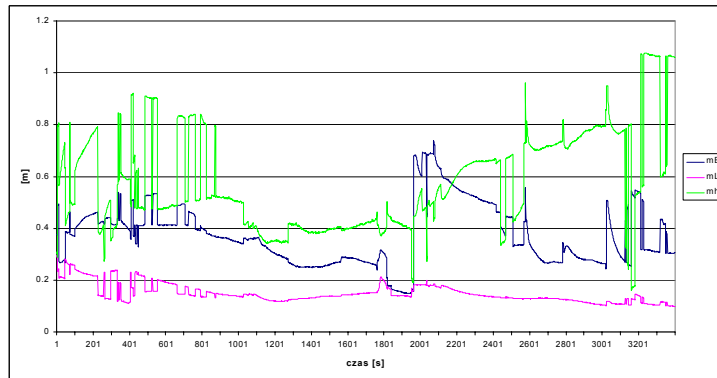


Fig. 35. Mean errors of geodetic co-ordinates B, L, h at one-second interval of the en-route phase - flight No.3.

Maximum error values of B, L and h in the en-route phase were as following: mB – 0.7 m, mL – 0.3 m, mh – 1.15 m (Fig.35).

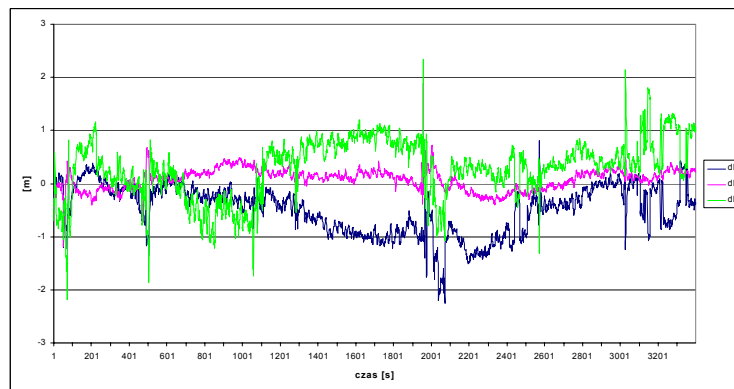


Fig. 36. Differences between the OTF aircraft co-ordinates and the DGPS „post-processing” co-ordinates in the en-route phase – flight No. 3.

Differences between OTF aircraft co-ordinates and the co-ordinates obtained by DGPS in the “post-processing” mode are considerably stable. The maximum error values are : dh –(+ 2 m), dL – (0,8 m), dB – (-2,2 m) (Fig.36)

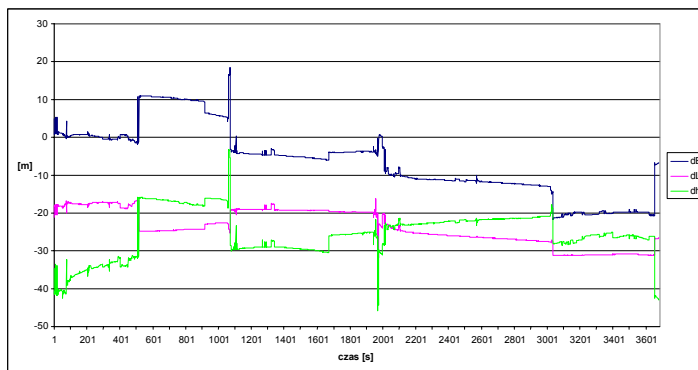


Fig. 37. Differences between OTF aircraft co-ordinates and co-ordinates obtained in autonomous positioning – flight No.3.

The differences between OTF aircraft co-ordinates and co-ordinates obtained in autonomous positioning are shown in Fig. 37.

The value of dh ranged from -45 m to -20 m, dL from -30 m to -16 m, dB from -20 m to $+19$ m.

2.4. ACCURACY OF TS-11 ISKRA AIRCRAFT TRAJECTORY DETERMINATION DURING EN-ROUTE FLIGHTS USING RADAR PROTOTYPE AND SATELLITE NAVIGATION SYSTEMS ACCORDING TO THE „BRDA” PROGRAM (EXPERIMENT DĘBLIN 2001)

New generation radars prepared for the experiment facilitated the analysis of aircraft positioning accuracy and comparison of the radar-obtained data with that recorded by the GPS receiver. The experiment, conducted in accordance to the „Brda” program²⁷, was carried out adopting the flight conditions from the previous experiments. Due to different tactical capabilities of the new radar, the following technical assumptions were made.

2.4.1. Materials and method

Due to the technical characteristics of the radar, the flight route in this experiment was planned as following:

Dęblin – Garwolin – Warka – Jedlnia – Zwoleń – Dęblin (according to the map of navigation).

Planned flight altitude:

- 1,000 m;
- 3,000 m;
- 5,000 m

Planned flight speed:

- 300-400 kph



Fig. 38. Navigation map used during the experiment

The objective of the experiment was to record the position of the flying aircraft, using two independent information sources, compare the obtained trajectories and assess the error value as well as to estimate the possibility of utilising the satellite technology in navigating the aircraft.

2.4.2. Schedule of the experiment

- Conducting three GPS observation sessions of the TS-11 Iskra aircraft during flights along the route: Dęblin – Garwolin – Warka – Jedlnia – Zwolen – Dęblin.
- Conducting radar observation sessions of the TS-11 Iskra aircraft.
Flight No.1 4th June, 2003, 11:09:01 – 12:55:00 (GPS time).
Flight No.2 5th June, 2003, 11:54:01 -13:45:00 (GPS time).
Flight No.3 6th June 16:45:01 -18:40:00 (GPS time).
- Computer processing of the GPS observations.
- Conducting comparative analyses of the radar- and GPS-recorded flight trajectories.

2.4.3. Description of the experiment

GPS measurements were carried out on 4th and 5th June, 2003 aiming to determine precise position of the Ashtech Z-Surveyor antenna allocated in the rear cockpit of the TS-11 Iskra aircraft, Fig. 39²⁸. During the flight, the aircraft positions were recorded at one-second time increment. Five reference points, with co-ordinates linked to ETRF'89 and allocated along the flight route were used in the experiment. The positions of the reference points were “denser” in the vicinity of an airfield (three reference stations), due to the much more strict accuracy requirements concerning the approach and landing flight phases. The network of reference points was made up of

five points with the following numbers. REF1 (Dęblin – roof of the Air Navigation Department), 0002 (Dęblin), 0003 (Dęblin), 2705 (Siedlce), REF0 (Siedlce) Tab. 1. The present work deals only with the results obtained in flight No. 2.

Tab. 2. Co-ordinates of reference points used in the experiment, in WGS'84

Point No.	Latitude (B)	Longitude (L)	Ellipsoidal Height (h)
REF1	51° 33' 16'',78093	21° 52' 03'',19967	167,875
0002	51° 34' 06'',14137	21° 54' 43'',31047	152,498
0003	51° 31' 40'',22063	21° 57' 16'',52972	152,508
2705	51° 37' 18'',58819	21° 12' 32'',62314	189,206
REF0	51° 37' 17'',16831	21° 12' 24'',58086	187,991



Fig. 39. Location of the GPS antenna during experimental flights.

Basic measurement parameters:

- minimum satellite elevation angle 5° over the horizon;
- measurement time interval -1 s;
- minimum number of satellites - 3;
- PDOP < 6.

Observation conditions (constellation, geometry, etc.) were analysed prior to the measurement sessions, using *Mission Planning* (Ashtech, 1998b) software. As we can see in Figure 40, the number of available satellites during flight No. 2 was sufficient to obtain the co-ordinates of the aircraft.

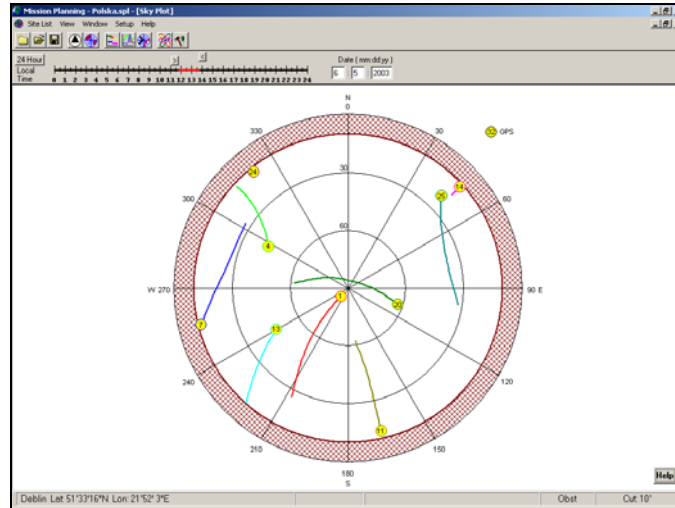


Fig. 40. Theoretical GPS satellite arrangement over the horizon during flight No.2 „BRDA” experiment

Post-processing of the observation data recorded by Ashtech Z-12 and Z-Surveyor was completed using AOSS v. 2.0 (Ashtech Office Suite for Survey) software. For each experimental session, independent calculations of aircraft positionings were performed for each of the five reference points – REF1, 0002, 0003, 2705 and REF0. Co-ordinates were received in the format containing measurement epoch, latitude and longitude, ellipsoidal height, position RMS and three velocity components in WGS84 reference system.

For the sake of the present study, flight No. 2 was analysed B, L and h coordinate errors obtained from both radar and GPS measurements are the main subject of analysis process .

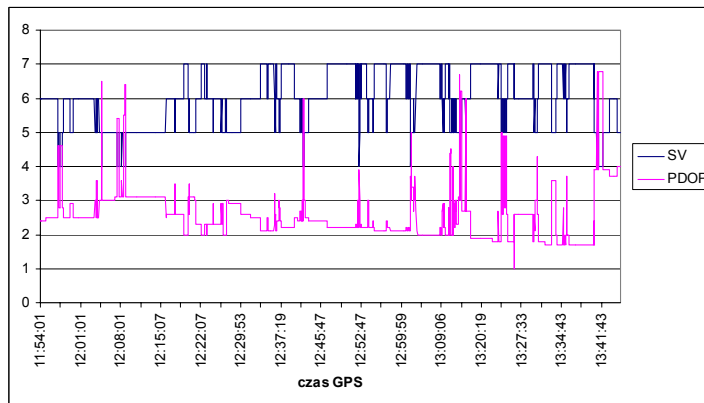


Fig. 41. Actual satellite conditions of the GPS system during flight No.2 „BRDA” experiment

As we can see (Fig. 40), the value of PDOP during flight No.2 was between 2 and 6 meters and the number of available satellites ranged from 5 to 7.

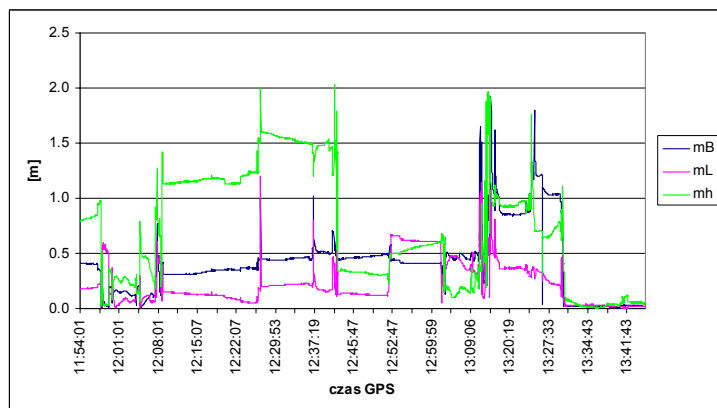


Fig. 42. Graphic representation of B, L and h co-ordinate errors calculated for each second of flight No.2

Figure 42 presents errors of all geodetic co-ordinates. Mean latitude error mB did not exceed 1.5 m while mean longitude error mL ranged between 0.25 and 1 m, reaching the latter value in two cases. Mean error of ellipsoidal height mh was the most interesting, falling between 0.25 and 2 m.

2.4.4. Conditions of experiment – flight No.2

Flight No.2 was conducted within the framework of qualification tests of the „BRDA” radar.

The data obtained with that radar went through the following operations:

- route calibration time shift (16 s);
- time base starting point shift (~ 400 m);
- route azimuth shift (0.7°);
- applying a filter, being a square window with the side length of 1,500 m, in order to eliminate the waypoints coming from landmark detection.

Flight No.2 results were analysed for the time interval 12:00:00 - 13:30:00

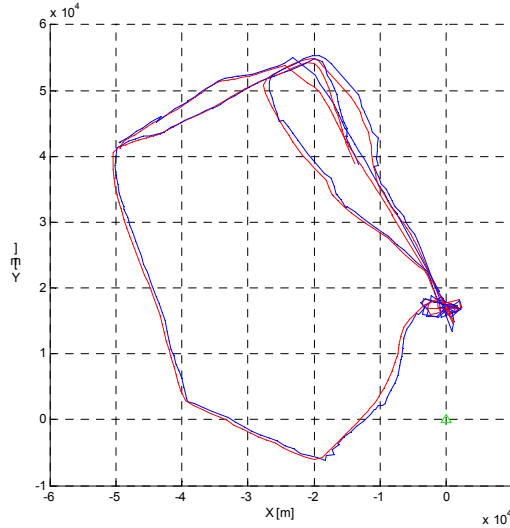


Fig. 43. 2D route recorded on 5th June, 2003 – flight No.2.

Fig. 43 shows the flight route of the TS-11 Iskra aircraft recorded on 5th June, 2003. The plot contains points recorded between 12:00:00 and 13:30:00 the red colour represents position fixes obtained from GPS measurements, while the radar-determined points are marked in blue. As it can be observed, both plots are placed very close to each other. Position differences are significantly small.

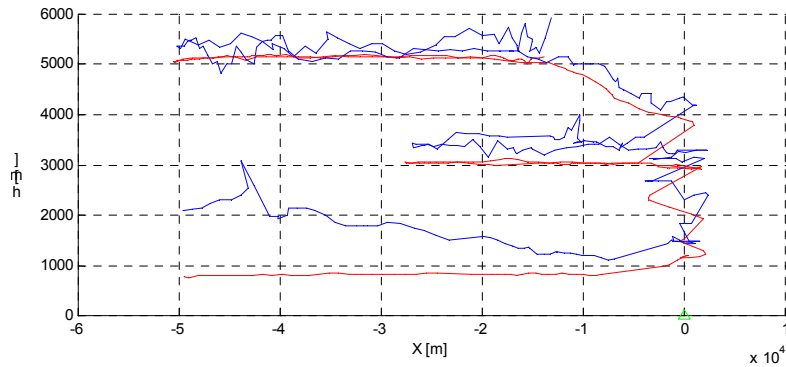


Fig. 44. Altitude estimation recorded by „Brda” radar.

Fig.44 illustrates flight altitude plots of the TS-11 Iskra aircraft recorded on 5th June, 2003 – flight No.2. The plots contain points that were recorded between 12:00:00 and 13:30:00. The red colour represents position fixes obtained from GPS measurements, while the radar-determined points are marked in blue. Radar-determined flight altitude part of the graph is characterised by instability. Altitude value differences reach even up to 1,000 m.

A 3D representation of flight No.2 is shown in Fig.45.

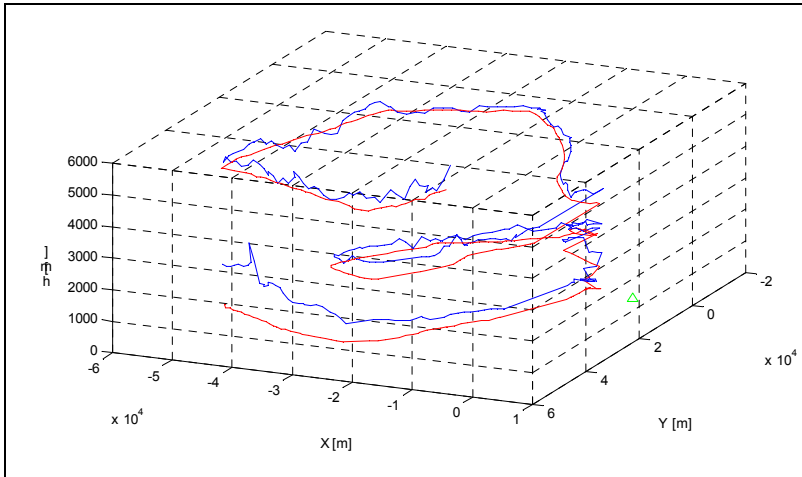


Fig. 45. 3D route recorded on 5th June, 2003 -flight No.2.

The graph contains the points recorded between 12:00:00 and 13:30:00; GPS – the red colour, radar-measured („BRDA”) – the blue colour.

