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METHODS TO GENERATE NUMERICAL MODELS OF TERRAIN FOR SPATIAL ENC PRESENTATION

ABSTRACT

In the paper the methods to generate Numerical Models of Terrain for spatial visualization have been shown. For spatial visualization the application has been created, uses TIN, GRID methods: inverse distance to a power, nearest neighbour and natural one and NURBS (Non-Uniform Rational B-Splines). Methods have been used for spatial presentation of ENC data.

Keywords:

INTRODUCTION

Once data are ready to be used several methods are available to generate and draw the expected seabed. All can be sorted in three general classes:

1. Grid methods [4]: Inverse Distance to a Power, Kriging, Minimum Curvature, Modifier Shepard's Method, Nearest Neighbour, Polynomial Regression, Radial Basis Function, Moving Average, Data metrics, Local Polynomial.
2. Triangulated Irregular Network.
3. Mathematical: B-splines, NURBS.

The first class is the most commonly used for its simplicity but it has been decided to use methods from all kinds of computation programs: Inverse Distance to a Power, Nearest Neighbour method, TIN, B-Spline and NURBS, will give us a good overview of what it is possible to do.

GRID METHODS

Grid method consists in creating a two-dimensional grid, whose size corresponds to the ENC size. The number of nodes which compose the grid is a variable which impacts on the accuracy of modelization: the more nodes there are, the better the accuracy will be. Of course, this number should be well chosen, because it is not use creating a 1000×1000 grid if the map only contains 3 sounding values, except to increase calculation time. Once the grid has been created, a third coordinate is assigned to each node, depending on the method used.

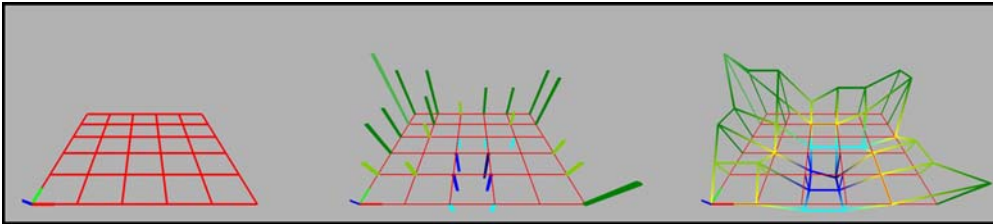


Fig. 1. Grid method steps

Inverse distance to a power

Inverse distance to a power method is the most used method for modeling terrains. To find a node z-coordinate, it consists in considering some known points around it. The unknown value is an average of these points depths, but a weighted one, which means the nearer they will be, the more impact they will have on the value.

If the distance between the interpolated point and the measured sounding considered is too important, compared to the distance between the interpolated point and other soundings, the impact will be very low, so only some points are needed to estimate the node's depth; otherwise it would increase the calculation time.

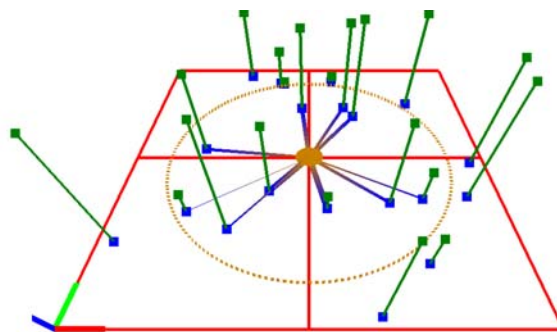


Fig. 2. Inverse distance to a power method principle

The general equation is the following one [7] :

$$Z_j = \frac{\sum_{i=1}^n \frac{Z_i}{h_{ij}^\beta}}{\sum_{i=1}^n \frac{1}{h_{ij}^\beta}} \quad \text{with} \quad h_{ij} = \sqrt{d_{ij}^2 + \delta^2}, \quad (1)$$

where:

- h_{ij} — effective separation distance between mesh node j and the neighbouring point i ;
- Z_j — interpolated value for mesh node j ;
- Z_i —neighbouring points;
- d_{ij} —distance between the grid node j and the neighbouring point i (2D projection);
- n —number of considered points;
- β — weighting power (the power parameter).

