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MANOEUVRING TO REQUIRED APPROACH PARAMETERS - DISTANCE AND TIME ABEAM

ABSTRACT Formulae for calculation of own speed and course to achieve a required distance and time abeam in respect to a selected object are derived and their graphical interpretation is provided.

INTRODUCTION

The predicted object distance abeam D_{ab} and the time interval to its occurrence T_{Dab} are sometimes used additional criteria for collision threat. The scope of this paper is aimed at the problem which although it can be and it is connected with collision avoidance manoeuvres, but it is rather reversed and can be applied for intentional approaches or in naval tactical manoeuvres - what own speed and/or course manoeuvre should be undertaken to achieve the required distance abeam D_{ab} and /or time to this distance T_{Dab} ?

ASSUMPTIONS AND INPUT PARAMETERS

For the purposes of this analysis, own vessel and extraneous objects of interest are regarded as if the mass of each object was concentrated at a point. It will be assumed that all moving external objects are travelling at constant speed and course. In the movable plane tangential to the Earth's surface Cartesian coordinates system Ox, Oy (Fig. 1) with Oy pointing North O is at the present position of own vessel. It will also be assumed that manual plots or the radar processing and tracking has yielded the present relative position of the extraneous object X, Y and components of its true V_{tx}, V_{ty} or relative V_{rx}, V_{ry} speed. The relationship of the own and the object speeds can be described by equations

$$V_{tx} = V_{rx} + V_x \tag{1}$$

$$V_{ty} = V_{ry} + V_y \tag{2}$$

where: V_x, V_y - own speed components,

$$V_x = V \sin \psi \tag{3}$$

$$V_y = V \cos \psi \tag{4}$$

$$V = \sqrt{V_x^2 + V_y^2} \tag{5}$$

ψ - own course (the angle measured clockwise from Oy to V).

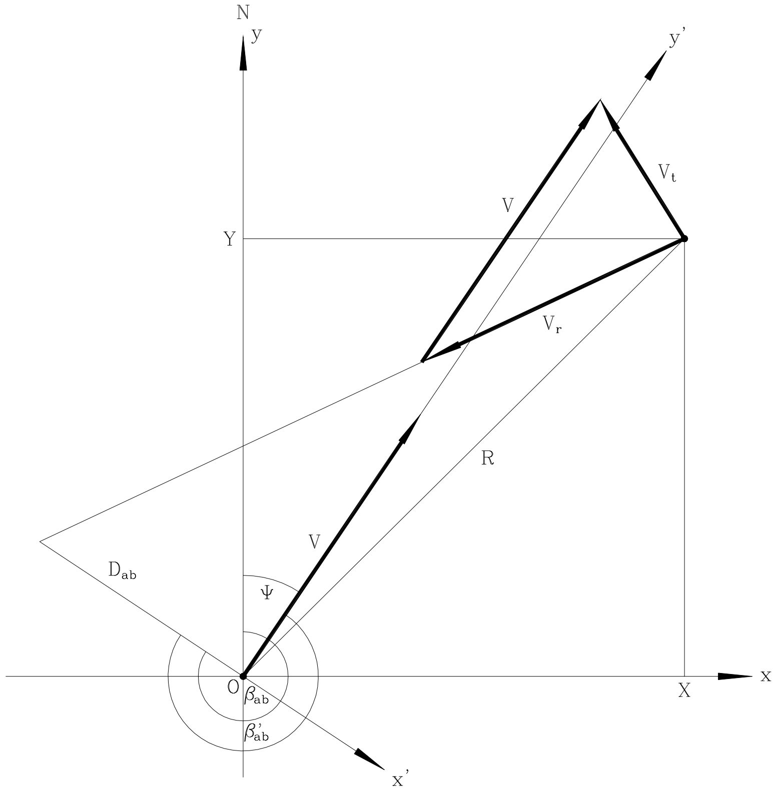

Fig. 1. Input parameters

The own and the object's motion parameters should be either ground or sea referenced and a drift angle is assumed to be zero. The relative position of an extraneous object, at time t, is given by

$$X(t) = X + V_{rx}\, t \tag{6}$$

$$Y(t) = Y + V_{ry}\, t \tag{7}$$

and then [Lenart, 1986]

$$D_{ab} = \frac{XV_{ry} - YV_{rx}}{V_{rx} \sin\psi + V_{ry} \cos\psi} \tag{8}$$

$$\tan\frac{\psi}{2} = \frac{A_{\psi TCPA} \pm \sqrt{A^2_{\psi TCPA} + B^2_{\psi TCPA} - C^2_{\psi TCPA}}}{B_{\psi TCPA} + C_{\psi TCPA}} \tag{9}$$

$D_{ab}>0$ means that the object will be abeam on starboard side of the own vessel, and if $D_{ab}<0$ the object will be abeam on port side.