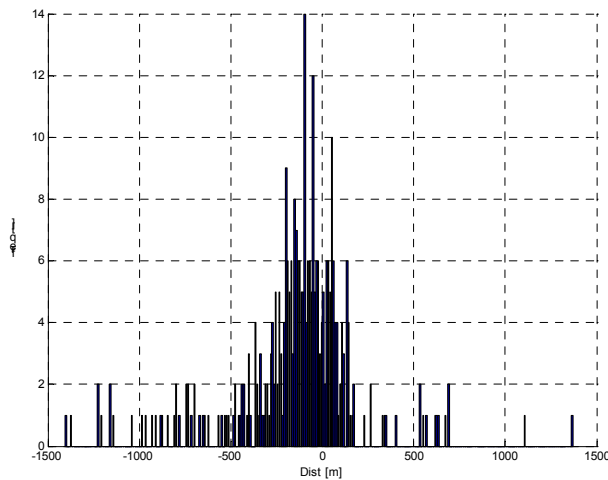


Fig. 46. Histogram of determination of distance accuracy– „BRDA” radar, 5th June, 2003. -flight No.2.

The graph contains the data of the points recorded between 12:00:00 and 13:30:00 (GPS time).

As we can see, the difference between the radar-determined altitude value and that measured with the GPS receiver was growing with the increasing distance between the aircraft and the radar. Fig. 46 shows the histogram of the distance determination accuracy in flight No.2. The following error values were recorded:

err_R_mean=-148.0559m,
 err_R_std=341.9696m,
 err_R_rms(Root Mean Square)=372.6443 m.

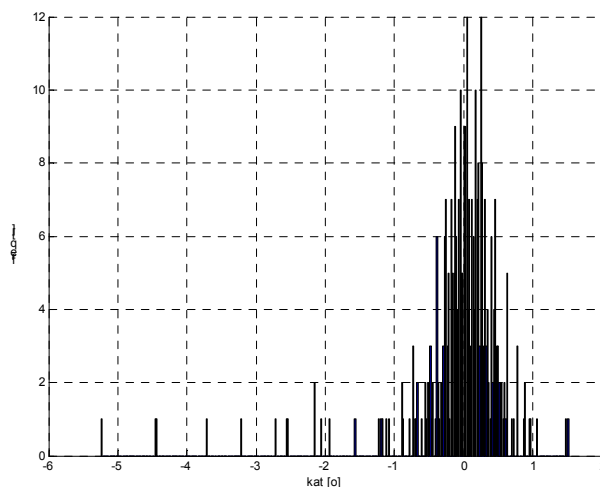


Fig. 47. Histogram of azimuth determination accuracy – „BRDA” radar, 5th June, 2003 – flight No.2.

The graph contains the data of the points recorded between 12:00:00 and 13:30:00. The azimuth accuracy was as following:

err_az_mean = -0.068907°, err_az_std = 0.741°, err_az_rms = 0.7442°.

while altitude was determined with the following accuracy:

err_h_mean = 401.0866 m,

err_h_std = 344.9444 m,

err_h_rms = 529.0152 m.

Conclusions

In-flight experiments proved the suitability of the GPS system in aircraft navigating. The old system of air traffic control supported full monitoring of a civilian aircraft in an air corridor. Such an organisation of flights is not effective due to high intensity of an air traffic. The entire air space is a national property and, consequently, it should be managed in a way to maximise financial benefits. Therefore, a new system of air traffic control organisation was proposed. First of all it requires precise determination of an aircraft position within the air space as well as continuous supply of air information. The latter condition is indispensable in ensuring safety during the approach phase and during descent phase on the glide path.

The next chapter presents the scope of experiments carried out during the approach and descent on the glide path.

3. APPROACH AND LANDING PHASE - AIRCRAFT POSITION ACCURACY ASSESSMENT

Approach and landing are the most important flight phases. Executing those phases of flight in adverse weather conditions, during the day or night is possible only with the assistance of uninterrupted work of radio navigation equipment. Existing experience in military aviation has proved that the landing system RSP-7T, consisting of DRŁ 7 and PRŁ 7 precision approach radars, and its modifications ensure safe landing (Tab. 8) so far. A very important parameter, characterising that system is time of data refreshment. In the case of the DRŁ -7 omni-directional radar, time of data refreshment is 10 s. During that time, light parameters of an approaching aircraft remain unknown, which with the vertical descent speed of 15m/s amounts to an altitude difference of 150 m. For DRŁ-7, distance determination accuracy is 450 metres, or 1% of the scale, and for PRŁ-7 from 150 to 200m. Azimuth determination accuracies are 1° and 3° respectively. Atmospheric artefacts in DRŁ and PRŁ radars, pose considerable difficulty for the air traffic controller (to make them visible on the screens of the control panels and display equipment). The precision approach landing system must also be characterised by good radio communications, since precision approach is carried out according to the air traffic controller's commands based on the data obtained by him from the precision approach radar's (PAR) displays.

Approach procedure reflects technical capabilities of the aircraft and the safety of its pilot (Fig. 48).

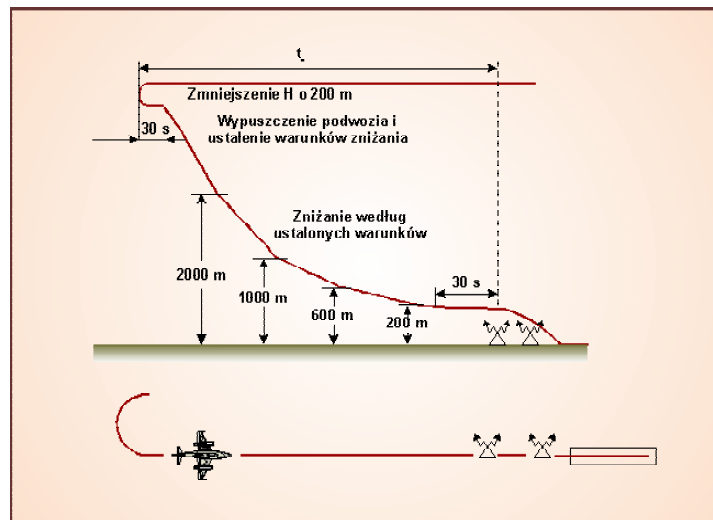


Fig. 48. Scheme of radar (PAR) assisted approach²⁹ [Niedziela 1975]

Due to the fact that most experiments involving the use of an onboard GPS receiver were carried out with the TS-11 Iskra aircraft, the scheme of a radar assisted approach concerns that particular type of aircraft. It was assumed that the aircraft would be descending in landing heading with the maximum deviation of $\pm 15^\circ$. These parameters set the air space limit for the aircraft. The value of vertical descent gradient varies as a function of the flight altitude. Vertical G load in such a flight is about 1, which points out to the fact that in that flight phase the pilot avoids abrupt manoeuvres.

If a different navigation system is used, e.g. ILS³⁰ [Loty 2003] pilot's work is based on the indications of the onboard instruments, in this case KNR.-634³¹ [Rypulak 2001]. The pilot's task is to observe the Cross Deviation Indicator (CDI), so that the needle representing the aircraft is in its centre. Such a location means that the aircraft is moving on his glide slope. The aircraft position is being given continuously. ILS is a standard system used in a civil aviation. However, the same system is used at military airfields as well.

Even though that system meets the standards of CAT I to III³² [ICAO 1995/96], it has numerous shortcomings. The most important of them is angularity of its radio beams, typically from 3° to 6° horizontally and 1.4° vertically. Consequently, the greater the distance between the aircraft and the runway threshold, the lower the position resolution for a given aircraft's CDI needle deflection can be obtained. If an airfield has two closely-placed parallel runways, then in a definite point of the approach, localiser beams of each runway will eventually overlap. For example, if the runways are separated by 750 ft. apart, the overlap at full CDI needle deflection can be equal up to 1.2 nm from the approach end of the runways. The fact that ILS system is stationary is also a very important limitation. Work continuity of landing systems is of the utmost importance for the Air Force. Therefore, the most effective landing system should be independent from ground equipment, and should support high accuracy of aircraft positioning on the glide path. It should provide continuity of work and high operational reliability.

Breakdowns in the transfer of data concerning the position of the aircraft in relation to the glide path may result in a crash. The most important parameter in the approach and landing phase is the absolute altitude of the aircraft (AA), which is its height above ground level (AGL). The second one is the error range of that parameter. The landing system based on satellite navigation systems is the best one because it can fully support requirements defined above. The basic disadvantage of using one of those systems, or all of them simultaneously, for flight monitoring is the fact that they are the property of the United States of America, Russia or the European Union. At present, the following navigation aids are commonly used in aviation: NDB (Non-Directional Radio Beacon), VOR (VHF Omni-Directional Radio Range) and DME (Distance Measuring Equipment). Those types of systems however are not reliable enough in assisting an aircraft approach in all weather conditions. Their relatively low accuracy limits the decision altitude.

Manufacturers of satellite receivers and international aviation companies set new directions in using satellite techniques for, among others, secure aircraft landing process. The GPS system is relatively cheaper and its work is independent from weather conditions³³ [Specht 1999]. When the European Satellite System (EGNOS)

and the Russian GLONASS come into life, the possibilities of assisting of an aircraft during landing phase of the flight will become considerably greater³⁴ [Instrukcja GG-24]. Experience³⁵ [Sasi 2001] gained so far points out to the possibility of developing precision GPS-assisted approach procedures.

Straight instrument approach corridors, which will be free of the angular dependence of ILS as a kind of limitation, can be created on the basis of un-augmented GPS position or on the basis of augmented GPS position from the Space Based Augmentation System. Un-augmented GPS positions suffer from the lack of integrity required in guidance for the approach and that is the main reason why they require extended monitoring. Therefore, constructors of receivers have proposed the Receiver Autonomous Integrity Monitoring (RAIM), in order to provide greater reliability of the GPS system.

The task of RAIM is to find the answers to two questions:

1. Is any GPS satellite not working properly?
2. If so, which of the satellites is the faulty one?

If GPS is regarded as a supplementary system for the classic navigation systems, it is sufficient to have a positive answer to the first question. If, however, GPS is to be the basic navigation system, both questions should be answered.

RAIM accuracy parameters have been shown in Tab. 3

Tab. 3.

Minimum operational performance standards for RAIM

Parameter	Departure	En-route	Terminal Control Area (TMA)	Initial Approach	Non-Precision Approach
Horizontal Precision Limit (HPL) [m]	220 m	740 m	740 m	220 m	220 m
Horizontal Alert Limit (HAL)	555 m	1,850 m	1,850 m	550 m	550 m
False Alert Limit [h^{-1}]	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
GPS Ephemerides Missed Detection Parameter	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$

Tab.4 contains accuracy requirements in precision (PA) and non-precision (NPA) approach.

Tab. 4.

Accuracy standards in precision and non-precision approach

Approach type/ decision height (DH)	Required position accuracy	
Non-precision approach (NPA)	Horizontal	Vertical
DH= 76.2 m	0.3 n.m.	500 m
Precision approach (PA)		
Cat I DH=30.5 m	18.2 m	7.7 – 4.4 m
Cat II DH=15.2 m	6.5 m	1.7 m
Cat III DH=0 m	4.1 m	0.6 m

What are, therefore, mean errors of geodetic co-ordinates B, L, h calculated at each second of the approach and landing phase?

During the „Powidz’99” experiment the position of the Su-22M4 aircraft on the glide path could not be recorded. Most data recorded by the satellite receiver concerned the en-route flight phase in the vicinity of the Powidz airfield.

**3.1. Approach and landing of the TS-11 Iskra aircraft
„Dunaj” project**

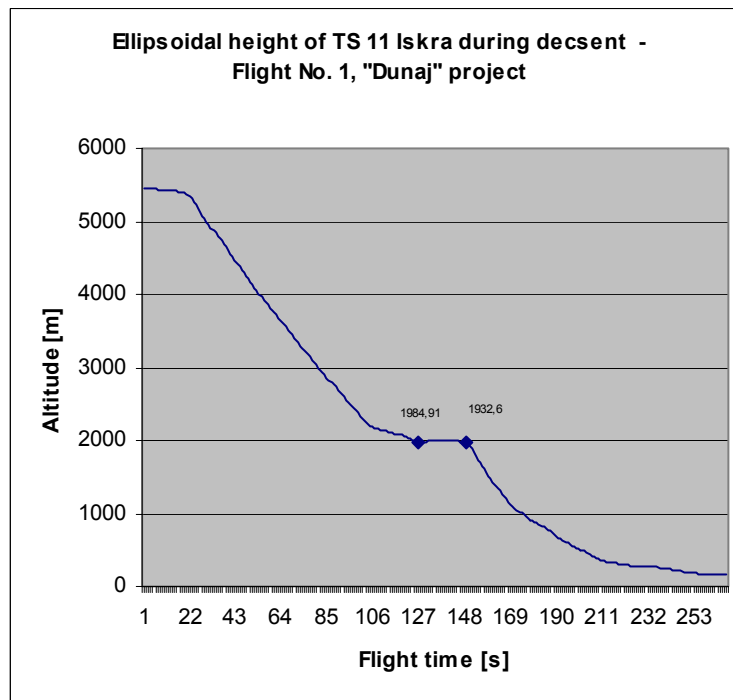


Fig. 49. Ellipsoidal height of TS-11 Iskra aircraft during descent – flight No.1 – „Dunaj” project

A total of five flights fully monitored by the ground stations were flown within „Dunaj” project. The present work describes the landing phases of flights No. 1, 3 and 5.

Fig.49 shows the glide path from the altitude of 5,500 m. The altitude is the ellipsoidal height as a function of time and the final point of the plot represents the touch-down moment. Aircraft position data was recorded at one-second interval. The plot contains the glide path segment from 1984.91 to 1932.6 m. The main co-ordinate errors within that segment were: dB - 0.09 m, dL - 0.16 m and dh - 1.12 m.

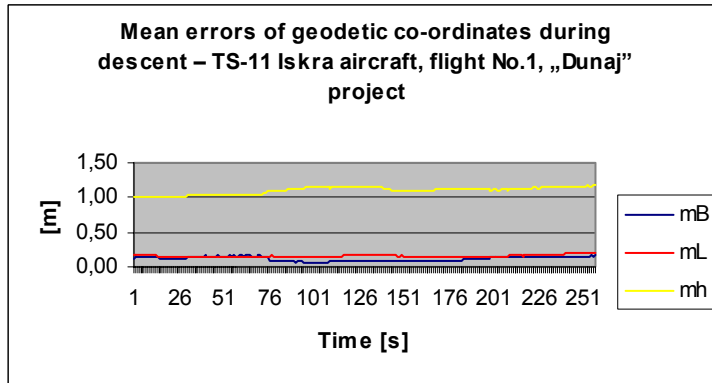


Fig. 50. Mean errors of geodetic co-ordinates during descent – TS-11 Iskra aircraft, flight No.1, „Dunaj” project

The landing phase of the flight No.1 was analysed using the following methods:

- DGPS „Real Time”.
- DGPS „Post Processing”.
- Autonomous
- DGPS/OTF

The DGPS/OTF method was assumed to be free of errors and therefore was taken as a reference for the other ones.

A) DGPS Real Time” positioning accuracy.

The main problem was the low output power of the radio modem which was sending differential corrections. The output power of the radio modem which was owned by the Polish Air Force Academy and used during an experiment was only 1.0 watt. Data files contained both DGPS position fixes and the autonomous position fixes did not include differential corrections. The lack of this portion of data means mainly the lack of aircraft’s altitude, which is the highly important factor especially on the glide path. When corrections were received, mean error values of B and L were 0.5 m. The values of mean error values of B and L increased up to 3 m when corrections were unavailable. The error of ellipsoidal height was stable and equal approximately to 1 m, with the maximum value of 2 m. When differential corrections were not

available its value increased up to 17 m. The achieved accuracy values do not meet ICAO CAT II requirements. The comparison of the co-ordinates obtained with that method with DGPS/OTF is shown in Fig. 51.

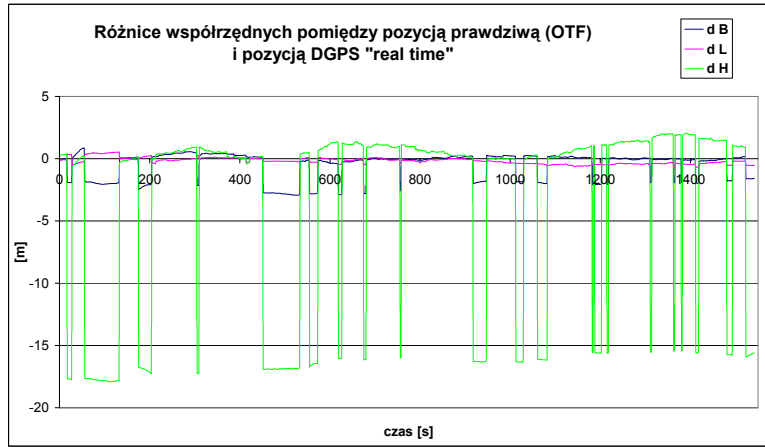


Fig. 51. Differences between the OTF aircraft co-ordinates and the DGPS „Real Time” co-ordinates.