Two parameters appear in this equation, independent from the sounding values: n and β . β is a smoothing parameter: the lower it is, the smoother the terrain will be, as all calculated values might be similar. The n value refers to the number of considered points for estimating the node's depth. In most cases, algorithms use a distance d as parameter, and the number of considered points — n — is the number of points whose distance from the node is lower than d .

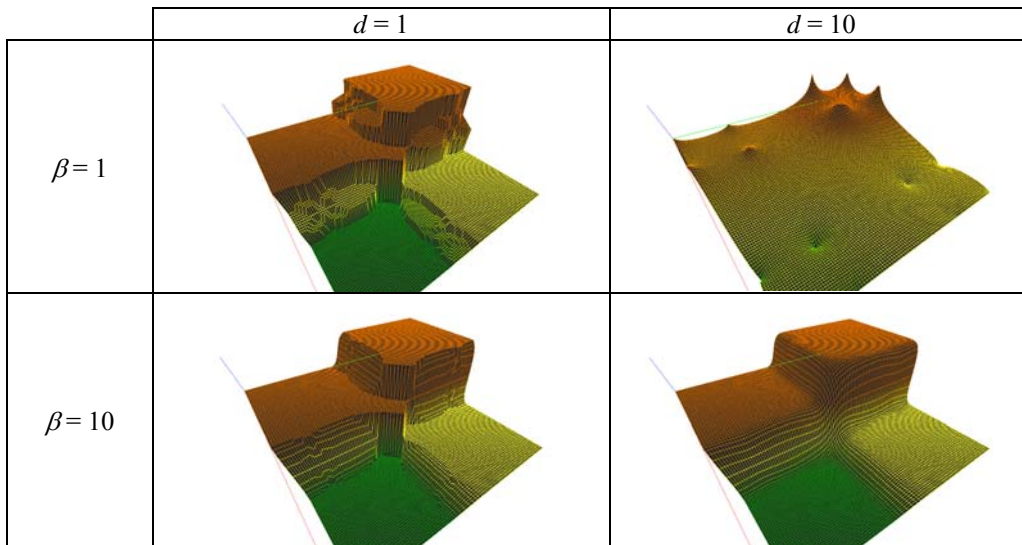


Fig. 3. Impact of inverse distance to a power method parameters

Changing β and d values may have a great impact on the smooth aspect of terrain. But the sea bottom isn't always very smooth, and a good way of choosing these parameters should depend on the composition of the seabed. Thus, changing β and d after reading "Seabed area" object class in ENC, could lead to a smooth modeling in sand areas, and a rough one in rock areas.

Nearest neighbour

Nearest neighbour method is based on the same idea as mapping a grid. It uses a simple grid made of square elements and only one point, the nearest, to determine the value of the node. Once the set of points is defined on the area a grid must be drawn: each node of the grid will receive the value of the nearest point next to it.

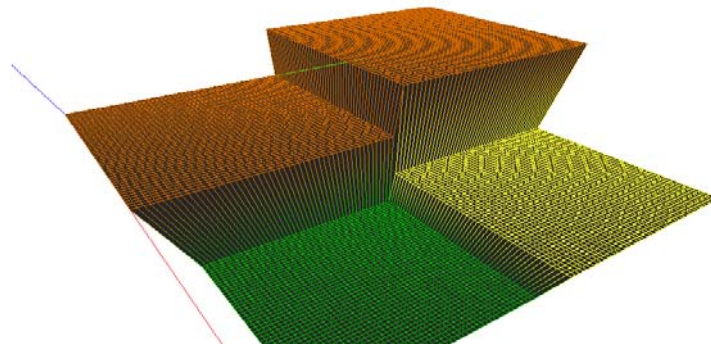


Fig. 4. Nearest neighbour method

This method can be first used, because of its simplicity. It can be quickly implemented and tested so as to check an approximate representation of the data used. But as given earlier this method is not a true way for interpolation and does not give the quality expected.