DERIVATION OF EQUATION $V = f(\psi, D_{AB})$

From equations (1) through (4)

$$V_{rx} = V_{tx} - V \sin \psi \qquad (10)$$

$$V_{ry} = V_{ty} - V \cos \psi \qquad (11)$$

Substitution in equation (8) and rearranging yields

$$V = \frac{A_{Dab} V_{tx} - V_{ty}}{A_{Dab} \sin\psi - \cos\psi} \qquad (12)$$

where:

$$A_{Dab} = \frac{Y + D_{ab} \sin\psi}{X - D_{ab} \cos\psi} \qquad (13)$$

Equation (12) gives the speed V which own vessel must adopt to achieve the required distance abeam D_{ab} (in respect to the selected object) for different assumed own courses ψ, but we should search for solution

$$V \geq 0 \qquad (14)$$

and V, ψ for which

$$T_{ab} \geq 0 \qquad (15)$$

Condition (15) means that the approach abeam is at present or will be in the future and not in the past. Equation (15) (from equations (9), (10) and (11)) can be rearranged to the form

$$(X \sin\psi + Y \cos\psi)(V - V_{tx} \sin\psi - V_{ty} \cos\psi) \geq 0 \qquad (16)$$

A conventional PPI displays the position of each object by plotting them in polar (r, ψ) or Cartesian (x, y) coordinates. If we apply a scaling factor τ to the speed coordinates (V, ψ) or (V_x, V_y) such that

$$r = V\,\tau \qquad (17)$$

$$x = V_x\,\tau \qquad (18)$$

$$y = V_y\,\tau \qquad (19)$$

then the position and speed coordinates can be plotted on a common display.

In the combined coordinates frame for plotting position and speed can also be plotted positions and speed vectors of objects and the own speed vector (real or simulated). Figure 2 illustrates a family of lines (12) for various required D_{ab} and an exemplary object.

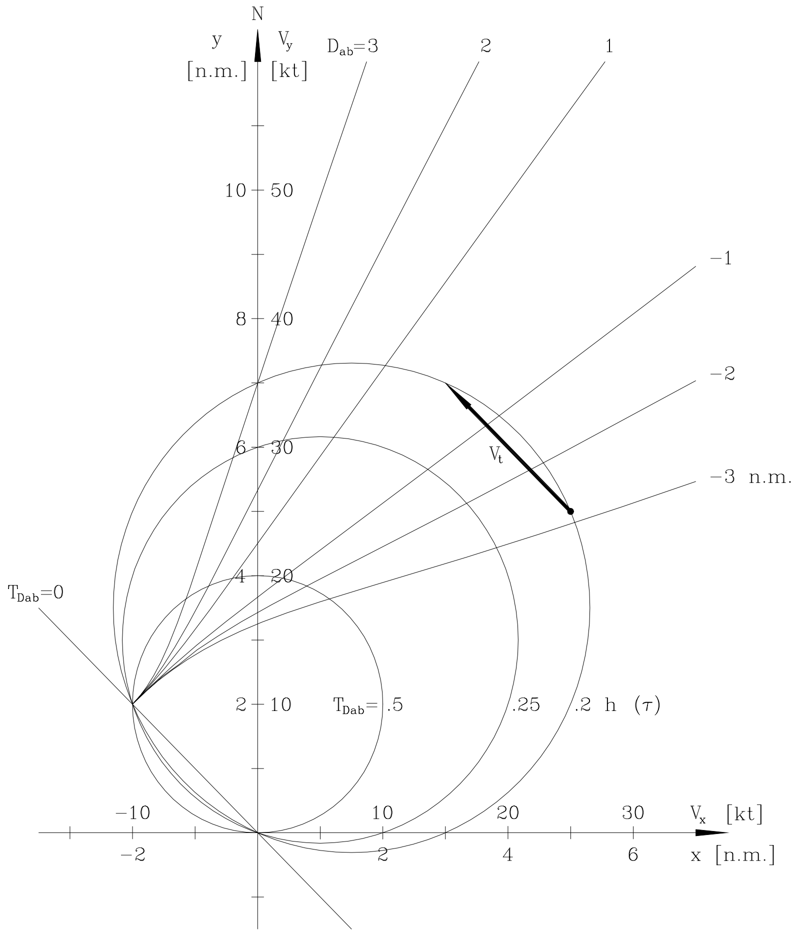

Fig. 2. Lines D_{ab}=const. and circles T_{Dab}=const.
τ=0.2 h, X=Y=5 n.m., V_{tx}= -10 kt, V_{ty}=10 kt