B) DGPS „Post Processing” positioning accuracy.

The error of geodetic co-ordinates B and L computed with the DGPS “Post Processing” method was almost constant. Its mean value was 0.4 m with the maximum of 0.7 m. The error value of h was considerably higher with the maximum of 2 m. Differences of co-ordinates obtained by DGPS in the “post-processing” mode with the OTF aircraft position are shown in Fig. 52.

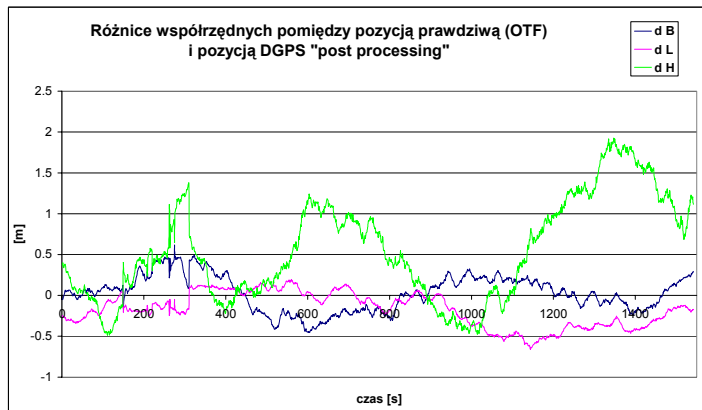


Fig. 52. Differences between the OTF aircraft co-ordinates and the DGPS „Post Processing” co-ordinates.

C) Autonomous positioning accuracy.

The values of results obtained using the autonomous method are stable. The mean value of L error was 7 m with the maximum value of 9.5 m. The mean value of B error was 0.5 m with the maximum of 1.5 m. The error of ellipsoidal height was however, very large having the mean value of 21 m with the maximum of 26 m. Such an error value does not meet the CAT II standards.

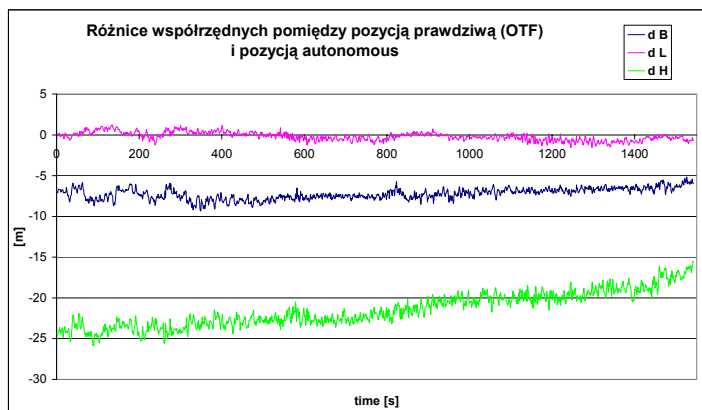


Fig. 53. Differences between the autonomous aircraft co-ordinates and the DGPS/OTF co-ordinates.

Conclusions:

- Measurement methods which can be applied during the approach phase are based on real time computations (DGPS “Real Time”).
- DGPS „Post Processing” method can be used in research studies and databases developing.
- Comparison with ICAO requirements (Tab.9) and conversion percentage values into numerical values for the characteristic points of the glide path (Tab.11) point out to the fact that the obtained error values do not exceed the PAR errors within 500 m from the runway threshold.
- Differential method may be regarded as a source of navigation data, alternative to precision approach radar (PAR).
- Autonomous method does not meet the ICAO standards concerning precision approach.

The Figures below present glide paths of the TS-11 Iskra aircraft in subsequent flights and mean error values of geodetic co-ordinates obtained during the approach phase.

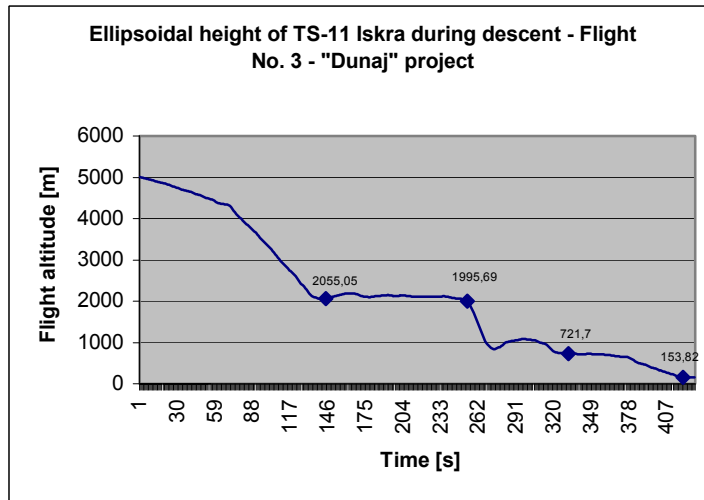


Fig. 54. Ellipsoidal height of TS-11 Iskra aircraft during descent phase– flight No.3 – „Dunaj” project

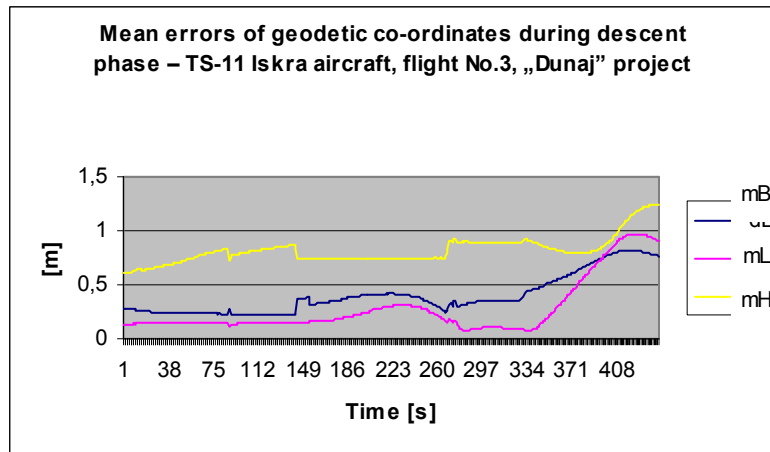


Fig. 55. Mean errors of geodetic co-ordinates during descent phase – TS-11 Iskra aircraft, flight No.3, „Dunaj” project

During flight No.3, the pilot descended to 2000 m and then carried out a two-minute horizontal flight. The following mean values of geodetic co-ordinates during the descent to 2000 m were recorded: mB – 0.21 m, mL – 0.17 m, mh – 0.85 m. During that phase of flight, the value of dH was stable at 0.78 m. The values of dB and dL had a growing tendency (maximum dB – 0.41 m; maximum dL – 0.30 m). Within the next stage of the descent, i.e. to 900 m dB and dL values were decreasing. In the 335th second of the flight (Fig. 54), the values of dB and dL increased rapidly: mB to 0.8 m and mL to 1.0 m. The value of mh also increased to 1.25 m. During flight No. 5

(Fig. 56), the pilot executed the en-route flight at 5000 m, and then a descent to 1000 m. Mean error of geodetic co-ordinates (Fig. 57) were $mB - 0.4$ m, $mL - 0.35$ m, and $dH - 0.9$ m. Between 1,000.23 m and 728.67 m, the mean error values remained unchanged. The mB , dL and dh plots (Fig. 57) are horizontal. Within the final stage of descent, from 728.67 m to 151.96 m, all mean error values were increasing; mh 1.1 m, $mB - 0.25$ m and $mL - 0.1$ m.

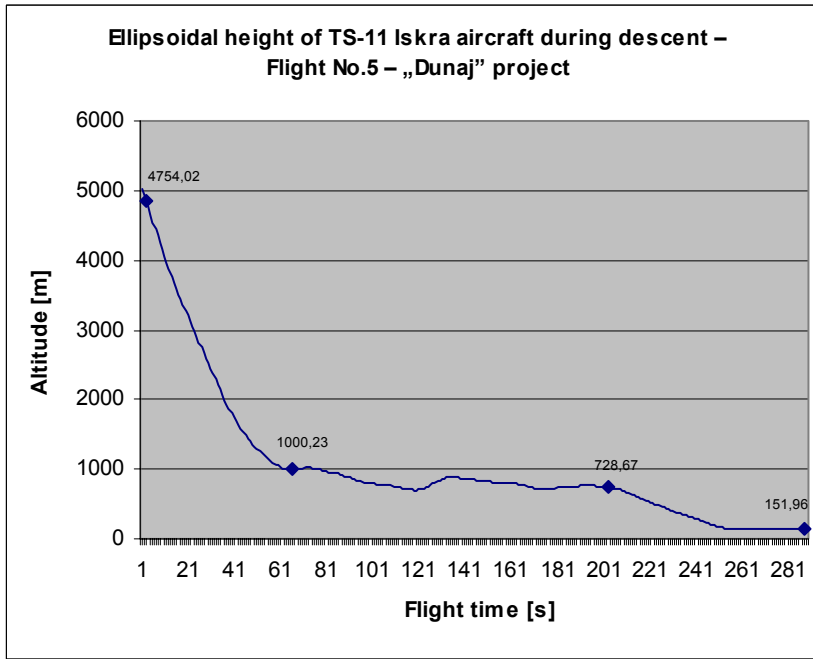


Fig. 56. Ellipsoidal height of TS-11 Iskra aircraft during descent – flight No.5 – „Dunaj” project

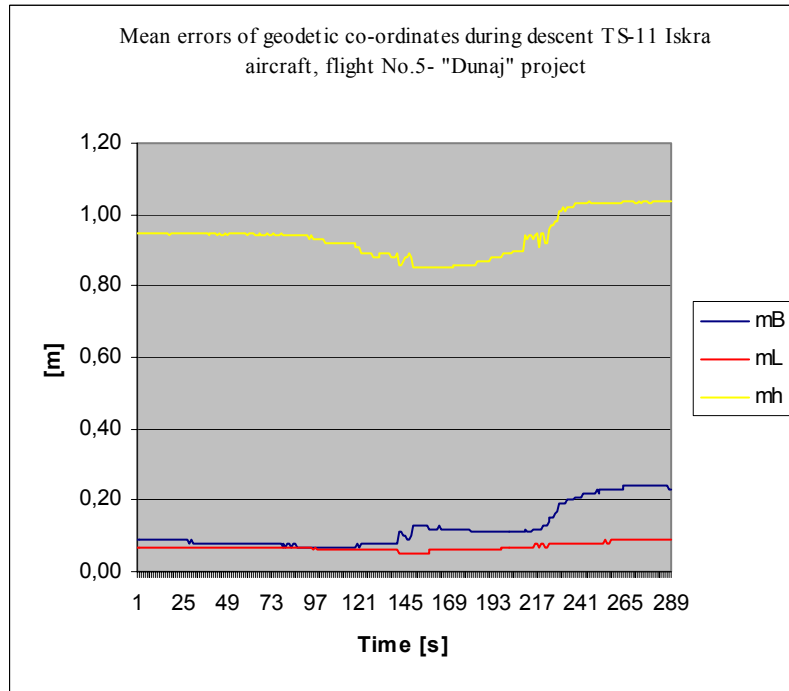


Fig. 57. Mean errors of geodetic co-ordinates during descent – TS-11 Iskra aircraft, flight No.5, „Dunaj” project

The obtained accuracy and high stability of positioning justifies the use of the GPS in approach guidance

Tab. 5.

Tolerance for GPS-assisted non-precision approach

Parameter	Tolerance (FAA/ICAO)
Intermediate Approach Segment (IAS)	
True bearing to the next navigation point	$\pm 2^0$
Distance to the next navigation point	+/- 0.5 NM
Final Approach Segment (FAS)	
True bearing to the next navigation point	$\pm 2^0$
Distance to the next navigation point	+/- 0.3 NM
Missed Approach Segment	
True bearing to the next navigation point	$\pm 2^0$
Distance to the next navigation point	+/- 0.5 NM

3.2. An-2 aircraft approach

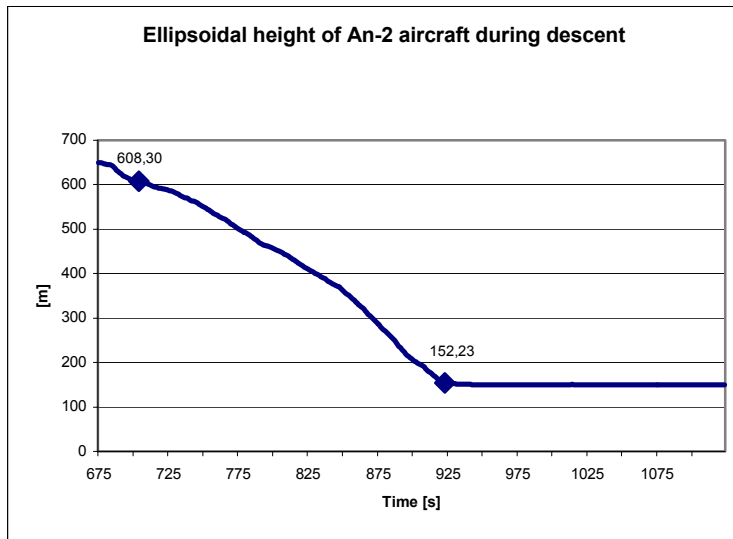


Fig. 58. Ellipsoidal height of An-2 aircraft during descent

Fig. 58 shows the vertical plot of the An-2 aircraft glide path.

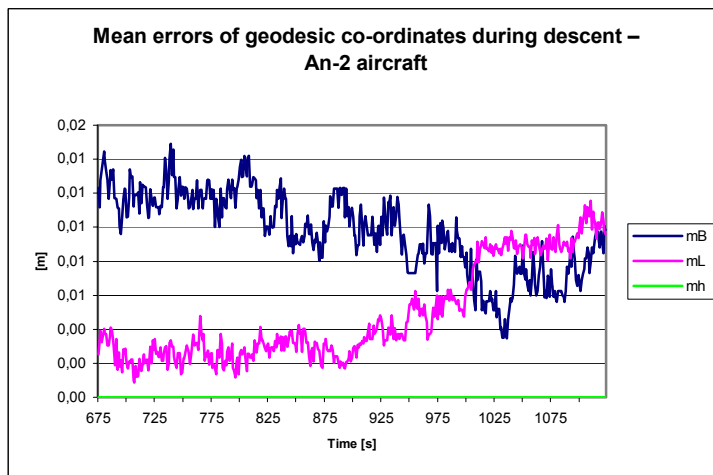


Fig. 59. Mean errors of geodesic co-ordinates during descent – An-2 aircraft

The flight of the An-2 aircraft in the vicinity of the Dęblin airfield was used for the experiment purposes.

Descent from 650 m to 150 m (ellipsoidal height) was recorded during the approach. The recorded segment of flight was considerably stable. Mean errors of

geodetic co-ordinates were: mB - 0.01 m in the initial phase tending to zero, whereas dL showed an upward trend. The obtained error values can be considered as exceptionally favourable. Such values result largely from the low rate of descent and exceptional flight stability.

3.3. Approach phase – project „BRDA”

A total of three TS-11 Iskra flights were flown along the route Dęblin-Garwolin-Waka-Jedlnia-Zwoleń-Dęblin during the experiment on utilising a new radar prototype (TRC-20 „BRDA”). Ashtech Z-Surveyor satellite receiver was also used in the experiment. The aircraft’s positions were recorded at one-second interval. Five reference points, with co-ordinates linked to ETRF'89 reference system, and situated along the flight route were used in the experiment. Three reference stations were allocated in the vicinity of the Dęblin airfield because of the requirements concerning the approach phase of the flight. Measurement data were recorded by the Ashtech Z-Surveyor satellite receiver and then processed using AOSSv.2.0 software. During each observation session the position of the aircraft was calculated by each reference station separately. The final position of the TS-11 Iskra aircraft was calculated for each second of the flight as an arithmetic mean of five independent OTF position fixes in the “post-processing” mode. Mean position errors were computed separately for each co-ordinate, B , L , h (ellipsoidal height) from formula 1.