TRIANGULATED IRREGULAR NETWORK

Creating a regular grid for modeling terrains is a simple solution, easy to store and manipulate, but some other solutions exist. Among them, the Triangulated Irregular Network (TIN) consists in considering the map as a set of triangles enclosing three points from ENC data.

Definition

A Triangulated Irregular Network (TIN) is a digital data structure used in a geographic information system (GIS) for the representation of a surface. A TIN is a vector based representation of the physical land surface or sea bottom, made up of irregularly distributed nodes and lines with three dimensional coordinates (X, Y, and Z) that are arranged in a network of no overlapping triangles.

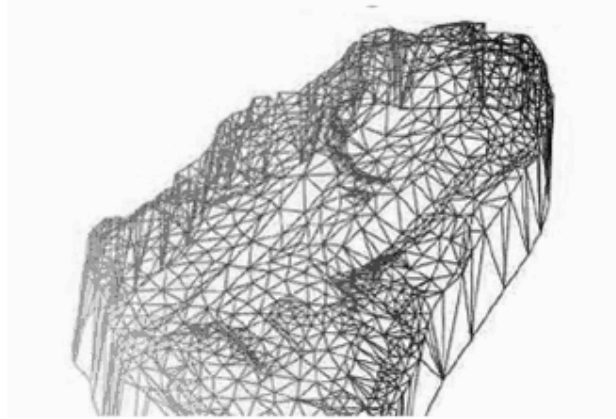


Fig. 5. TIN example

A TIN comprises a triangular network of vertices, known as mass points, with associated coordinates in three dimensions connected by edges to form a triangular tessellation. Three-dimensional visualizations are readily created by rendering of the triangular facets. In regions where there is little variation in surface height, the points may be widely spaced whereas in areas of more intense variation in height the point density is increased. A TIN is typically based on a Delaunay triangulation.

Although usually associated with three-dimensional data (X, Y, and Z) and topography, TINs are also useful for the description and analysis of horizontal (X and Y) distributions and relationships.

Comparison with other methods

Other methods used for meshing, as the grid method, maybe one of the most commonly used, bring easiness to use and to store all the data created next to Triangulated Irregular Network, moreover it provides a more natural representation of the terrain.

Compared to the grid method, Triangulated Irregular Network provides the advantage of a variable precision. When needed, for example when the relief gets

very chaotic on a special area of the map, the TIN enables to increase the precision by using a bigger number of points in this area. Due to this adaptability the TIN method is more efficient and uses less memory.

Domain of use and sub-method

Triangulated Irregular Network can be used as an independent method to draw a Numerical Model of Terrain but can also be a part of a bigger method as for Natural Neighbour. In this case it stands for the creation of a triangulated mesh, whose nodes coefficients will be later interpolated.

Different algorithms can be used to define triangles based on a set of points as the Pitteway or Delaunay Triangulation but to make a reliable mesh the Delaunay's one is the most commonly used because of its specification.

Delaunay Triangulation

For a set P of points in the (d -dimensional) Euclidean space, a Delaunay triangulation is a triangulation $DT(P)$ such that no point in P is inside the circum-hypersphere of any simplex in $DT(P)$.

It is known that there exists a unique Delaunay triangulation for P , if P is a set of points in general position; that is, no three points are on the same line and no four are on the same circle, for a two-dimensional set of points, or no $n + 1$ points are on the same hyperplane and no $n + 2$ points are on the same hypersphere, for an n -dimensional set of points.

The Delaunay triangulation consists in finding for a set of points in a two-dimensional space, the way to draw triangles linking all points provided that no triangle gets any point of one's neighbour in its circumcircle. What can also be defined by a triangulation which maximizes the minimum angle.

Proprieties

1. Every circumcircle contains no other point than the three of its triangle (condition necessary and sufficient). This propriety enables to define a Delaunay triangulation.
2. Each internal edge satisfies the local criteria of minimum angle.

Delaunay triangulation advantages over other triangulation methods

1. Triangles are as equi-angular as possible, thus reducing potential numerical precision problems created by long skinny triangles.

2. Ensures that any point on the surface is as close as possible to a node.
3. The triangulation is independent of the order the points are processed.