DERIVATION OF EQUATION $\psi = g\,(V, D_{AB})$

Substitution in equation (12) equation (13) and rearranging yields

$$A_{VDab} \sin\psi - B_{VDab} \cos\psi + C_{VDab} = 0 \qquad (20)$$

where:

$$A_{VDab} = YV - D_{ab}V_{tx} \qquad (21)$$

$$B_{VDab} = XV + D_{ab}V_{ty} \qquad (22)$$

$$C_{VDab} = D_{ab}V + XV_{ty} - YV_{tx} \qquad (23)$$

If we search for own course ψ which will lead to the required distance abeam D_{ab} at an assumed own speed V then we can get an inverse function $\psi=g(V, D_{ab})$ to the function $V=f(\psi, D_{ab})$ by substitution in equation (20) trigonometric identities

$$\sin\psi = \frac{2\tan\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{24}$$

$$\cos\psi = \frac{1-\tan^2\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{25}$$

and after solving

$$\tan\frac{\psi}{2} = \frac{-A_{Dab} \pm \sqrt{A_{Dab}^2 + B_{Dab}^2 - C_{Dab}^2}}{B_{Dab} + C_{Dab}} \tag{26}$$

Real solutions exist if

$$A_{Dab}^2 + B_{Dab}^2 \geq C_{Dab}^2 \tag{27}$$

and equation (26) can give up to two own courses ψ which will lead to the required distance abeam D_{ab} at an assumed own speed V if they additionally fulfil condition (16).

DERIVATION OF EQUATION $V = f(\psi, T_{DAB})$

Substitution in equation (9) equations (15) and (16) results in equation

$$T_{Dab} = -\frac{X\sin\psi + Y\cos\psi}{V_{tx}\sin\psi + V_{ty}\cos\psi - V} \tag{28}$$

hence

$$V = A_{TDab}\sin\psi + B_{TDab}\cos\psi \tag{29}$$

where:

$$A_{TDab} = V_{tx} + \frac{X}{T_{Dab}} \tag{30}$$

$$B_{TDab} = V_{ty} + \frac{Y}{T_{Dab}} \tag{31}$$

Equation (29) can yield the speed $V \geq 0$ which own vessel must adopt to achieve the required time to distance abeam T_{Dab} (in respect to the selected object) for different assumed own courses ψ.

A graphical interpretation of solutions given by equation (29) can be obtained in Cartesian coordinates of own speed V_x, V_y substituting in equation (29) equations (3) through (5)

$$(V_x - \tfrac{1}{2}A_{TDab})^2 + (V_y - \tfrac{1}{2}B_{TDab})^2 = \left(\tfrac{1}{2}A_{TDab}\right)^2 + \left(\tfrac{1}{2}B_{TDab}\right)^2 \qquad (32)$$

The above equation reveals that the locus of points for which T_{Dab} is a constant is a circle centred at $(\tfrac{1}{2}A_{TDab}, \tfrac{1}{2}B_{TDab})$ and crossing the origin of coordinates. Fig. 2 illustrates a family of circles for various values of T_{Dab}.

DERIVATION OF EQUATION ψ = g(V, T_{DAB})

If we search for own course (which will lead to the required time to distance abeam T_{Dab} at an assumed own speed V then we can get an inverse function ψ=g(V, T_{Dab}) to the function V=f(ψ, T_{Dab}) by substitution in equation (29) identities (24) and (25) which after solving yields

$$\tan\frac{\psi}{2} = \frac{A_{TDab} \pm \sqrt{A_{TDab}^2 + B_{TDab}^2 - V^2}}{B_{TDab} + V} \qquad (33)$$

Real solutions exist if

$$A_{TDab}^2 + B_{TDab}^2 \geq V^2 \qquad (34)$$

and equation (34) can give up to two own courses (which will lead to the required time to distance abeam T_{Dab} at an assumed own speed V.