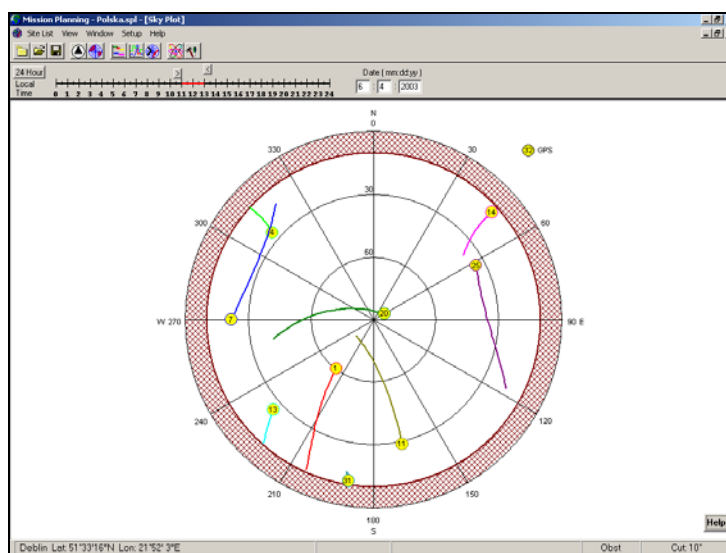


Fig. 60. Theoretical GPS satellite arrangement over the horizon during flight No.1

Fig. 60 shows satellite conditions of the GPS system during flight No. 1 from 11:09:00 to 12:55:00 (GPS time) on 4th June, 2003. The number of available satellites was sufficient for computing the position of the flying aircraft.

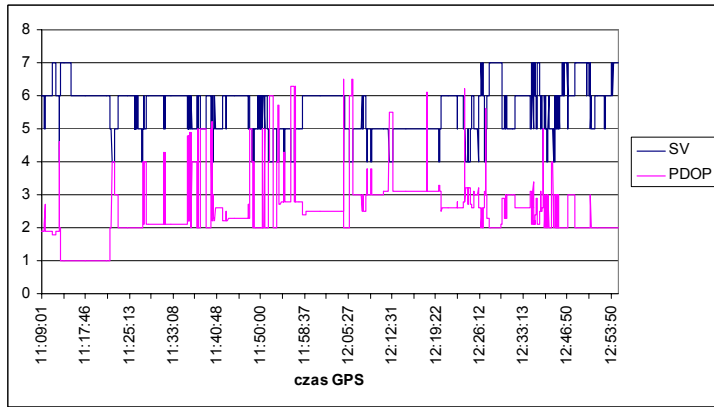


Fig. 61. Actual satellite conditions of the GPS system during flight No.1 – „BRDA” experiment

The actual satellite conditions during the flight are shown in Fig. 61. The value of PDOP ranged from 2 to 6.

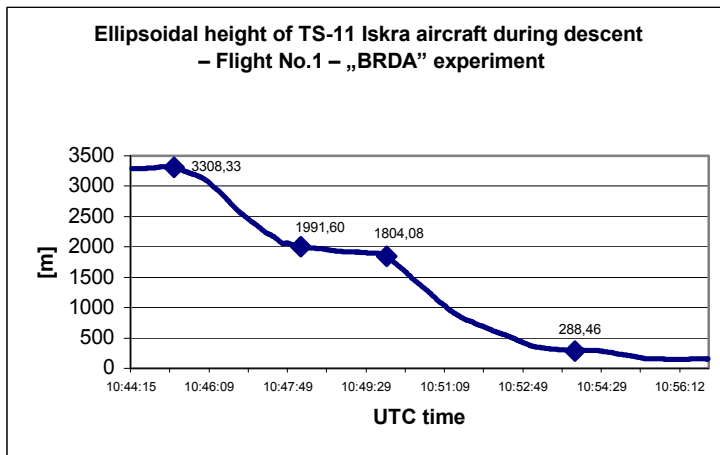


Fig. 62. Ellipsoidal height of TS-11 Iskra aircraft during descent – flight No.1 – „BRDA” experiment

Fig. 62 shows the vertical plot of the TS-11 Iskra glide path from 3,300 m to the aerodrome altitude. The error values of geodetic co-ordinates (Fig. 63) during the descent were: mB – 1.3 m, mL – 0.25 m, mh – 1.0 m. During the level flight at 2,000 m, the error values were stable: mB -0.5 m, mL -0.20 m and mh -1 m. Throughout the flight however, the value of mh was varying. As it can be seen in Fig. 63 its value ranged from 0.0 to 3.5 m.

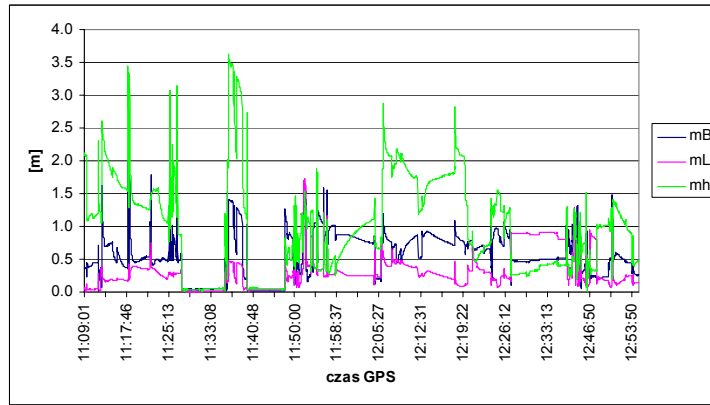


Fig. 63. Graphic representation of B, L and h co-ordinates calculated for each second of flight No.1

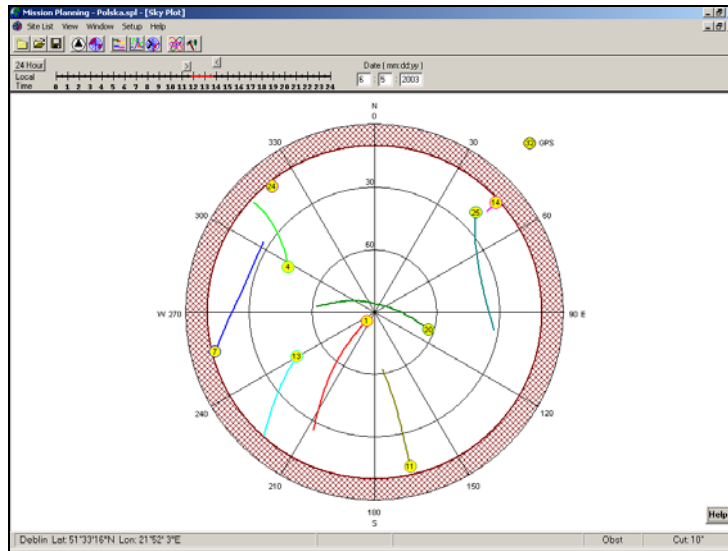


Fig. 64. Theoretical satellite conditions of the GPS system during flight No.2 – „BRDA” experiment

Fig. 64 shows theoretical satellite conditions of the GPS system during flight No.2 of the „BRDA” experiment, from 11:54:00 to 13:54:00 (GPS time) on 5th June, 2003. Data processing was completed with the same Ashtech software. The number of available satellites (Fig. 64) was sufficient for computing the position of the flying aircraft.

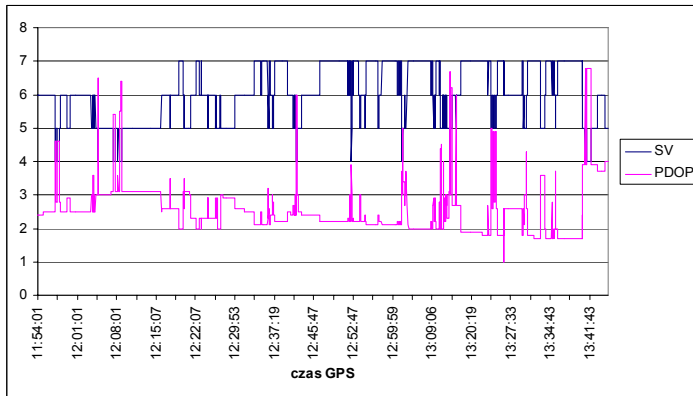


Fig. 65. Actual satellite conditions of the GPS system during flight No.2 – „BRDA” experiment

The PDOP value, also in that instance, ranged from 2 to 6.

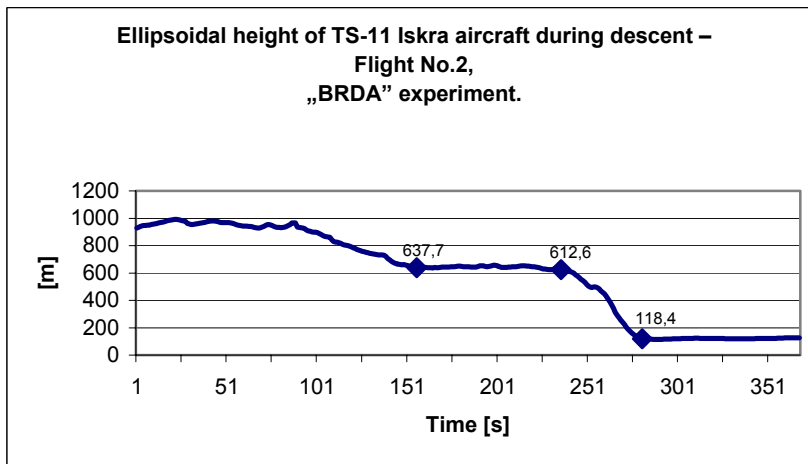


Fig. 66. Ellipsoidal height of TS-11 Iskra aircraft during descent – flight No.2, „BRDA” experiment.

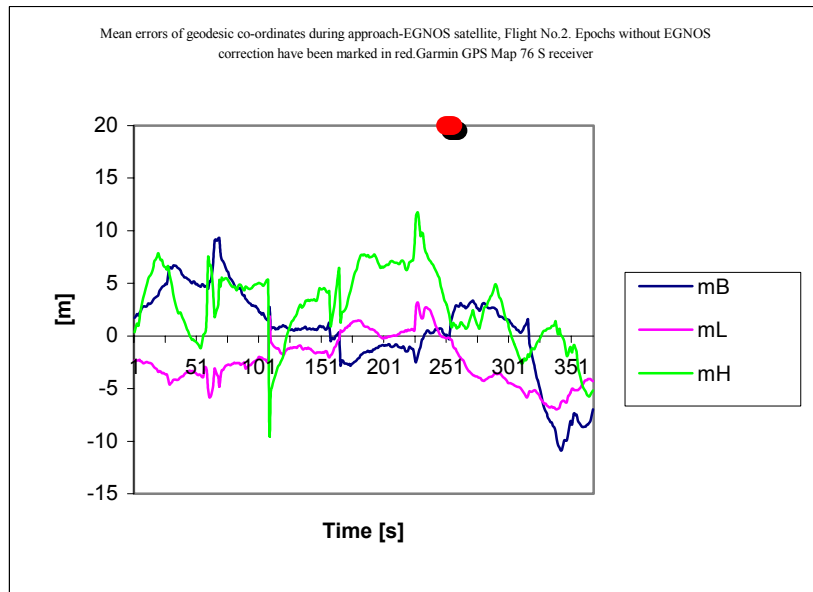


Fig. 67. Mean errors of geodetic co-ordinates during approach – EGNOS satellite, flight No.2

Fig. 66 shows the vertical plot of the TS-11 Iskra glide path. The recorded glide path is stable. Navigation data were supplied throughout the entire descent phase.

Fig. 67 presents the mean errors of geodetic co-ordinates recorded using the EGNOS satellite. The epochs without EGNOS corrections have been marked in red. Garmin GPS Map 76S satellite receiver was used during the experiment. The following mean error values of geodesic co-ordinates were recorded: mh-max 10 m, mB- max 9 m, mL from -8 m to +4 m.

Tab. 6.

NATO Minimum Aviation System Performance Standards (MASPS) for Global Positioning System

Accuracy	Threshold	Objective
		Equivalent to ILS CAT II requirements
Integrity and Continuity	Equivalent to ICAO Annex 10 Level 3	Equivalent to ICAO Annex 10 Level 4
Integrity: Alarm Limit	5.4 m (vertical axis, most stringent)	
Integrity: Time to Alarm	2 seconds	1 second
Availability	99.8%	99.9%

Tab. 7

ICAO Minimum Operational Performance Standards for PAR systems

Equipment type		Data refreshment time	Effective coverage		Range	Accuracy of target co-ordinate determination			Resolution		
			Horizontal	Vertical		Azimuth	Distance	Altitude (elevation)	Azimuth	Distance	Altitude (elevation)
PAR SYSTEM	SRE	4 s	360°	10°	20NM 37 km	2°	150 m or 5% of the observation distance	Not available(on the basis of pilot's informatio n)	4°	230 m or 1% of the observat ion distance	—
	PAR	1s	±10°	7°	Minim um 9NM 16,7 km	0.6% of the distanc e		0.4% of the distance	1,2°	120 m	1°

Tab. 8.
Parameters of basic air traffic control radars

Equipment type		Data refreshment time	Effective coverage		Range	Accuracy of target co-ordinate determination			Resolution		
			Horizontal	Vertical		Azimuth	Distance	Altitude (elevation)	Azimuth	Distance	Altitude (elevation)
RSP-7T	DRL-7	10 s	360°	10°	H=1000m 3 - 50 km	1°	450 m or 1% of the display scale range	Not available (on the basis of pilot's information)	6°	675 m or 1.5% of the display scale range	-
	PRL-7		±15°	-1° ÷ 8°	20/40* km	3°	150-200* *m	100-300** m	1,5°	240 m	1°
RSP-10	DRL-10	10s	360°	10°	H=1000 m 5 -50 km	3°	450 m or 1% of the display scale range	Not available (on the basis of pilot's information)	5°	675 m or 1.5% of the display scale range	-
	PRL-10		±15°	1° ÷ 8°	20/40* km	0,4°*	40 m*	0,25°*	1,5°	150 m	1°
SKRL	AVIA-V	10/15s	360°	45°	H≥1800 m 100 km	±1°	1% of time base range	Not available (on the basis of pilot's information)	1,5°	300 m or 0.6% of the range	-
SWiN	Jawor-M2		360°		H-1000 m 110 km	±1°	600 m	-	±2°	1,500 m	-
	PRW-13		360°		H-1000 m 100 km	-	1,000 m	300 m	-	-	-

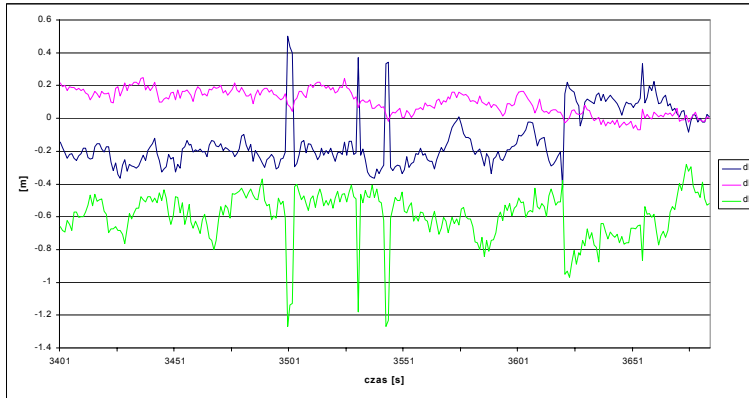


Fig. 68. Differences between the OTF aircraft co-ordinates and the DGPS „post-processing” co-ordinates in the approach phase – „BRDA” experiment.

Tab. 9

IAO Minimum Operational Performance Standards for Air Navigation Systems³⁶
[Fellner 1999]

Flight Phase	Accuracy 95%		Flight altitude interval (m.)	Availability	Integrity		Remarks	
	Position (m)	Altitude (m)			Time to alarm (s)	Loss of Integrity Risk		
Trans-oceanic flights	23000	50	8400-12200	0,9977	30	-	Controlled air space, IFR flights	
En-route flight	1000	50	150-18000	0,9975	10	-	Controlled air space, IFR flights	
Approach	500	50	75-900	0,9975	10	$3,3 \times 10^{-4}$	Controlled and uncontrolled air space, IFR and VFR flights	
Landing approach	Non-precision	100	50	75-900	0,9977	10	$3,3 \times 10^{-4}$	Controlled and uncontrolled air space, IFR and VFR flights
	Precision	17,1	1,7	30-900	0,9999	6	$3,3 \times 10^{-4}$	Controlled air space, IFR flights
CAT I Landing	17,1	1,7	60-900	0,9999	6		Controlled air space, IFR flights	
CAT II Landing				0,9999	2	$3,3 \times 10^{-4}$	Controlled air space, IFR flights	
CAT III Landing				0,9999	2	$3,3 \times 10^{-4}$	Controlled air space, IFR flights	

Tab. 10.