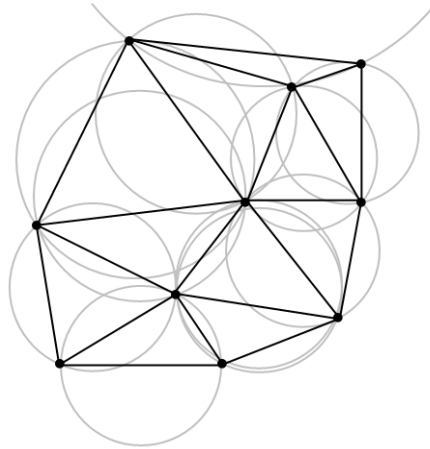


Fig. 6. Delaunay triangulation with circumcircle

Duality of Delaunay and Voronoi methods

The Voronoi diagram or tessellation consists in meshing a set of points by defining the polygons, limits of the region where any place would be closer to the point studied than others. The Delaunay triangulation and the Voronoi tessellation are dual, meaning that one is equivalent to the other. For example it is possible to switch from Delaunay to Voronoi by using the centre of the triangle circumcircle as the apexes of the polygon.

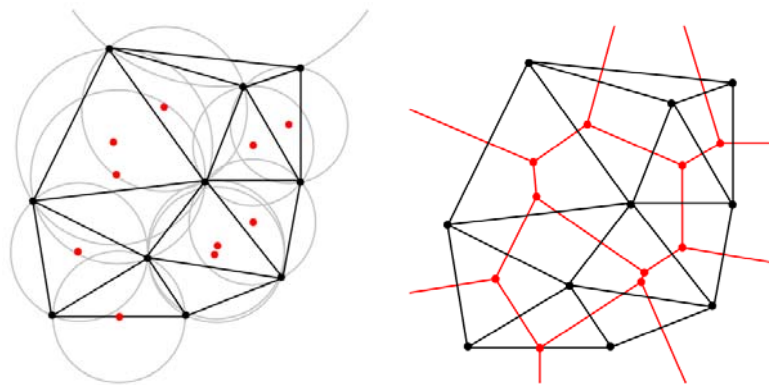


Fig. 7. Duality of Delaunay and Voronoi diagrams

Delaunay triangulation algorithm

Since the set of points to the drawing of all triangles the implementation is divided into several main steps which consist in adding a new point in the structure already established, checking for its belonging to a circumcircle of other triangles, deleting those triangles and rebuilding others adding the new point to satisfy the property that rules the method [15].

First step: delimiting the region which contains the set of points and making the first triangle. By analyzing the set of points it is possible to define a rectangle containing all elements. Then a first point, randomly chosen, can be added into the rectangle to create four triangles.

Choosing a new point. After the first four triangles the main part of the algorithm can be run, it firstly consists in adding a new point. Instead of picking a point it has been chosen to pick a triangle and then search an unplaced point in it. At the beginning, several points can be taken in the same element reducing the algorithm duration and its complexity. But to be advantageous the triangle must not be taken randomly, in order to improve the fastness it is better to add the new element in the biggest triangle. This enables to make a homogeneous structure and so to reduce the number of triangles to suppress and rebuilt after adding the new point [1].

Deleting triangles. The new point added, the circumcircle condition can be applied. The existing triangles are checked, and if they include the point in their circle, their points are stored in a temporary memory and then the triangle is suppressed.

Adding new triangles. Since the unsatisfying triangles have been deleted a polygon-shaped hole contains the new point. The method consists in sorting the points of this polygon, which are those stored in the temporary memory, according to their angular position around the new point. Then each edge of the polygon and the new point constitute the minimum elements needed to create one of the new triangles.