DERIVATION OF EQUATIONS V, ψ = f(D_{AB}, T_{DAB})

In the rotated by angle (coordinates frame x', y' (Fig. 1) equations (6) and (7) transform to

$$X'(t) = X\cos(- Y\sin(+ (V_{rx}\cos(- V_{ry}\sin\psi)\, t \qquad (35)$$

$$Y'(t) = X\sin(+Y\cos(+ (V_{rx}\sin(+ V_{ry}\cos\psi)\, t \qquad (36)$$

Since

$$X'(T_{Dab}) = D_{ab} \qquad (37)$$

$$Y'(T_{Dab}) = 0 \qquad (38)$$

therefore after substitution of equations (10) and (11) equation (35) can be rearranged to

$$B_{TDab} \sin\psi - A_{TDab} \cos\psi + C_{ab} = 0 \tag{39}$$

where:

$$C_{ab} = \frac{D_{ab}}{T_{Dab}} \tag{40}$$

and equation (36) to equation (29).
Equation (39) has solutions

$$\tan\frac{\psi}{2} = \frac{-B_{TDab} \pm \sqrt{A_{TDab}^2 + B_{TDab}^2 - C_{ab}^2}}{A_{TDab} + C_{ab}} \tag{41}$$

Real solutions exist if

$$A_{TDab}^2 + B_{TDab}^2 \geq C_{ab}^2 \tag{42}$$

and substitution of equation (41) to equation (29) (with application of equations (24) and (25)) yields

$$V = \pm\sqrt{A_{TDab}^2 + B_{TDab}^2 - C_{ab}^2} \tag{43}$$

We search for solutions $V \geq 0$ and therefore finally

$$V = \sqrt{A_{TDab}^2 + B_{TDab}^2 - C_{ab}^2} \tag{44}$$

$$\tan\frac{\psi}{2} = \frac{V - B_{TDab}}{A_{TDab} + C_{ab}} \tag{45}$$

Equations (44) and (45) can give own speed V and own course ψ which will lead to the required distance abeam D_{ab} of the selected object at time to this distance T_{Dab}.

POSITION OF DISTANCE ABEAM

When the object is abeam then the relative position of the object (in respect to our vessel) is (X_{ab}, Y_{ab}) or in polar coordinates (D_{ab}, β_{ab}) or (D_{ab}, β'_{ab}) where β_{ab} and β'_{ab} are the object's true and relative bearings abeam respectively. These parameters are given by equations

$$X_{ab} = X + V_{rx} T_{Dab} = X + (V_{tx} - V \sin\psi)\, T_{Dab} \tag{46}$$

$$Y_{ab} = Y + V_{ry} T_{Dab} = Y + (V_{ty} - V \cos\psi)\, T_{Dab} \tag{47}$$

$$D_{ab} = \sqrt{X_{ab}^2 + Y_{ab}^2} \tag{48}$$

$$\beta'_{ab} = 90° \text{ and } \beta_{ab} = \psi + 90° \text{ if } D_{ab} > 0 \tag{49}$$

$$\beta'_{ab} = 180° \text{ and } \beta_{ab} = \psi + 180° \text{ if } D_{ab} < 0 \tag{50}$$

and D_{ab} and T_{Dab} are either required or calculated, T_{Dab} from equation (28) and D_{ab} from equation (48) or (8) transformed to true speeds by substitution equations (10) and (11)

$$D_{ab} = \frac{XV_{ty} - YV_{tx} - (X\cos\psi - Y\sin\psi)V}{V_{tx}\sin\psi + V_{ty}\cos\psi - V} \tag{51}$$

TIME TO MANOEUVRE

It has to be emphasised that the calculated above manoeuvres are kinematics and should be undertaken immediately. If we require to have the time lapse Δt for calculations, for the decision to initiate a manoeuvre and for the execution of the calculated manoeuvre then X, Y in the above equations should be replaced by $X_{\Delta t}$, $Y_{\Delta t}$ respectively, given by equations

$$X_{\Delta t} = X + V_{rx}\Delta t \tag{52}$$

$$Y_{\Delta t} = Y + V_{ry}\Delta t \tag{53}$$

REFERENCES

1. Lenart A. S.: Some selected problems in analysis and synthesis of shipboard collision avoidance systems. Zeszyty Naukowe Politechniki Gdańskiej Nr 405 - Budownictwo Okrętowe Nr XLIV, Gdańsk 1986 (in Polish).

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