Parameters of specific points on the glide path³⁷ [Ćwiklak 2002]

No.	Specific glide path point	Distance to RWY threshold (km)	Maximum permissible error		Approximate aircraft altitude	Remarks
			horizontal	vertical		
1	Initial Approach Fix	25	150	100	2500	
2	Intermediate Approach Fix	10	60	40	400-300	
3	Final Approach Fix	4	24	16	200	

3.4 Approach Phase – Project „ODRA”

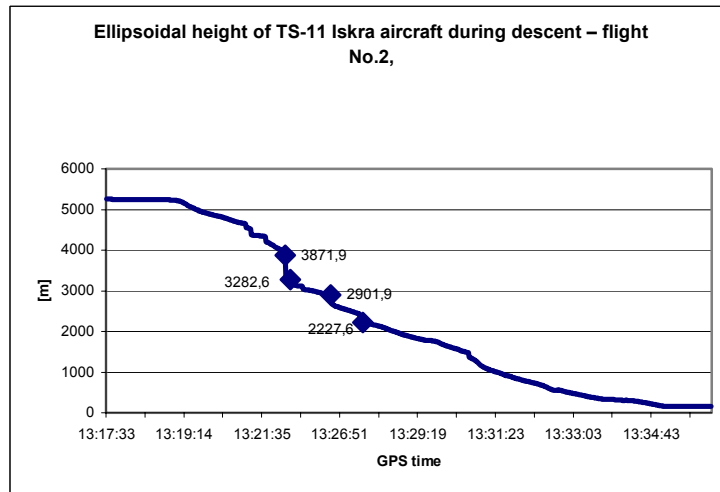


Fig. 69. Glide path from 5,250 m as a function of GPS time (ellipsoidal height)

The approach executed by the TS-11 aircraft was analysed from the altitude of 5,200 m to the moment of landing.

Points 3871,9; 3282,6; 2901,9; 2227,6 representing the aircraft altitude plotted in Fig. 69 show the fragments of the flight route with rapid altitude changes. At those points, the aircraft descended while doing the circuit work in the vicinity of the airfield. Such character of the flight is clearly reflected in the plot of mean errors of geodetic co-ordinates (Fig. 71). The values of mB increased from 0.0 m to 1.4 m, mL to 1.2 m and mH also to 1.2. The trajectory of that descent segment from 500 m is

presented in Fig. 70. Points 306 m and 287.2 m mark a 30-second segment of the horizontal flight carried out over the outer marker. During the flight, the pilot did not follow the PAR procedures that describe the flight altitude of 200 m to be kept over that point. For that reason, the final segment of the flight (4000 m) was executed with the descent rate higher than planned. That fact did not affect the error value, but constituted a serious breach of flight safety conditions.

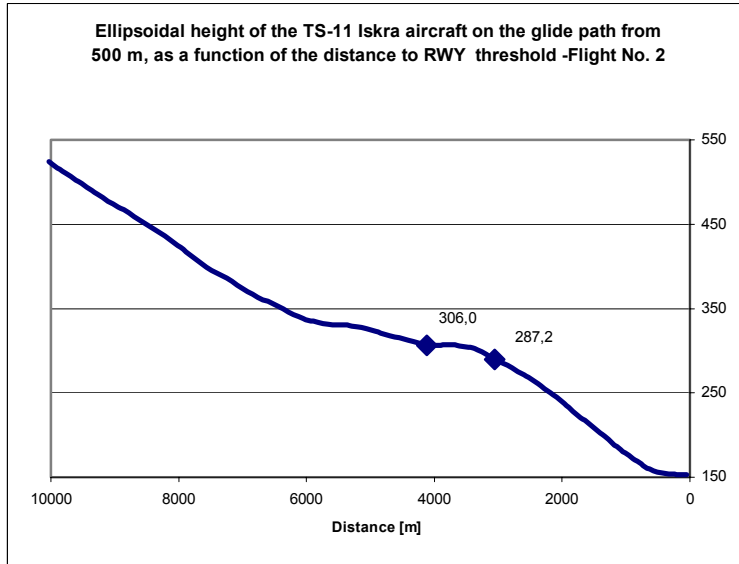


Fig. 70. Ellipsoidal height of the TS-11 Iskra aircraft on the glide path from 500 m, as a function of the distance to runway threshold.

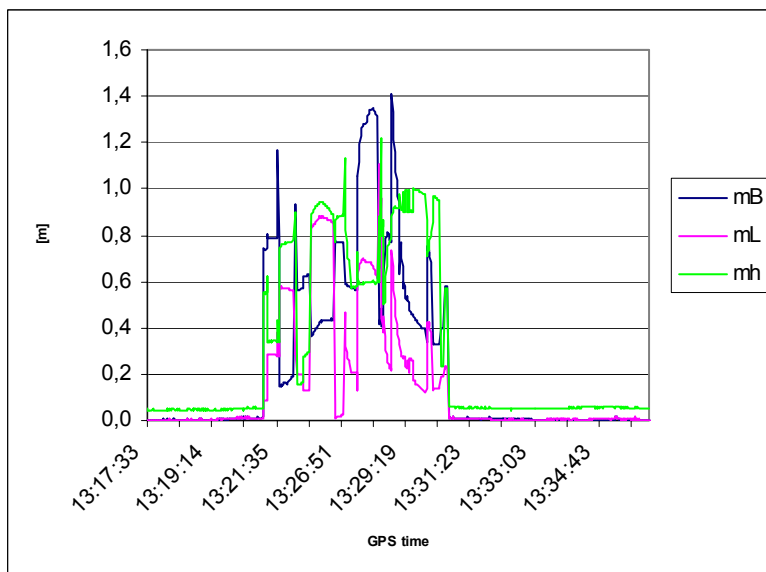


Fig. 71. Mean errors of geodetic co-ordinates during descent, flight No.2, ODRA experiment

3.5. Satellite navigation technique utilisation for civilian aircraft landing procedures support.

The ECAC plan concerning an employment of GNSS in aircraft landing assumes the implementation of GPS from the year 2000, the SBAS CAT I system – from 2004 and GBAS CAT I/II/III/ from 2008. The benefits from the utilisation of GPS in uncontrolled airspace are as following:

- Aircraft position determination – geographical orientation.
- Maintaining assigned flight conditions, e.g. MRT.
- Providing Air Traffic Control - FIS only

GNSS/GPS systems are already in use in airfield operations, i.e. instrumental departure and landing procedures. The benefits from utilising GNSS are as following³⁸[Marczewski 2004]:

- Lack of necessity of installing radio navigation aids for GNSS – based procedures.
- A single ground monitoring station for all aircraft within the reception range, for both en-route and approach phases of flight.
- Very low unit cost of a monitoring station in relation to the cost of installation of ILS system (about €1 million per unit) in all airfields within its reception range.

FedEx is an example of GNSS/GPS implementation.

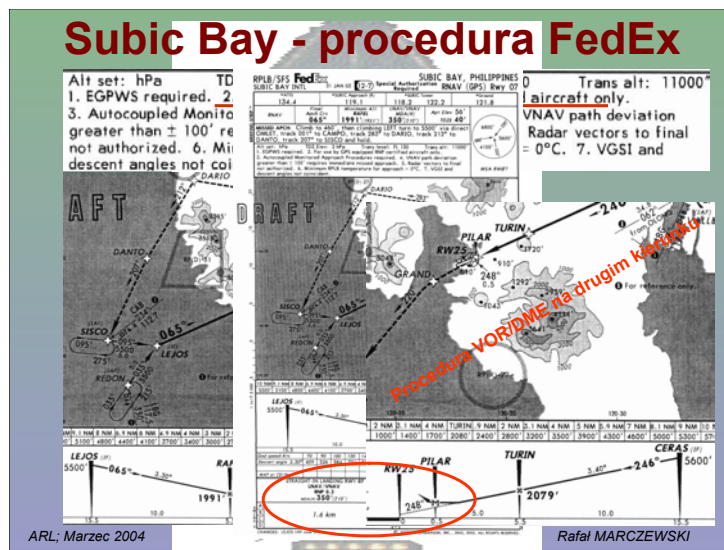


Fig. 72. Subic Bay Procedure – FedEx

Basic principles of the Subic Bay Procedure:

- Status of satellite navigation – primary. Alternative procedure – VOR/DME for aircraft not equipped with GPS receivers or in the event of GPS signal unavailability.

- Satellite availability and constellation information – current (RAIM) and pre-take-off forecast. Actual navigation performance alarm (ANP) below required navigation performance (RNP) of 0.3 n m.
- Missed approach at ANP below 0.3 n m. Second VOR/DME approach or flight to the other airfield.

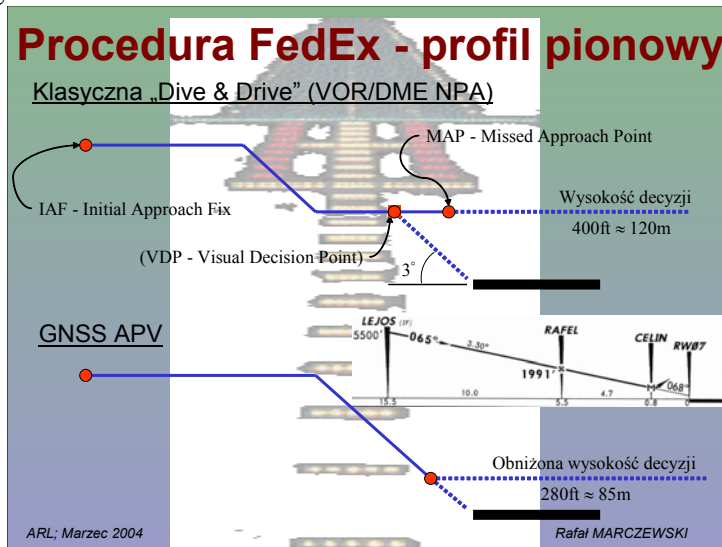


Fig. 73. Vertical profile of a classical glide path and GNSS APV procedure.

Fig. 73 illustrates the difference between the classical „Dive & Drive” VOR/DME non-precision approach and the GNSS APV approach. As a result of the implementation of satellite navigation techniques, the decision height was lowered from 400 ft (about 120 m) to 280 ft (about 85 m).

Conclusions:

- GNSS-based landing procedures have better parameters than non-precision VOR/DME approach procedures.
- Statistical data concerning visibility conditions at Polish airfields justify the necessity of developing GNSS procedures as an alternative to ILS.
- Implementation of GNSS will bring costs reduction related to purchasing, installing and maintaining ground navigation aids in area operations, TMA and airfield operations, especially at uncontrolled ones.
- Implementation of GNSS enhances airspace capacity in area operations (RNAV, Random RNAV air routes)
- Implementation of GPS techniques gives a positive influence on the improvement of flight safety in uncontrolled airspace (position determination and maintaining assigned flight conditions in the event of unavailability of radar coverage).
- According to ECAC, only the implementation of GNSS and DMA leads to achieving RNP1 in TMA.
- Achieving RNP1 in TMA facilitates the introduction of P-RNAV STAR which increases TMA capacity.

4. NAVIGATING BY MEANS OF A POSITION POTENTIAL IN SPACE

The number of factors which influence obtaining complete navigation information and the character of such factors result in the fact that experimental data do not always constitute complete information for precise assessment of the location of a given aircraft. Even when the satellite system is fully available and its configuration is proper, some factors may emerge which prevent its utilisation. The proper arrangement of satellite antennas in a given points onboard the aircraft is of crucial importance with that regard. Wrong location of those antennas may exert a decisive influence on finding the position in space. Factors resulting from abrupt changes of flight parameters e.g. due to rapid changes of weather conditions or manoeuvres of the aircraft are also to be taken into consideration. The above-mentioned factors necessitate working out additional methods which supplement an aircraft. One of the most common ways to solve such kind of problems are mathematical methods.

An alternative navigation filter³⁹ [Vanicek 2000] has been developed, relying mainly on the following measurements connected to the motion of the aircraft: positions, velocities and position error statistics. In reality, a cluster of observed position fixes contains all vital kinematic information of the aircraft. While constructing the program, “position potential” was used based on the data received from the navigation services. According to Newton’s law of universal gravitation, a free mass particle is attracted by another mass. The acceleration of a particle of a given mass is proportional to the gravitation constant G and inversely proportional to the squared distance from the attracting mass. *Per analogiam*, we consider the statistical confidence regions (position error ellipsoid) of the position fixes as sources of force, which should attract the trajectory passing through these regions. The potential field of a single particle (in our case an aircraft), should reflect the observed position fix and the required force on the particle, which should monotonically decrease when the particle approaches the assumed position fix.

When the particle enters the position error ellipsoid, it is attracted with a force whose magnitude is proportional to the intensity of the surrounding potential, which monotonically decreases with the decreasing distance between the particle and the assumed fix. Moreover, in order to allow the particle to continue moving after the position fix appears, the potential of the attracting source will be dissipated exponentially in time. By selecting properly the position density function (“position potential”), which contains dissipation exponent α and the parameter G (positioning uncertainty, corresponding to Newton’s gravitation constant), we assume that the path of the particle will represent the real trajectory of the aircraft.

The first attempt at developing a navigation filter based on the above assumptions was made by Inzinga and Vanicek. Their study assumes that the force of the position potential field affecting the particle is connected with the probability, which describes the fact that the position fix falls in the two-dimensional error ellipse.

Since the equation of the motion of the particle is extremely difficult to be treated analytically, the following model was selected:

- selection of a position potential function;
- formulation of a temporal dissipation of the potential field;
- description of a motion equation;
- solving the motion equation;
- determining α and G parameters;
- finding the final solution of the navigation problem.

A proper potential function is the first to be selected. Using the selected position density function, the time-related position potential field can be established for a sequence of position fixes. Subsequently, a model of motion of an individual particle can be set up so as to find the solution of the equation of its motion. In order to reflect changing navigational environment, the potential function contains variable parameters, α and G .

When these parameters and the initial conditions are not known, the number of possible positions of the particle is infinite and further data are required to select a proper trajectory. Consequently, a „self-learning” procedure for the filter is to be set up so that parameters and conditions from earlier observations will be taken into consideration.

Parameters and initial motion conditions in the motion model are related to previous observations by “motion equations”. At this point, the motion equation is solved with the assumption that the initial conditions are known. Then, the parameters α and G have to be determined and optimised by means of the least-square criterion.

The prediction of a given future particle (aircraft) position is possible from the model of motion with the assumed parameters and its present position. Error detection is possible at the moment of obtaining a new position fix using the concordance of the predicted and actual positions, i.e. checking whether the new position is located within the position error ellipsoid.

4.1 The navigation model

We assume that the position density function⁴⁰ [Kački 1975] for a three-dimensional vector of random position is:

$$\phi_{\mathbf{r}_0} = \frac{1}{K} \exp\left[-\frac{1}{2}(\mathbf{r} - \mathbf{r}_0)^T \mathbf{C}^{-1}(\mathbf{r} - \mathbf{r}_0)\right] \quad (4.1)$$

where:

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (4.2)$$

- particle position vector at the present moment “ t ”;

$$\mathbf{r}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad (4.3)$$

- particle position vector at the set moment “ t_0 ”;

$$K = (2\pi)^{\frac{3}{2}} (\det \mathbf{C})^{\frac{1}{2}} \quad (4.4)$$

\mathbf{C} – symmetric covariance matrix⁴¹ [Plucinska 200]

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} D^2X & \text{cov}(X, Y) & \text{cov}(X, Z) \\ \text{cov}(Y, X) & D^2Y & \text{cov}(Y, Z) \\ \text{cov}(Z, X) & \text{cov}(Z, Y) & D^2Z \end{bmatrix} \quad (4.5)$$

Let us assume that the particle potential U is the basis for its position fix determination:

$$U_i(t) = G(\mathbf{r} - \mathbf{r}_{oi})^T \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_{oi}) e^{-\alpha(t-t_i)} \quad (4.6)$$

where:

$$(\mathbf{r} - \mathbf{r}_{oi})^T \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_{oi}) \quad (4.7)$$

is the quadratic form of the error ellipsoid at the moment “ t ”, $t \geq t_0$

\mathbf{C}_i is the positive covariance matrix of the i -th particle positively determined. Whereas positive parameters α and G are to be determined and represent, respectively α - dissipation parameter;

G – uncertainty of the position fixes being a counterpart of the gravitational constant in Newton’s attraction.

In relation to potential determined in (4.6), time-varying (t) potential resulting from “ n ” position fixes of a particular particle may be expressed as:

$$U = \sum_{i=1}^n U_i = G e^{-\alpha t} \sum_{i=1}^n (\mathbf{r} - \mathbf{r}_{oi})^T \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_{oi}) e^{\alpha t_i} \quad (4.8)$$

where:

$$t \geq t_n$$

It should be noted that the filter (potential) keeps the pace with the kinematics of the particle (aircraft). Time-varying potential field is constantly updated by each new position fix of the particle in order to incorporate the most recent information as soon as possible.

