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JAKUB MONTEWKA
Maritime University of Szczecin

MATHEMATICAL MODEL OF UNCERTAINTY OF SHIP'S SAFETY MANEUVERING AREA DETERMINATION

ABSTRACT

The paper presents uncertainty model of the original method based on optoelectronic measurements used for ship's safety maneuvering area determination. The method takes advantage of land borne laser rangefinder and encoder measurements. The results obtained from the method are compared with results obtained from GNSS RTK method and the uncertainty model is evaluated and implemented.

Keywords:
safety maneuvering, uncertainty.

INTRODUCTION

Presented model concerns original method of ship's safety area determination, based on advanced geodetic measuring techniques and instruments like land borne laser rangefinder with encoder attached. The method has been worked out in the Institute of Marine Traffic Engineering at MU Szczecin. The algorithms adopted in the method for determination the ship's safety maneuvering area and probability of ship's collision with obstacle are based on ship paths' distribution [2, 4]. The detailed description of discussed method one may find in the following publications [1, 7], therefore author took the liberty of omitting this part and focus more on the subject of the paper, which is the uncertainty model. The aims of experiments which were conducted along the approach channel to port of Świnoujście in western coast of Poland were:

- to determine ship's safety maneuvering area by means of original method;
- to determine value of uncertainty of above method.

As a standard ship m/f Jan Śniadecki was adopted, which was a tailor made ship for Świnoujście — Ystad line, presented in figure 1.

Fig. 1. The investigated ship, m/f Jan Śniadecki

To determine the value of proposed method's uncertainty, results obtained from the method were compared with results obtained from GNSS RTK measurements, which were adopted as the standard results. Two experiments were conducted simultaneously, one from the shore, and another on board the ship m/f Jan Śniadecki. The aim of both experiments was to evaluate safety maneuvering area parameters. The layout investigated waterway is presented in figure 2.

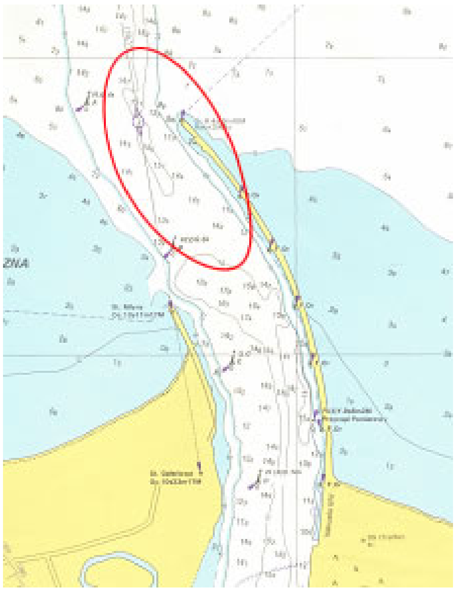

Fig. 2. The layout of investigated area

The hydro meteorological conditions encountered during experiment were classified into three groups, as shown in table 1. The primary factors which determined the classification were wind direction and speed.

Table 1. Hydro meteorological conditions observed during experiment

Variant	Wind — direction	Wind velocity [m/s]	Current — direction	Current — velocity [kn]
1	NE, E, SE	3–6	–	0
2	W	5–7	Towards Sea	0.2
3	NW, W, SW	3–5	Towards Harbor	0.4

EVALUATING AND EXPRESSING THE UNCERTAINTY

In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement, that is, the measurand, and thus the result is complete only when accompanied by a quantitative statement of its uncertainty [3]. The uncertainty of measurement is defined as difference between measured values obtained from experiment and the true value. The difference may be computed by the following formula [5]:

$$\delta = y - y_{TRUE}, \tag{1}$$

where: y — results of measurement;
y_{TRUE} — true value.

The result of measurement is a value of measured parameter, which is usually obtained from the survey. In case of original method, it was assumed that result of this method would be treated as result of measurement (y).

The true value determines explicitly the analyzed parameter, for the conditions encountered during surveys. It was assumed, that results obtained from GNSS RTK survey, would be the true value (y_{TRUE}).

There are several methods for evaluating uncertainty and uncertainty's model determination, one of them is the comparative method. The essence of this method is not to identify the causes of uncertainty, but investigate its consequences by identification the differences between data from the survey and adopted standard. The advantage of this method is its simplicity, and no need to be familiar with all uncertainty's components [6].

Expressing the results of measurement

As a result of measurement (y) the maximum distance of vessel hull outline's to left and right from the adopted reference line (eg.: center line of the fairway) in each sector in single passage obtained from original method, was adopted (fig. 3).