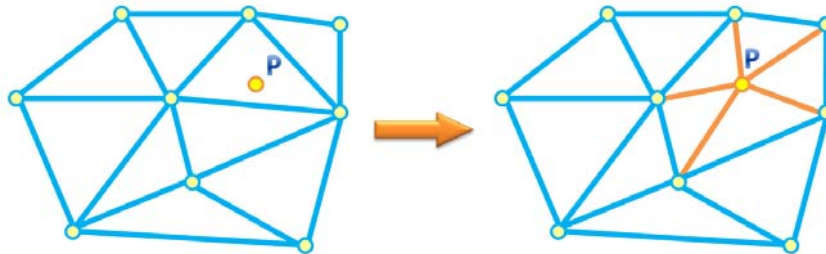


Fig. 8. Insertion of a point in a TIN structure

Sub-functions

Sub-functions have been implemented in the algorithm for area calculus, belonging to a triangle, and belonging to a circumcircle.

Belonging to a triangle. The method consists in studying two cases according to the position of the point in the first half part or the other of the triangle (cut by a vertical line at the middle point abscissa) modeled by the equation of two affine lines carried by the two edges. Then the position of the point must be between the two lines to belong to the triangle. Many special cases must be taken into account due to the spatial position of this middle point, at the vertical of another point, up or down the other edges of the triangle.

Belonging to a circumcircle. The method consists in comparing the distance between the centre of the circle and the point studied, and the radius of the circumcircle. All information about the circumcircle is defined thanks to its definition as the intersection of the two edges perpendicular bisectors. As in the previous method many special cases have to be studied due to the spatial orientation of the two perpendicular bisectors.

To go beyond

As noticed in the new point selection section it has been chosen to work with a method picking the biggest triangle and looking for a point inside. This method remains advantageous as far as others use full iterative search. Otherwise a better but more complex method can be generated by picking a new point randomly and then doing an oriented search among the triangles.

Moreover totally different approaches called global approaches can be used. They are named 'Divide and conquer' or 'Divide to build', which consist in building several associations of triangles at the same time and gather the assemblies, or the flipping method which consists in flipping triangle to fit one to another and satisfy the condition.

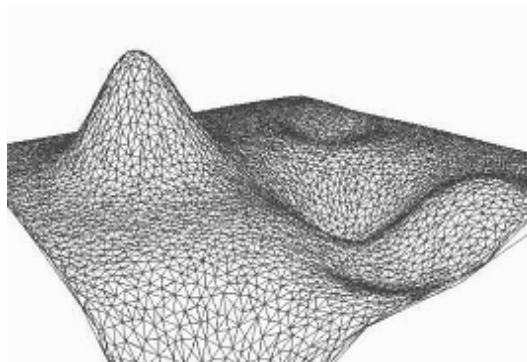


Fig. 9. Example of NMT made with TIN

MATHEMATICAL METHODS

Both grid method and Triangulated Irregular Network create models linking points with straight lines. It is of course a quick way to draw seabed, but to be consistent with reality, curves used for modeling should not only be continuous curves, but their derivative functions should also be continuous.

As a consequence, B-Splines and NURBS are sometimes used for modeling numerical models of terrain.

B-Splines

Creating B-Splines implies to deal with two kind of elements: on the one hand, a set of 3-dimensional points called control points P_0, \dots, P_m belonging to \mathfrak{R}^3 , which form the Control Polygon, and on the other hand a nondecreasing sequence of real numbers called knots $t_0 \leq \dots \leq t_m$ [14].

A first step consists in creating a curve $X_1(t)$ made of straight lines passing through the control points:

$$\forall t \in [t_i, t_{i+1}] X_1(t) = \left(1 - \frac{t - t_i}{t_{i+1} - t_i}\right) P_{i-1} + \frac{t - t_i}{t_{i+1} - t_i} P_i. \tag{2}$$

When t goes from t_i to t_{i+1} , $X_1(t)$ goes from P_{i-1} to P_i . This first approach leads to a continuous, but generally non-derivable function.