Since the co-ordinate system X, Y, Z is orthogonal, it may be assumed that random variables X, Y, Z are uncorrelated.

In the case of uncorrelated random variables it is proved that:

$$\text{cov}(X,Z) = \text{cov}(Y,Z) = \text{cov}(X,Y) = 0 \quad (4.9)$$

In case when the matrix \mathbf{C} is not diagonal, it can be transform to be diagonal by means of an isometry.

After using (4.8) and adopting the following notation:

$$D^2X = \sigma_{11}^2 \quad D^2Y = \sigma_{22}^2 \quad D^2Z = \sigma_{33}^2 \quad (4.10)$$

Covariance matrix (4.5) is a diagonal matrix:

$$\mathbf{C} = \begin{bmatrix} \sigma_{11}^2 & 0 & 0 \\ 0 & \sigma_{22}^2 & 0 \\ 0 & 0 & \sigma_{33}^2 \end{bmatrix} \quad (4.11)$$

Using (4.11), we have:⁴² [Berlin 1996]

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{1}{\sigma_{11}^2} & 0 & 0 \\ 0 & \frac{1}{\sigma_{22}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_{33}^2} \end{bmatrix} \quad (4.12)$$

$$K = (2\Pi)^{\frac{3}{2}} (\det \mathbf{C})^{\frac{1}{2}} \quad (4.13)$$

$$\det \mathbf{C} = \sigma_{11}^2 \sigma_{22}^2 \sigma_{33}^2 \quad \text{- determinant of a covariance matrix.} \quad (4.14)$$

Positive form (4.7) is expressed as follows in the co-ordinates :

$$(\mathbf{r} - \mathbf{r}_o)^T \mathbf{C}^{-1} (\mathbf{r} - \mathbf{r}_o) = \frac{(x - x_o)^2}{\sigma_{11}^2} + \frac{(y - y_o)^2}{\sigma_{22}^2} + \frac{(z - z_o)^2}{\sigma_{33}^2} \quad (4.15)$$

Substituting (4.13) – (4.15) into (4.1), function of position density being Gaussian random reads:

$$f(x, y, z) = \frac{1}{\sqrt{(2\Pi)^3 \sigma_{11} \sigma_{22} \sigma_{33}}} e^{-\frac{1}{2} \left[\frac{(x-x_o)^2}{\sigma_{11}^2} + \frac{(y-y_o)^2}{\sigma_{22}^2} + \frac{(z-z_o)^2}{\sigma_{33}^2} \right]} \quad (4.16)$$

Writing $U(t)$ (4.8) by means of co-ordinates (see (4.5) in [3]) we obtain for a three-dimensional problem:

$$U(t) = \sum_{i=1}^n U_i(t) = G e^{-\alpha t} \sum_{i=1}^n \left[\frac{(x - x_{oi})^2}{\sigma_{11i}^2} + \frac{(y - y_{oi})^2}{\sigma_{22i}^2} + \frac{(z - z_{oi})^2}{\sigma_{33i}^2} \right] e^{\alpha t_i}; \quad t \geq t_n \quad (4.17)$$

where:

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathbf{r}_{oi} = \begin{bmatrix} x_{oi} \\ y_{oi} \\ z_{oi} \end{bmatrix}$$

$$U_i(t) = G e^{-\alpha t} \sum_{i=1}^n (\mathbf{r} - \mathbf{r}_{oi})^T \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_{oi}) e^{\alpha t_i}$$

and the equation of the motion (see (4.6) w [3])

$$\ddot{\mathbf{r}} = -\frac{\partial U(t)}{\partial \mathbf{r}} \quad (4.18)$$

By using (4.8), we will obtain:

$$\frac{\partial U}{\partial \mathbf{r}} = 2Ge^{-\alpha t} \sum_{i=1}^n \mathbf{C}_i^{-1} (\mathbf{r} - \mathbf{r}_{oi}) e^{\alpha t_i} \quad (4.19)$$

or in an alternative form:

$$\frac{\partial U}{\partial \mathbf{r}} = 2Ge^{-\alpha t} \left[\sum_{i=1}^n \mathbf{C}_i^{-1} \mathbf{r} e^{\alpha t_i} - \sum_{i=1}^n \mathbf{C}_i^{-1} \mathbf{r}_{oi} e^{\alpha t_i} \right] \quad (4.20)$$

After substituting (4.20) into the motion equation (4.18), we get the following equation:

$$\ddot{\mathbf{r}}(t) = -2Ge^{-\alpha t} \left[\sum_{i=1}^n \mathbf{C}_i^{-1} \mathbf{r} e^{\alpha t_i} - \sum_{i=1}^n \mathbf{C}_i^{-1} \mathbf{r}_{oi} e^{\alpha t_i} \right] \quad (4.21)$$

Assuming the following notation:

$$\mathbf{A} = 2 \sum_{i=1}^n e^{\alpha t_i} \mathbf{C}_i^{-1} \quad \text{- matrix (3x3)} \quad (4.22)$$

$$\mathbf{B} = 2 \sum_{i=1}^n e^{\alpha t_i} \mathbf{C}_i^{-1} \mathbf{r}_{oi} \quad \text{-vector} \quad (4.23)$$

the equation of motion (4.21) becomes:

$$\ddot{\mathbf{r}}(t) = e^{-\alpha t} \mathbf{G}(\mathbf{A}\mathbf{r} - \mathbf{B}); \quad t \geq t_n \quad (4.24)$$

being the equation of motion of the particle (aircraft) in the time-varying (t) position potential field after the occurrence of “n” position fixes.

For simplicity let us write the matrix \mathbf{C}_i^{-1} as:

$$\mathbf{C}_i^{-1} = \begin{bmatrix} \frac{1}{\sigma_{11i}^2} & 0 & 0 \\ 0 & \frac{1}{\sigma_{22i}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_{33i}^2} \end{bmatrix} = \begin{bmatrix} p_{xi} & 0 & 0 \\ 0 & p_{yi} & 0 \\ 0 & 0 & p_{zi} \end{bmatrix} \quad (4.25)$$

Now, computing the product of the matrix \mathbf{Ar} (see in [2]), the equation of motion of the particle (4.24) can be written as follows in the co-ordinates :

$$\begin{aligned} \ddot{x}(t) &= -G(A_x x - B_x) e^{-\alpha t} \\ \ddot{y}(t) &= -G(A_y y - B_y) e^{-\alpha t} \\ \ddot{z}(t) &= -G(A_z z - B_z) e^{-\alpha t} \end{aligned} \quad t \geq t_n \quad (4.26)$$

In order to avoid problems with computer calculation, we always move the initial time “ t ” to the time of occurrence of the latest point „ t_n ”. Then, vectors \mathbf{Ar} and \mathbf{B} are represented in co-ordinates as:

$$\mathbf{Ar} = [A_x x, A_y y, A_z z]$$

$$\begin{aligned} A_x &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{xi} \\ A_y &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{yi} \\ A_z &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{zi} \end{aligned} \quad (4.27)$$

and the co-ordinates of the vector \mathbf{B} are represented as:

$$\begin{aligned} B_x &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{xi} x_{oi} \\ B_y &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{yi} y_{oi} \\ B_z &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{zi} z_{oi} \end{aligned} \quad (4.28)$$

To solve the set of inhomogeneous linear differential equations of the second order (4.26), we can write this set in an alternative way:

$$\begin{aligned}\ddot{x}(t) &= -GA_x \left(x - \frac{B_x}{A_x}\right) e^{-\alpha t} \\ \ddot{y}(t) &= -GA_y \left(y - \frac{B_y}{A_y}\right) e^{-\alpha t} & t \geq t_n \\ \ddot{z}(t) &= -GA_z \left(z - \frac{B_z}{A_z}\right) e^{-\alpha t}\end{aligned}\tag{4.29}$$

which is a set of inhomogeneous linear differential equations of the second order. As an example, let us solve the equation (4.29), to that end making a substitution:

$$x(t) - \frac{B_x}{A_x} = U(t),\tag{4.30}$$

then

$$\dot{x}(t) = \dot{U}(t)\tag{4.31}$$

and

$$\ddot{x}(t) = \ddot{U}(t)\tag{4.32}$$

Equations (4.29) – (4.32) imply the following equation:

$$\ddot{U}(t) + GA_x e^{-\alpha t} U(t) = 0\tag{4.33}$$

Substituting the independent variable (time)

$$s = \frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x}\tag{4.34}$$

we have

$$\frac{ds}{dt} = \frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \left(-\frac{\alpha}{2}\right) = -e^{-\frac{\alpha}{2}t} \sqrt{GA_x}$$

$$\frac{ds}{dt} = -\frac{\alpha}{2}s \quad (4.35)$$

$$\frac{du}{dt} = \frac{du}{ds} \cdot \frac{ds}{dt} = -\frac{\alpha}{2}s \cdot \frac{du}{ds} \quad (4.36)$$

$$\begin{aligned} \frac{d^2u}{dt^2} = \ddot{U}(t) &= \frac{d}{dt} \left(\frac{du}{dt} \right) = \frac{d}{dt} \left(-\frac{\alpha}{2}s \frac{du}{ds} \right) = \frac{d}{ds} \left(-\frac{\alpha}{2}s \frac{du}{ds} \right) \cdot \frac{ds}{dt} = \\ & \left[-\frac{\alpha}{2} \frac{du}{ds} - \frac{\alpha}{2}s \frac{d^2u}{ds^2} \right] \cdot \left(-\frac{\alpha}{2}s \right) \end{aligned} \quad (4.37)$$

$$\frac{d^2u}{ds^2} = \frac{\alpha^2}{4}s^2 \frac{d^2u}{ds^2} + \frac{\alpha^2}{4}s \frac{du}{ds} \quad (4.38)$$

From (4.29):

$$GA_x e^{-\alpha t} = \frac{\alpha^2}{4}s^2 \quad (4.39)$$

Substituting (4.38) and (4.39) into (4.33), we get:

$$\begin{aligned} \frac{\alpha^2}{4}s^2 \frac{d^2u}{ds^2} + \frac{\alpha^2}{4}s \frac{du}{ds} + \frac{\alpha^2}{4}s^2 u = 0 / : \frac{\alpha^2}{4}s^2 \\ \frac{d^2u}{ds^2} + \frac{1}{s} \frac{du}{ds} + u = 0 \end{aligned} \quad (4.40)$$

Equation (4.40) is a Bessel⁴³ [Kącki 1972] differential equation, whose solution is found to be

$$U(s) = a_1 J_0(s) + a_2 N_0(s) \quad ^{44} \quad [Kącki 1972] \quad (4.41)$$

where $J_0(s)$, $N_0(s)$ are Bessel functions (cylinder functions) of the first and second kind respectively with index zero.

When we get back to the initial variables, i.e. x and t

$$\begin{cases} U = x - \frac{B_x}{A_x} \\ s = \frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \end{cases} \quad \text{see: (4.30) and (4.34)}$$

the solution (4.41) of equation (4.29) is expressed as

$$\text{follows: } x(t) = \frac{B_x}{A_x} + a_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) + a_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \quad (4.42)$$

where the constant parameters a_1 and a_2 are to be determined by the initial condition, which gives the initial position $x(t)$ at the initial moment and instantaneous velocity at the same moment.

Following the same procedure as in equation (4.29)₁ when solving equations (4.29)₂ and (4.29)₃, the solution of the system of equations (4.29) will be;

$$\begin{cases} x(t) = \frac{B_x}{A_x} + a_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) + a_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \\ y(t) = \frac{B_y}{A_y} + b_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) + b_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) \\ z(t) = \frac{B_z}{A_z} + c_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) + c_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) \end{cases} \quad (4.43)$$

Where:

the first addend $\frac{B_x}{A_x}$ is to be regarded as a particular solution of a inhomogeneous differential equation, and both elements containing constants a_1 and a_2 as general solutions of a homogeneous differential equation.

In order to determine the integration constants $a_1, a_2; b_1, b_2; c_1, c_2$, initial conditions have to be used, i.e. initial conditions must be added to the system of equations (4.24) (formulation of the Cauchy problem).

To simplify the determination of these constants, the following properties of Bessel function⁴⁶[Smirnow 1967] should be used :

$$\begin{cases} \frac{dJ_0(t)}{dt} = -J_1(t) \\ \frac{dN_0(t)}{dt} = -N_1(t) \end{cases} \quad (4.44)$$

and

$$J_1(t)N_0(t) - J_0(t)N_1(t) = \frac{2}{\Pi t} \quad (4.45)$$

For instance, we have:

$$\frac{d}{dt} \left[J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \right] = \sqrt{GA_x} J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) e^{-\frac{\alpha}{2}t} \quad (4.46)$$

$$\frac{d}{dt} \left[N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \right] = \sqrt{GA_x} N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) e^{-\frac{\alpha}{2}t}$$

J_1, N_1 - Bessel functions of the first and second kind respectively, of the first order.

Using (4.11) in (4.38) we have:

$$\begin{cases} \dot{x}(t) = \sqrt{GA_x} e^{-\frac{\alpha}{2}t} \left[a_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) + a_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \right] \\ \dot{y}(t) = \sqrt{GA_y} e^{-\frac{\alpha}{2}t} \left[b_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) + b_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) \right] \\ \dot{z}(t) = \sqrt{GA_z} e^{-\frac{\alpha}{2}t} \left[c_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) + c_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) \right] \end{cases} \quad (4.47)$$

Equations (4.43) and (4.47) with known parameters (G and α) are basic equations for following studies [see: (4.17)].

Using the initial condition for $t_n = 0$

$$\mathbf{r}(0) = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \quad \text{i} \quad \dot{\mathbf{r}}(0) = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix}, \quad (4.48)$$

in equations (4.43) and (4.47) and the dependence (4.45), we compute the constants:

$a_1, a_2, b_1, b_2, c_1, c_2$.

Let us, for example, compute a_1 . To that end, we will write equations (4.43)₁, (4.47)₁ and (4.45), after adopting for simplicity, the following notation:

$$T = e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \quad \text{i} \quad z = \frac{2}{\alpha}T \quad \text{w (4.40)}, \quad (4.49)$$

in the following form

$$\begin{cases} x(t) = \frac{B_x}{A_x} + a_1 J_0\left(\frac{2}{\alpha}T\right) + a_2 N_0\left(\frac{2}{\alpha}T\right) \\ \dot{x}(t) = a_1 T J_1\left(\frac{2}{\alpha}T\right) + a_2 T N_1\left(\frac{2}{\alpha}T\right) \\ J_1\left(\frac{2}{\alpha}T\right) \cdot N_0\left(\frac{2}{\alpha}T\right) - J_0\left(\frac{2}{\alpha}T\right) \cdot N_1\left(\frac{2}{\alpha}T\right) = \frac{\alpha}{\Pi T} \end{cases} \quad (4.50)$$

Multiplying (4.50)₁ by $-TN_1\left(\frac{2}{\alpha}T\right)$ and (4.50)₂ by $N_0\left(\frac{2}{\alpha}T\right)$ we obtain

$$\begin{aligned} -TN_1\left(\frac{2}{\alpha}T\right)x(t) &= -\frac{B_x}{A_x}TN_1\left(\frac{2}{\alpha}T\right) - a_1TJ_0\left(\frac{2}{\alpha}T\right)N_1\left(\frac{2}{\alpha}T\right) \\ &- a_2TN_0\left(\frac{2}{\alpha}T\right)N_1\left(\frac{2}{\alpha}T\right) \\ N_0\left(\frac{2}{\alpha}T\right)\dot{x}(t) &= a_1TJ_1\left(\frac{2}{\alpha}T\right)N_0\left(\frac{2}{\alpha}T\right) + a_2TN_0\left(\frac{2}{\alpha}T\right)N_1\left(\frac{2}{\alpha}T\right) \end{aligned} \quad (4.51)$$

Adding the sides of equations (4.51), we will obtain:

$$\begin{aligned}
 & -\text{TN}_1\left(\frac{2}{\alpha}T\right)x(t) + \text{N}_0\left(\frac{2}{\alpha}T\right)\dot{x}(t) = -\frac{\text{B}_x}{\text{A}_x}\text{TN}_1\left(\frac{2}{\alpha}T\right) \\
 & + a_1\text{T}\left[\left(\text{J}_1\frac{2}{\alpha}T\right)\text{N}_0\left(\frac{2}{\alpha}T\right) + \text{J}_0\left(\frac{2}{\alpha}T\right)\text{N}_1\left(\frac{2}{\alpha}T\right)\right],
 \end{aligned}$$

and, after using (4.50), we have

$$\frac{\alpha}{\Pi}a_1 = \frac{\text{B}_x}{\text{A}_x}\text{TN}_1\left(\frac{2}{\alpha}T\right) - \text{TN}_1\left(\frac{2}{\alpha}T\right)x(t) + \text{N}_0\left(\frac{2}{\alpha}T\right)\dot{x}(t),$$

hence:

$$a_1 = -\frac{\Pi}{\alpha}\left[\left(x(t) - \frac{\text{B}_x}{\text{A}_x}\right)\text{TN}_1\left(\frac{2}{\alpha}T\right) - \text{N}_0\left(\frac{2}{\alpha}T\right)\dot{x}(t)\right]. \quad (4.52)$$