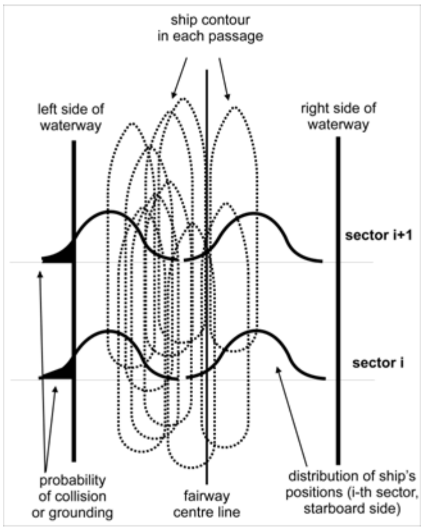

Fig. 3. Distributions of vessel hull outline's maximum distances from the reference line

Therefore, the results of the survey may be expressed as a random variable and represented by the vector. The number of vector's elements is equal to number of sectors, into which the waterway was formerly divided. One vector describes left side of fairway, and another describes right side, as follows:

$$Y_L = [y_{L1}, y_{L2} ... y_{Ln}];$$
$$Y_P = [y_{P1}, y_{P2} ... y_{Pn}], \tag{2}$$

where: y_{L1} — maximum distance of vessel's hull outline to the left from adopted reference line, in single passage;
y_{P1} — maximum distance of vessel's hull outline to the right from adopted reference line, in single passage;
n — number of sectors.

As the random variable for the series of k ship's passages through the investigated area the mean value of maximum distance of vessel's hull outline to the left and right from reference line, computed for all passages may be adopted, and expressed as follows:

$$\overline{Y_L} = [\overline{y_{L1}}, \overline{y_{L2}} ... \overline{y_{Ln}}]; \tag{3}$$

$$\overline{Y_P} = [\overline{y_{P1}}, \overline{y_{P2}} ... \overline{y_{Pn}}], \tag{4}$$

where: $\bar{y}_{L1}$ — mean value of maximum distances of vessel's hull outline to the left from adopted reference line, for series of k passages;
$\bar{y}_{P1}$ — mean value of maximum distances of vessel's hull outline to the right from adopted reference line, for series of k passages;
n — number of sectors.

Expressing the true value

As a true value (y_{TRUE}) the maximum distance of vessel hull outline's to left and right from the adopted reference line (eg.: center line of the fairway) in each sector in single passage obtained from GNSS RTK survey was adopted. Therefore, the results of the survey may be expressed, in a comparable manner to methodology described in previous chapter, as a random variable represented by the following vectors:

$$\begin{aligned} Y_{TL} &= [y_{TL1}, y_{TL2} ... y_{TLn}]; \\ Y_{TP} &= [y_{TP1}, y_{TP2} ... y_{TPn}], \end{aligned} \tag{5}$$

where: y_{TL1} — maximum distance of vessel's hull outline to the left from adopted reference line, in single passage, true value;
y_{TP1} — maximum distance of vessel's hull outline to the right from adopted reference line, in single passage, true value;
n — number of sectors.

The random variable for the series of k ship's passages may be described as a mean value for all passages, and expressed as follows:

$$\begin{aligned} \overline{Y}_{TL} &= [\overline{y}_{TL1}, \overline{y}_{TL2} ... \overline{y}_{TLn}]; \\ \overline{Y}_{TP} &= [\overline{y}_{TP1}, \overline{y}_{TP2} ... \overline{y}_{TPn}], \end{aligned} \tag{6}$$

where: $\bar{y}_{L1}$ — mean of maximum distances of vessel's hull outline to the left from adopted reference line, for series of k passages;
$\bar{y}_{P1}$ — mean of maximum distances of vessel's hull outline to the right from adopted reference line, for series of k passages;
n — number of sectors.

Expressing the uncertainty

On the basis of formula derived above, the uncertainty may be expressed as a random variable described by the following vectors:

$$\Delta_{mL} = [\overline{\delta}_{L1}, \overline{\delta}_{L2} \ldots \overline{\delta}_{Ln}]; \qquad (7)$$
$$\Delta_{mP} = [\overline{\delta}_{P1}, \overline{\delta}_{P2} \ldots \overline{\delta}_{Pn}],$$

where: $\overline{\delta}_{L1}$ — mean value of uncertainty for series of n passages;

$\overline{\delta}_{P1}$ — mean value of uncertainty for series of n passages;

n — number of sectors.