Consequently, another function $X_2(t)$ might be used. This new function is a C^l class function, but it doesn't passes through the control points anymore. However, if t belongs to $[t_i, t_{i+1}]$, $X_2(t)$ is located in the convex hull of the 3 vertices P_{i-2}, P_{i-1}, P_i . A way of defining $X_2(t)$ is to consider it as a combination of functions. It is possible to generalize this definition for a C^k class function by:

$$X_k(t) = \sum_i B_{i,k}(t) P_i. \tag{3}$$

$B_{i,k}(t)$ is the k^{th} B-Spline basis function of k -degree, defined thanks to the following algorithm, called Cox-De Boor algorithm:

$$B_{i,0}(t) = \begin{cases} 1 & \text{if } t_i \leq t \leq t_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{i,k}(t) = \frac{t - t_i}{t_{i+k} - t_i} B_{i,k-1}(t) + \left(1 - \frac{t - t_{i+1}}{t_{i+k+1} - t_{i+1}}\right) B_{i+1,k-1}(t). \tag{4}$$

The computation of the k^{th} -degree functions should be done by the following way:

$$\begin{array}{cccccc}
 B_{0,0}(t) & & B_{1,0}(t) & & B_{2,0}(t) & & B_{3,0}(t) & & B_{4,0}(t) \dots \\
 & B_{0,1}(t) & & B_{1,1}(t) & & B_{2,1}(t) & & B_{3,1}(t) \dots & \\
 & & B_{0,2}(t) & & B_{1,2}(t) & & B_{2,2}(t) \dots & & \\
 & & & B_{0,3}(t) & & B_{1,3}(t) \dots & & &
 \end{array}$$

Non-Uniform Rational Basis Splines

Non-Uniform Rational Basis Splines (NURBS) are quite similar to B-Splines as these functions generalize B-Splines. Using NURBS, it is possible to draw curves that cannot be drawn with B-Splines.

NURBS can be defined with the same B-Splines, as those described in algorithm (3). But the k^{th} -degree NURBS curve itself is defined by [8, 14]:

$$C_k(t) = \frac{\sum_{i=0}^n B_{i,k}(t) \omega_i P_i}{\sum_{i=0}^n B_{i,k}(t) \omega_i}, \tag{5}$$

where:

- P_j — control points,
- ω_i — weights,
- $B_{i,k}(t)$ — k^{th} B-Spline basis functions.

It is also possible to generate NURBS surfaces, in order to model not curves but surfaces. In this case, the NURBS surface should be written as [10]:

$$C_{p,q}(u,v) = \frac{\sum_{i=0}^n \sum_{j=0}^m B_{i,p}(u) B_{j,q}(v) \omega_{i,j} P_{i,j}}{\sum_{i=0}^n \sum_{j=0}^m B_{i,p}(u) B_{j,q}(v) \omega_{i,j}}. \tag{6}$$

In C++, and more particularly in OpenGL, a specific function can be used to draw NURBS: `gluNurbsSurface()` will compute a surface thanks to the different parameters mentioned above.

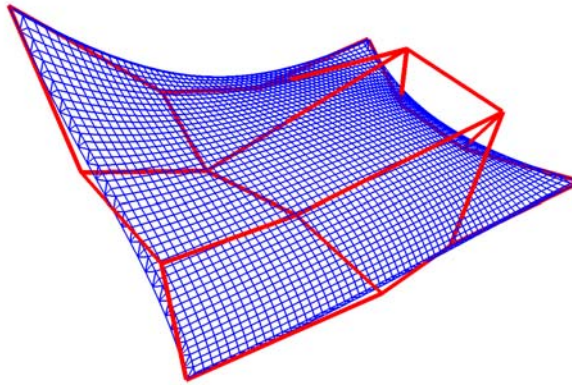


Fig. 10. NURBS surface and control polygon drawn with OpenGL

Using such a function is also a way to draw the surface for visualizing it (Fig. 10), as visualization is the main purpose of generating of digital sea bottom model.

CONCLUSIONS

Numerical modeling of sea bottoms in 3D enables seamen to better perceive their surrounding environment. Several methods can be used to create numerical models of terrain, but they should be adapted to ENC data. There are several ways of computing a surface that could model land or sea bottom, and it is worth it, to analyze some of them so that the models they generate can be compared. GRID methods, Triangulated Irregular Networks and mathematical methods enabled to draw this comparison.

Results presented in this paper have been obtained during realization the project 'Generating of digital sea bottom model by means ENC data' by Ronan Dupuy and Roman Sassais in 2009.

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