Because of the initial condition (4.48) in the time interval $[t_n, t_{n+1}]$ for $t = t_n = 0$, we have:

$$\left. \begin{aligned}
 & x(0) = x_n \\
 & \dot{x}(0) = \dot{x}_n \\
 & e^{-\frac{\alpha}{2} \cdot 0} = 1 \\
 & T = \sqrt{\text{GA}_x}
 \end{aligned} \right\} \quad (4.53)$$

Taking into consideration (4.53) in (4.52), a_1 will read :

$$a_1 = -\frac{\Pi}{\alpha}\left[\left(x_n - \frac{\text{B}_x}{\text{A}_x}\right)\sqrt{\text{GA}_x}\text{N}_1\left(\frac{2}{\alpha}\sqrt{\text{GA}_x}\right) - \dot{x}_n\text{N}_0\left(\frac{2}{\alpha}\sqrt{\text{GA}_x}\right)\right] \quad (4.54)$$

The remaining integration constants a_2, b_1, b_2, c_1, c_2 are determined in the same way. Finally, the integration constants are represented by the following formulas:

$$\left. \begin{aligned}
 a_1 &= -\frac{\Pi}{\alpha} \left[\left(x_n - \frac{B_x}{A_x} \right) \sqrt{GA_x} N_1 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) - \dot{x}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) \right] \\
 a_2 &= \frac{\Pi}{\alpha} \left[\left(x_n - \frac{B_x}{A_x} \right) \sqrt{GA_x} J_1 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) - \dot{x}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) \right] \\
 b_1 &= -\frac{\Pi}{\alpha} \left[\left(y_n - \frac{B_y}{A_y} \right) \sqrt{GA_y} N_1 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) - \dot{y}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) \right] \\
 b_2 &= \frac{\Pi}{\alpha} \left[\left(y_n - \frac{B_y}{A_y} \right) \sqrt{GA_y} J_1 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) - \dot{y}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) \right] \\
 c_1 &= -\frac{\Pi}{\alpha} \left[\left(z_n - \frac{B_z}{A_z} \right) \sqrt{GA_z} N_1 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) - \dot{z}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) \right] \\
 c_2 &= \frac{\Pi}{\alpha} \left[\left(z_n - \frac{B_z}{A_z} \right) \sqrt{GA_z} J_1 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) - \dot{z}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) \right]
 \end{aligned} \right\} \quad (4.55)$$

Because of the initial condition, the computed integration constants $a_1, a_2, b_1, b_2, c_1, c_2$ represented by (4.55) exist within the time interval $[t_n, t_{n+1}]$. Therefore, the position and velocity at next stage t_{n+1} can be determined from equations (4.43) and (4.47) for $t = t_{n+1} - t_n$.

Functions present in the above problem are described by the following units:

- $\alpha [s^{-1}]$ - parameter;
- $U [m^2 s^{-2}]$ - potential;
- $G [m^2 s^{-2}]$ - parameter present in determining the potential U ;
- $A [m^2]$ - matrix;
- $B [m^{-1}]$ - vector.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} [m] \text{ - vector of the particle (aircraft) position;}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} [m \cdot s^{-1}] \text{ - velocity of the particle (aircraft) position;}$$

Example of α and G parameters optimisation

In order to optimise parameters α and G , we will analyse the following problem, consisting in solving the equation of motion:

$$\begin{aligned}\ddot{x}(t) &= -G(A_x x - B_x) e^{-\alpha t} \\ \ddot{y}(t) &= -G(A_y y - B_y) e^{-\alpha t} \\ \ddot{z}(t) &= -G(A_z z - B_z) e^{-\alpha t}\end{aligned} \quad t \geq t_n \quad (4.56)$$

where the vector \mathbf{A} is represented by:

$$\begin{aligned}A_x &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{xi} \\ A_y &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{yi} \\ A_z &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{zi}\end{aligned} \quad (4.57)$$

and the co-ordinates of the vector \mathbf{B} are represented as:

$$\begin{aligned}B_x &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{xi} x_{oi} \\ B_y &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{yi} y_{oi} \\ B_z &= 2 \sum_{i=1}^n e^{\alpha(t_i - t_n)} p_{zi} z_{oi}\end{aligned} \quad (4.58)$$

with the initial conditions for $t_n = 0$

$$\mathbf{r}(0) = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \quad \dot{\mathbf{r}}(0) = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix} \quad (4.59)$$

The particle position vector is described by the following formulas:

$$\begin{cases} x(t) = \frac{B_x}{A_x} + a_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) + a_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \\ y(t) = \frac{B_y}{A_y} + b_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) + b_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) \\ z(t) = \frac{B_z}{A_z} + c_1 J_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) + c_2 N_0 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) \end{cases} \quad (4.60)$$

whereas the velocity vector is described by the following equations:

$$\begin{cases} \dot{x}(t) = \sqrt{GA_x} e^{-\frac{\alpha}{2}t} \left[a_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) + a_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_x} \right) \right] \\ \dot{y}(t) = \sqrt{GA_y} e^{-\frac{\alpha}{2}t} \left[b_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) + b_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_y} \right) \right] \\ \dot{z}(t) = \sqrt{GA_z} e^{-\frac{\alpha}{2}t} \left[c_1 J_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) + c_2 N_1 \left(\frac{2}{\alpha} e^{-\frac{\alpha}{2}t} \sqrt{GA_z} \right) \right] \end{cases} \quad (4.61)$$

and the integration constants by the relation:

$$\left. \begin{aligned}
 a_1 &= -\frac{\Pi}{\alpha} \left[\left(x_n - \frac{B_x}{A_x} \right) \sqrt{GA_x} N_1 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) - \dot{x}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) \right] \\
 a_2 &= \frac{\Pi}{\alpha} \left[\left(x_n - \frac{B_x}{A_x} \right) \sqrt{GA_x} J_1 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) - \dot{x}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_x} \right) \right] \\
 b_1 &= -\frac{\Pi}{\alpha} \left[\left(y_n - \frac{B_y}{A_y} \right) \sqrt{GA_y} N_1 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) - \dot{y}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) \right] \\
 b_2 &= \frac{\Pi}{\alpha} \left[\left(y_n - \frac{B_y}{A_y} \right) \sqrt{GA_y} J_1 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) - \dot{y}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_y} \right) \right] \\
 c_1 &= -\frac{\Pi}{\alpha} \left[\left(z_n - \frac{B_z}{A_z} \right) \sqrt{GA_z} N_1 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) - \dot{z}_n N_0 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) \right] \\
 c_2 &= \frac{\Pi}{\alpha} \left[\left(z_n - \frac{B_z}{A_z} \right) \sqrt{GA_z} J_1 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) - \dot{z}_n J_0 \left(\frac{2}{\alpha} \sqrt{GA_z} \right) \right]
 \end{aligned} \right\} \quad (4.62)$$

Let us consider the case where the parameter α is large and where $t_i - t_{i-1} = 1$ [s] $i = 1, 2, \dots, n$,

Then, following (4.57)₁, we have:

$$\begin{aligned}
 A_x &= 2 \left[e^{-n\alpha} p_{x_1} + \dots + e^{-1\alpha} p_{x_{n-1}} + e^{-0\alpha} p_{x_n} \right] = 2 \left[e^{-n\alpha} p_{x_1} + \dots + e^{-1\alpha} p_{x_{n-1}} \right] \\
 &+ 2p_{x_n} \leq 2 \max_{1 \leq i \leq n-1} p_{x_i} \left[e^{-\alpha} + e^{-2\alpha} + \dots + e^{-n\alpha} \right] + 2p_{x_n} = \\
 &= 2e^{-\alpha} \frac{1 - (e^{-\alpha})^{n-1}}{1 - e^{-\alpha}} \max_{1 \leq i \leq n-1} p_{x_i} + 2p_{x_n} \approx 2p_{x_n}
 \end{aligned}$$

If we analyse (4.57) and (4.58) analogously, we will obtain:

$$\begin{cases}
 A_x = 2p_{x_n} \\
 A_y = 2p_{y_n} \\
 A_z = 2p_{z_n}
 \end{cases} \quad (4.63)$$

and

$$\begin{cases} B_x = 2p_{x_n} x_{0n} \\ B_y = 2p_{y_n} y_{0n} \\ B_z = 2p_{z_n} z_{0n} \end{cases} \quad (4.64)$$

and

$$\begin{cases} x_n - \frac{B_x}{A_x} = x_n - x_{0n} = \Delta x_n \\ y_n - \frac{B_y}{A_y} = y_n - y_{0n} = \Delta y_n \\ z_n - \frac{B_z}{A_z} = z_n - z_{0n} = \Delta z_n \end{cases} \quad (4.65)$$

Assuming the following symbols:

$$\begin{cases} T_{01} = \frac{2}{\alpha} \sqrt{2Gp_{x_n}}; & T_1 = e^{-\frac{\alpha}{2}t} T_{01} \\ T_{02} = \frac{2}{\alpha} \sqrt{2Gp_{y_n}}; & T_2 = e^{-\frac{\alpha}{2}t} T_{02} \\ T_{03} = \frac{2}{\alpha} \sqrt{2Gp_{z_n}}; & T_3 = e^{-\frac{\alpha}{2}t} T_{03} \end{cases} \quad (4.66)$$

where: $t \in [t_n, t_{n+1}]$

the vector of the position and velocity described by systems of equations (4.60) and (4.61), respectively, where the integration constants are described by (4.62), will read:

$$\begin{cases} x(t) = x_{0n} + W_{11}\Delta x_n + W_{21}\dot{x}_n \\ y(t) = y_{0n} + W_{12}\Delta y_n + W_{22}\dot{y}_n \\ z(t) = z_{0n} + W_{13}\Delta z_n + W_{23}\dot{z}_n \end{cases} \quad (4.67)$$

$$\begin{cases} \dot{x}(t) = \dot{x}_n + W'_{11} \Delta x_n + W'_{21} \dot{x}_n \\ \dot{y}(t) = \dot{y}_n + W'_{12} \Delta y_n + W'_{22} \dot{y}_n \\ \dot{z}(t) = \dot{z}_n + W'_{13} \Delta z_n + W'_{23} \dot{z}_n \end{cases} \quad (4.68)$$

where: $t \in [t_n, t_{n+1}]$

$$\begin{cases} W_{1i} = \frac{\Pi T_{0i}}{2} [J_1(T_{0i})N_0(T_i) - N_1(T_{0i})J_0(T_i)] \\ W_{2i} = \frac{\Pi}{\alpha} [N_0(T_{0i})J_0(T_i) - J_0(T_{0i})N_0(T_i)] \\ W'_{1i} = \frac{\Pi \alpha}{4} T_i T_{0i} [J_1(T_{0i})N_1(T_i) - N_1(T_{0i})J_1(T_i)] \\ W'_{2i} = \frac{\Pi}{2} T_i [N_0(T_{0i})J_1(T_i) - J_0(T_{0i})N_1(T_i)] - 1 \end{cases} \quad (4.69)$$

$i = 1, 2, 3$

T_{0i} and T_i are determined by (4.66) and, J_1 , N_0 , N_1 are Bessel functions.

W_{1i} and W_{2i} can be interpreted as weights of the vector of position, whereas W'_{1i} and W'_{2i} are those of the vector of velocity.

W_{1i} , W_{2i} , W'_{1i} and W'_{2i} are functions of parameters α , G and the parameter relating to variance. They are also functions of time, where $t \in [t_n, t_{n+1}]$.

Our objective is to optimise parameters α and G so that the function of two variables:

$$f(\alpha, G) = \sum_{i=1}^n [\vec{r}(t_i) - \vec{r}_{0i}]^2 = \text{minimum}$$

or, written in co-ordinates:

$$f(\alpha, G) = \sum_{i=1}^n \left\{ [x(t_i) - x_{0i}]^2 + [y(t_i) - y_{0i}]^2 + [z(t_i) - z_{0i}]^2 \right\} = \text{minimum} \quad (4.70)$$

where:

x_{0i}, y_{0i}, z_{0i} – position of the particle (aircraft) at the instance t_i (data obtained from a satellite)

$x(t_i), y(t_i), z(t_i)$ – position of the particle (aircraft) at the instance t_i computed from (4.67)

and

$x_{0n}, y_{0n}, z_{0n}, \dot{x}_{0n}, \dot{y}_{0n}, \dot{z}_{0n}$ - particle position and velocity data and W_{1i}, W_{2i} computed from (4.69) following the computing of the value of Bessel function (tabulated).

INITIAL VERIFICATION OF THE MATHEMATICAL MODEL

The proposed algorithm of the alternative navigation filter was implemented due to the lack of constructive solutions and the high level of the problem complexity. Measurement data obtained as a result of a number of experiments, as well as flight trajectory profiles or the number of factors affecting the level of accuracy and the assumed probability level of correct position determination, point out to the necessity of working out an innovatory method of data (position) prediction in the event of their lack in the population of experimental data.

The study is based on the defined function of a position potential which requires optimisation of that function by means of seeking its minimum, which, from the character of the function, guarantees the maximum approximation of estimated positions and the minimum error (3D position). Special software, based on evolutionary algorithm theory⁴⁶ [Ombach 2004], was prepared by the author in order to analyse the process of seeking the minimum of the function. Its implementation became necessary since the traditional methods of finding global extremes of functions, such as:

- elementary method (partial derivatives);
- linear programming method
- maximum probability method

or, finally,

- the method of solving complex equation systems

did not guarantee obtaining satisfactory results. Because of the above-mentioned facts, using an evolutionary algorithm has become a necessity, since the mathematical apparatus applied in it has turned out to be indispensable in achieving the set aim. It was possible thanks to the implementation of the concepts included in the study concerning the utilisation of evolutionary algorithms, "Theory of Evolutionary Algorithms", mentioned above.

In order to present the effectiveness of the method used, the algorithm was used to process data obtained during the descent phase of Flight 1 of TS-11 Iskra aircraft in "ODRA" experiment, on No. 7th, 2003.

Fig. 74 shows the vertical plot of the glide path from the altitude of 700 m to the airfield elevation. Optimisation was carried out for discrete time intervals $s, s+1, s+2$ in the following way:

Potential (formula 4.17) is calculated for instances $s, s+1$ and the following function is optimised:

$$f(\alpha, G) = \sum_{i=1}^n \left\{ [x(t_i) - x_{0i}]^2 + [y(t_i) - y_{0i}]^2 + [z(t_i) - z_{0i}]^2 \right\}$$

The values of the dissipation exponent $\alpha = 2.84[1/s]$ and of the parameter $G = 0.35 [m^2/s^2]$ (positioning uncertainty, corresponding to Newton's gravitation constant) were obtained as a result of optimisation. The vertical plot of the glide path for the above parameters is presented in Fig. 75. The solid line shows the optimised trajectory of the aircraft and the markers on the graph show the real position of the aircraft in a given time interval.

It can be observed in Fig. 74 that analysed phase of the flight was stable. The optimised spatial position (Fig. 75) is close to the real position of the aircraft. It should be stressed that the assumed method proves to be useful in a stable-flight-phase, in a short time interval and at time increments = 2 s .

While analysing Fig. 74, we can notice a specific moment of flight parameter disturbance – the vertical descent rate decreased. It is especially visible in the 58th second of the flight – Fig. 76 - when the correctness of the optimisation was disturbed. The plot representing the real flight trajectory (markers) deviates considerably from the solid line obtained as a result of the approximation by means of an aircraft trajectory potential function. The values of both parameters ($\alpha = 3.82$ [1/s]), ($G = 0.54$ [m²/s²]) prove that a rapid change of flight parameters causes the increase of the difference between the real flight trajectory and the optimised one. As soon as the flight parameters stabilise – Fig. 77 – the optimisation is practically identical with the real trajectory and that is visible to the altitude of 290 m.