Due to descriptive character of above notation, one may not use it for prediction purposes. To implement the model of uncertainty into the algorithm of original method of ship's safety maneuvering area determination, it is necessary to change the notation from vectorial into functional.

Model of uncertainty estimation, based on regression functions

To express the uncertainty as a regression function it is crucial to define the operands and dependent variables. For the purposes of uncertainty model two operands were adopted:

1) x_d — distance from measuring station to the observed vessel;
2) x_b — an angle between main axis of observed vessel and line connecting her with measuring station.

The dependent variable is the uncertainty of original method of vessel's safety maneuvering area determination for given part of the fairway. The uncertainty is described by two equations. One represents uncertainty for left side of fairway, another for its right side. The model may be expressed by the following general formulae:

$$\delta_m = \begin{cases} \delta_{mL} = \beta_0 + \beta_1 \cdot x_d + \beta_2 \cdot x_b + \beta_3 \cdot x_d \cdot x_b + \beta_4 \cdot x_d^2 + \beta_5 \cdot x_b^2 + \varepsilon \\ \delta_{mP} = \beta'_0 + \beta'_1 \cdot x'_d + \beta'_2 \cdot x'_b + \beta'_3 \cdot x'_d \cdot x'_b + \beta_4 \cdot x'^2_d + \beta_5 \cdot x'^2_b + \varepsilon' \end{cases}, \qquad (8)$$

where: δ_{mL} — uncertainty for the left side of the fairway;
δ_{mP} — uncertainty for the right side of the fairway;
β_j — model's parameters describing influence of given operand;
x_d — oparand — distance;
x_b — oparand — angle;
ε — residuals.

To determine the type of dependence and the regression function's parameters nonlinear estimation was adopted. The assessment of model parameters was done on the basis of the least square method. As the estimation method, the Levenberg — Marquardt method was adopted, which is highly efficient and time saving. The loss function, which describes the divergences between empirical values of dependent variable and its theoretical values obtained from regression function, may be expressed as follows:

$$L = \sum_{i=1}^{n} \left(\delta_{(i)} - \hat{\delta}_{(i)}\right)^2 , \tag{9}$$

where: $\delta_{(i)}$ — observed uncertainty value;
$\hat{\delta}_{(i)}$ — uncertainty value obtained from regression function.

As the results of estimation, two regression functions were obtained:

$$\delta_{mL} = 90{,}0173 - 10{,}31913 \cdot x_d - 1{,}5423 \cdot x_b + 0{,}00278 \cdot x_d \cdot x_b + 0{,}00682 \cdot x_b^2 ;$$

$$\delta_{mL} = 99{,}24635 - 0{,}32049 \cdot x_d - 1{,}99016 \cdot x_b + 0{,}00302 \cdot x_d \cdot x_b + 0{,}01012 \cdot x_b^2 ,$$

where: x_d — oparand — distance;
x_b — oparand — angle.

Model of uncertainty verification

After the model parameters' estimation is completed, verification of its correctness should be conducted. The verification process may be stated as follows:

1) verification of the model parameters correctness;
2) verification of the whole model correctness.

The graphs on the figures below represent the empirical uncertainties as dots and fitted regression functions, as planes. The axis X, Y represent operands (x_d, x_b), while axis Z represents dependent variable (y). The graph presented in figure 4 concerns right side of the fairway, while the graph presented in figure 5 describes its left side. One may notice that estimated regression functions fit quite well to empirical data, and regressions' parameters were estimated correctly. However statistical tests need to be conducted to confirm this proposition.

Verification of model parameters correctness were conducted on the basis of t-Student test. The following hypothesis were constructed:

$\mathbf{H_0}$: $\beta_i = 0$
$\mathbf{H_1}$: $\beta_i \neq 0$

The verification of the whole model correctness took advantage of Fisher test, with following hypothesis constructed:

$\mathbf{H_0}$: $\beta_j = 0$ for $j = 0, 1, 2, \ldots, k$
$\mathbf{H_1}$: there exists such j, that $\beta_j \neq 0$

The tests were conducted for left and right side of the fairway separately. The results of both tests pointed out, that there are no statistical reasons, at the significant level α = 0.05, to adopt any of the $\mathbf{H_0}$ hypothesis. Therefore the model's parameters as well as the model in whole are statistically correct.

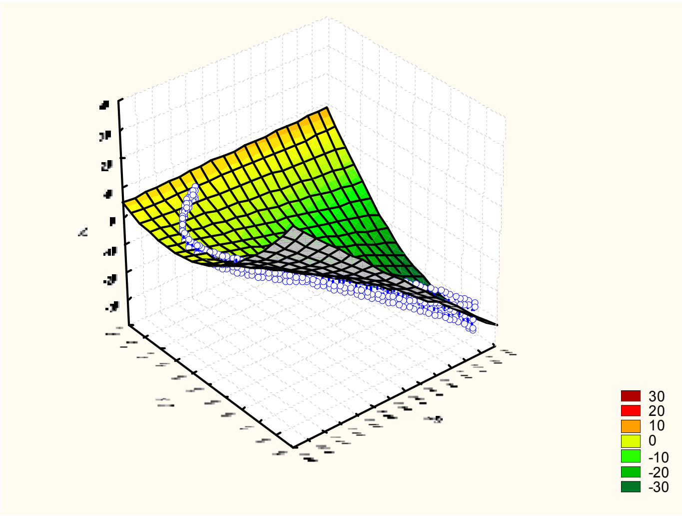

Fig. 4. Three dimensional scatter plot of empirical uncertainty values with fitted plane of regression function — right side of fairway

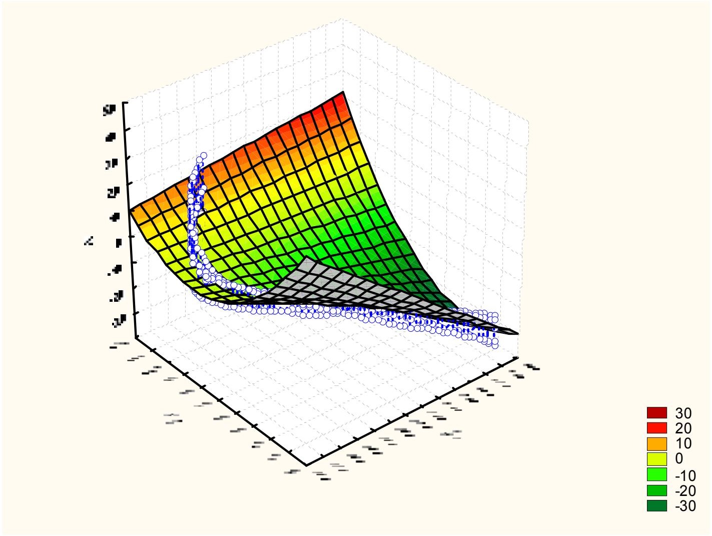

Fig. 5. Three dimensional scatter plot of empirical uncertainty values with fitted plane of regression function — left side of fairway

MODEL IMPLEMENTATION — RESULTS

Two graphs presented in Figure 6 show the safety maneuvering areas obtained by means of original method prior to uncertainty model implementation, and GNSS RTK method. One may notice the significant divergences in both areas' layout with reference to the fairway axis. The most considerable divergences are at the end on analyzed area, obtained from original method. It is due to adopted measure technique and some approximations during ship's course determination.

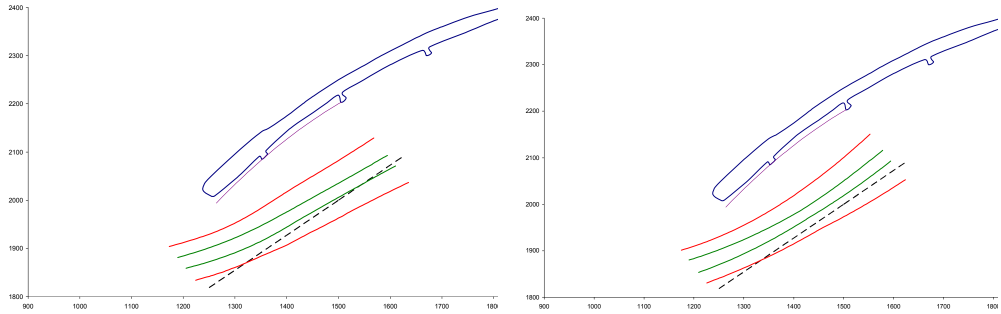

Fig. 6. Safety maneuvering area for m/f Jan Śniadecki, obtained by means of original method before uncertainty model implementation (right) and GNSS RTK method (left)

The histograms placed in figure 7 represent original methods' uncertainties distributions prior to model implementation. The uncertainties are expressed as an absolute value. The most common values for left side of fairway are within range of (1–6) meters. Analogical values for right side occur within (2–10) meters range.