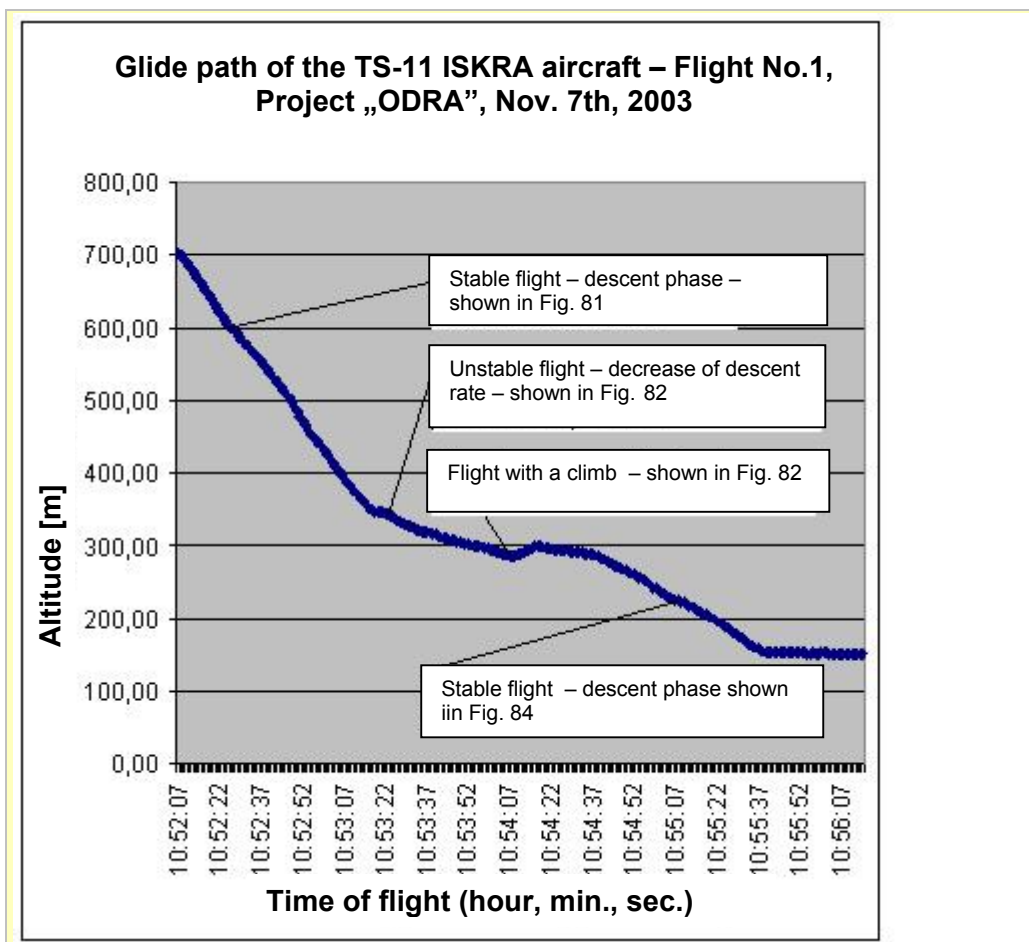


Fig. 74. Glide path of the TS-11 ISKRA aircraft.

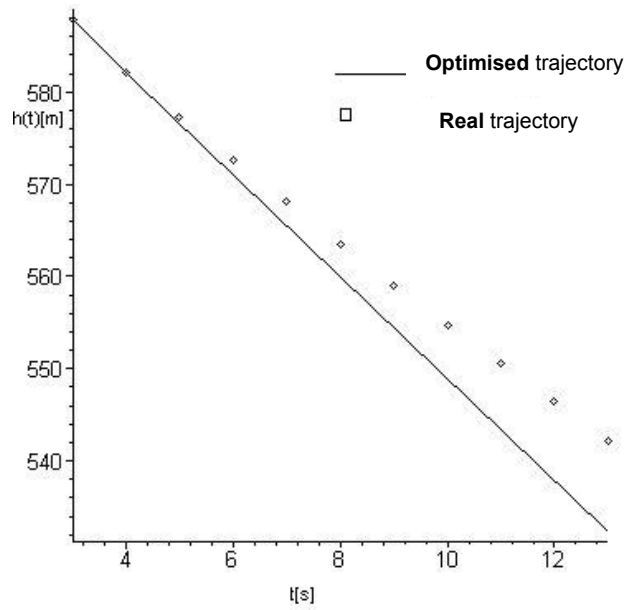


Fig. 75. Descent phase – stable flight.

Below that altitude there is a transition to horizontal flight, which is illustrated by the fact that the optimised plot deviates from the real trajectory. The values of both parameters were $\alpha = 2.84$ [1/s] and $G = 0.35$ [m²/s²].

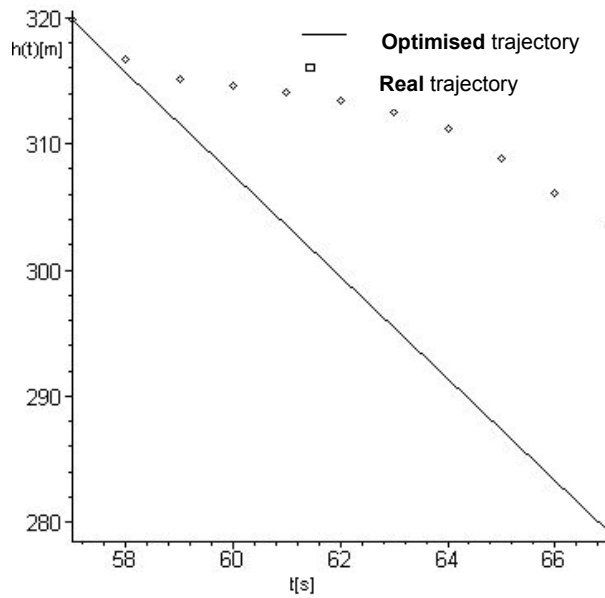


Fig. 76. Descent phase – illustration of flight parameter disturbance.

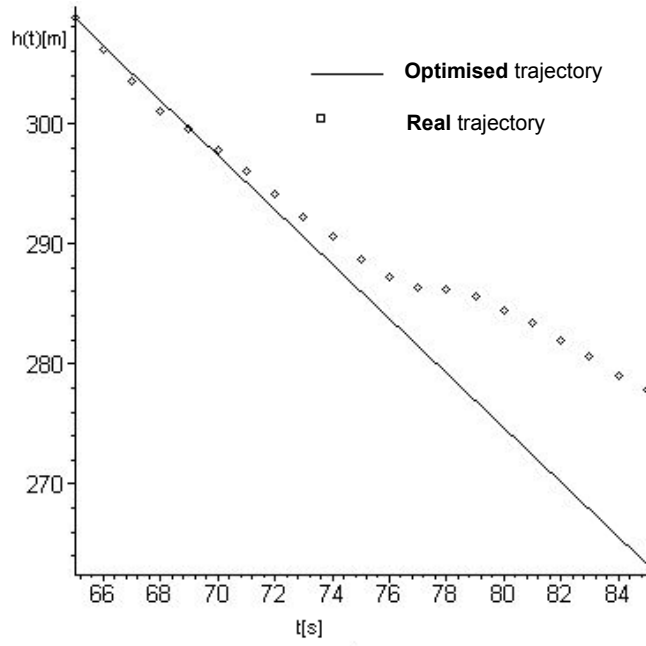


Fig. 77. Descent phase – subsequent flight parameter disturbance (flight with a climb).

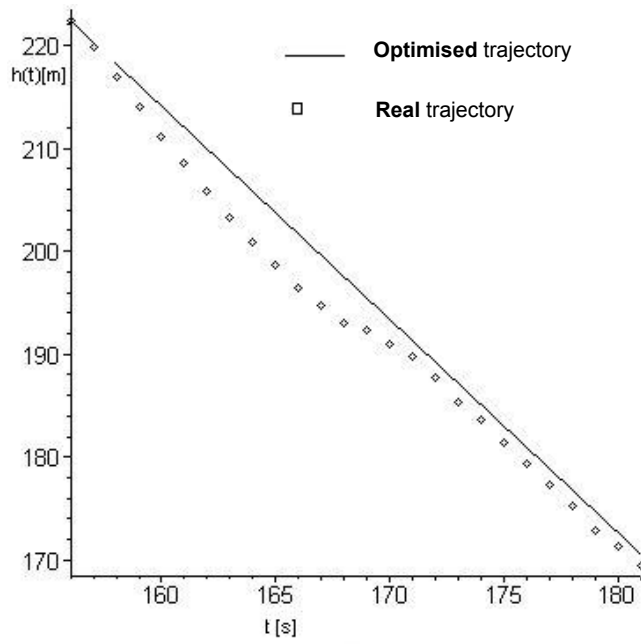


Fig. 78. Descent phase – return to stable parameters.

Optimisation of the last flight phase, from the altitude of 220 m to 170 m shows the concordance between the optimised trajectory and the real one. At that altitude there is a transition to horizontal flight, which is illustrated by the fact that the optimised plot deviates from the real trajectory. The values of both parameters were ($\alpha = 2.67$ [1/s] and $G = 0.23$ [m²/s²], for $t = 2$ [s].

The above analysis of various flight phases during Flight No.1 shows that applying a new alternative navigation filter as an expansion of the solution proposed by Vanicek and Inzinga is justified.⁴⁷ [Inzinga 1985]

The effectiveness of the proposed solution may be further justified by a great divergence of the obtained parameters of the aircraft position when the parameters α and G were selected randomly without the application of the evolution algorithm of the function potential optimisation. In the presented example: $\alpha = 0.2$ [1/s] and $G = 5.0$ [m²/s²] for $t = 2$ [s]. The illustration of that phenomenon is Fig. 79.

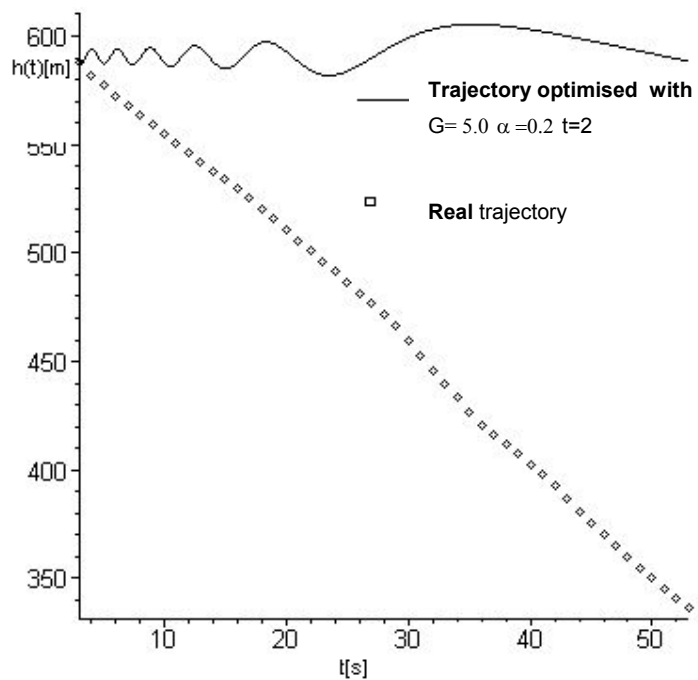


Fig. 79. Illustration of the divergence between the optimised and the real flight trajectory with randomly selected parameters α and G .

SUMMARY:

The main purpose of the submitted dissertation was to develop an alternative navigation filter. The basic idea of the mathematical model based on the position potential theory, presented within the scope of the present work was born during scientific research carried out at the Polish Air Force Academy.

Scientific activities mentioned above were focused on the problem of estimating the aircraft positioning precision during en-route flights as well as in approaching and landing phases of flight. In addition, theoretical works by Inzinga and Vanicek concerning the two-dimensional space (X, Y) were taken into consideration.

During scientific activities, the main effort was put on the problem of landing safety as landing is the most complicated phase of the flight to be completed, especially in difficult atmospheric conditions – with no visibility of the ground. The scope of studies was very broad and all the experiments were performed with very intensive usage of GPS, GLONASS and EGNOS systems in various weather conditions.

Many analyses were also performed within the phase of practical, in-flight experiments. They covered the problems of the quality and fixation of GPS antennas as well as the antennas of data transmission systems. The results obtained during the experiments were the first ones in Polish Air Force and unique in this field. They constituted the fundamentals for preparing new solutions and have been implemented in the form of the new equipment of the Su-22 M4 cockpit. Databases created during many phases of different scientific projects, provided very broad scope of knowledge to be used for implementing satellite receivers as a main navigation tool to support aircraft navigation in space.

The primary reasons for elaborating the subject of the present work were problems of GPS data exchange continuity, closely related to the flight safety problem. Continuous GPS data exchange and its availability is of vital importance from the flight safety perspective. The main thesis of the project has been fully confirmed.

The scientific goal of the project - developing a new navigation filter as a tool supporting an aircraft landing has been achieved. The mathematical model, i.e. the new navigation filter went through preliminary verification process, which fully justified the correctness of the thesis taken as a basis for theoretical and experimental stages of the study.

It is worth mentioning that the presented dissertation, includes the results of practical in-flight experiments, obtained during experimental flights, which were carried out for the first time in Poland in such a broad extend and that the theoretical assumptions were successfully verified by a simulation program.

The results obtained within the scope of the study secure the work continuity of the system. The present study is also a valuable starting point for many new projects and research programs which are to bring new solutions – new navigation filters in the form of modules, being integrated parts of satellite navigation receivers. Additionally, the scope of future developments should and will include the correctness verification of operations performed by new, filter based, navigation tools during all phases of flight.

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ABBREVIATIONS

AA - Absolute Altitude
ACARS - Aircraft Communications Addressing and Reporting System;
ACP - Azimuth Count Pulse;
AFTN - Aeronautical Fixed Telecommunication Network;
AGL - Above Ground Level
AIP - Aeronautical Information Publication;
AIRAC - Aeronautical Information Regulation and Control;
APW - Area Proximity Warning;
ASR - Approach Surveillance Radar;
ATC - Air Traffic Control;
ATIS - Automatic Terminal Information Service;
ATM System - Air Traffic Management System;
CAVOK – Ceiling and Visibility OK;
CVOR - Conventional Very High Frequency Omnidirectional Radio Range;
DGPS- Differential Global Positioning System;
DME – Distance Measuring Equipment
DRL – Omni-directional radar (Russian: *dookružnyj radiolokator*);
DVOR - Doppler Very High Frequency Omnidirectional Radio Range;
EGNOS - European Geostationary Navigation Overlay Service;
ETRF – European Terrestrial Reference Frame
FMG - Frequency Management Group;
GFSK - Gaussian Frequency Shift Keying;
GLONASS – Global Navigation System;
GPS - Global Positioning System;
HDOP – Horizontal Dilution of Precision (Latitude, Longitude);
ICAO – International Civil Aviation Organization;
ILS – Instrument Landing System;
INS – Inertial Navigation System;
ITP – Aeroplane Flight Manual (Polish: *Instrukcja Techniki Pilotowania*);
JPALS – Joint Precision Approach and Landing System;
MLS – Microwave Landing System
MOC –Minimum Obstacle Clearing;
MOR - Meteorological Optical Range;
MSAW - Minimum Safe Altitude Warning;
MTBO - Mean Time Between Outages;
NATO – North Atlantic Treaty Organization
NDB - Non Directional Beacon;
NOTAM - Notice To Airmen;
OLDI - On-line Data Interchange;
OTF – On- the Fly;
PAR – Precision Approach Radar;
PDOP – Position Dilution of Precision
PNAV – Precision Seekerless Guidance PNAV
POLREF – POLish REFerence Frame;

PPDB – Initial point/push point
PRL – Landing Radar (Russian: *pasadocznyj radiolokator*);
PRN – Pseudo Radon Noise;
PSR - Position Search Radar;
QFE - Atmospheric Pressure at Field Elevation ;
QNH -Atmospheric Pressure at Nautical Height;
RAIM - Receiver Autonomous Integrity Monitoring;
RH - relative humidity;
RMS – Root Mean Square. The standard deviation of the terror In the GPS location;
RNP – Required Navigation Performance;
RSL – Radar-Based Landing System (Polish: *Radiolokacyjny System Lądowania*);
RSP – . RSP 7 Landing Radar;
RVR - Runway Visual Range;
RTK – Real-Time Kinematic;
RTCM – Radio Technical Commission for Maritime Services;
SAASM – Selective Availability Anti-Spoofing;
SBAS - Space Based Augmentation System;
Site Acceptance Test;
SPI - Special Position Identifier;
SSR - Secondary Surveillance Radar;
STC - Sensitivity Time Control;
TMA - Terminal Control Area;
UPS - Uninterruptible Power Supply;
VDL -VHF Digital Link;
VHF - Very High Frequency;
VOLMET - Meteorological information for aircraft in flight;
VOR – Very High Frequency Omnidirectional Range;
WASS – Wide Area Augmentation System;
WGS-84 – World Geodetic System 1984;

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