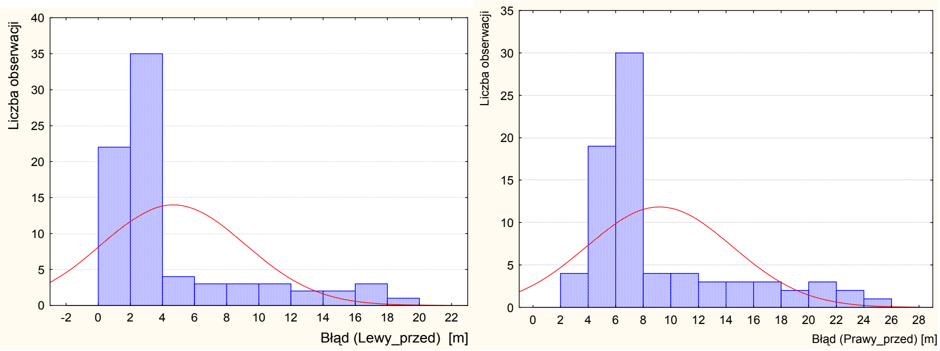

Fig. 7. Distributions of the absolute value of original method uncertainty priori to model implementation for left side (left) and right side (right) of fairway

Two graphs presented in Figure 8 show safety maneuvering areas obtained by means of original method after uncertainty model implementation, and GNSS RTK method. The differences are almost unnoticeably.

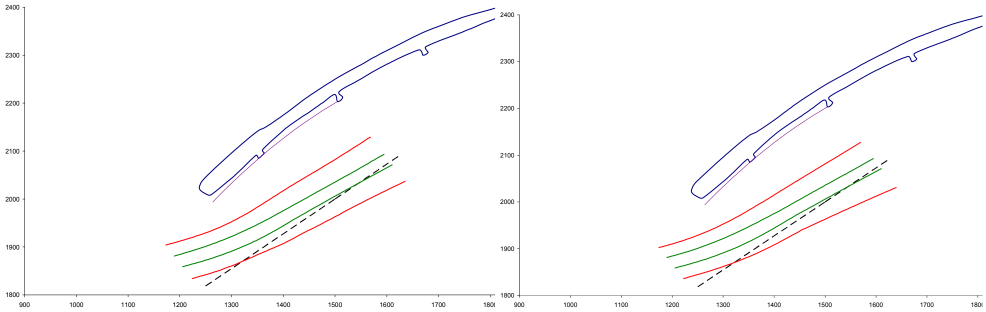

Fig. 8. Safety maneuvering area for m/f Jan Śniadecki, obtained by means of original method after uncertainty model implementation (right) and GNSS RTK method (left)

The histograms placed in figure 9 represent original methods' uncertainties distributions after model implementation.

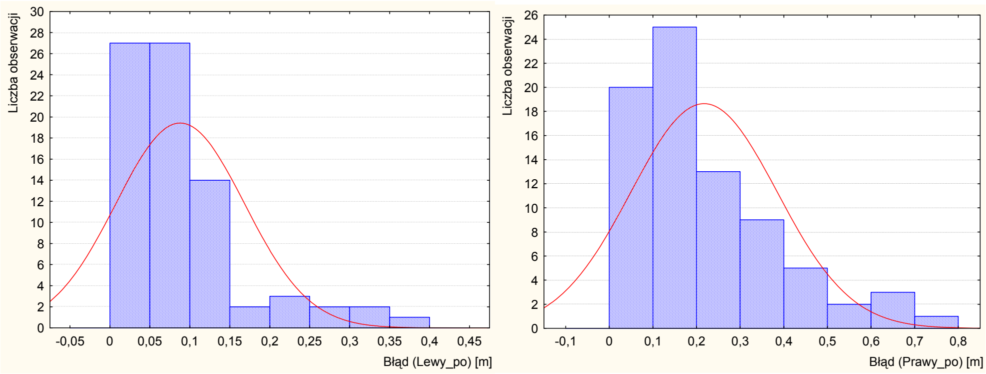

Fig. 9. Distributions of the absolute value of original method uncertainty after model implementation for left side (left) and right side (right) of fairway

The uncertainties are expressed as an absolute value. The most common values for left side of fairway are within range of (0.1–0.25) meters. Analogical values for right side occur within (0.1–0.5) meters range.

CONCLUSIONS

The paper presents an original method's uncertainty model evaluation and its implementation into method's algorithm of ship's safety maneuvering area determination.

The model was validated by means of comparative method, and its correctness was proved by means of the appropriate statistical tests. The model's response improves quality of original method's results, reducing significantly the value of method's uncertainty. The model has practical application for further survey conducted with the same technique.

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