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AN INFLUENCE OF PASSING BAN ON FAIRWAY UNITS TRAFIC STREAM

ABSTRACT

The present report concerns the disturbed traffic of vessels, the passage through a fairway section with passing ban. To determine parameters of vessel traffic stream a convolution operation has been used. Four cases of entering of two successive units to the fairway section with the passing ban have been examined.

Keywords: Fairway Traffic,

INTRODUCTION

A very important traffic restriction in narrow fairways is the passing ban on certain sections of the fairway. Therefore, a vessel approaching a special fairway section (for example section P_1P_2), where the ban is in force, stops before the point P_1. She waits for the vessel in section P_1P_2 to leave it and next continue sailing.

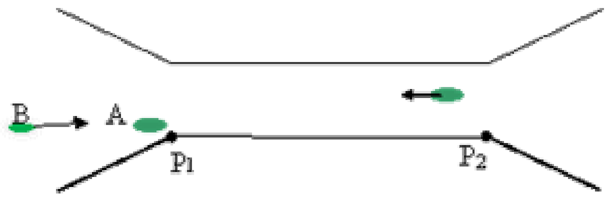

Fig. 1. Passing ban on section P_1P_2

Considering the particular phases of the movement a general form of density function was determined of the waiting time for the reporting of the successive vessel leaving the area with the passing ban in force:

$$T = X_B + (Y_B - Y_A) \tag{1}$$

where: X denotes the time between reporting of fairway vessels in undisturbed vessel traffic,
Y_B and Y_A are realisations (for units A and B) of variable Y, which denotes the waiting time for entering section P_1P_2.

PROCESS OF REPORTING

When the fairway unit arrives to the beginning of fairway section with passing ban, opposite unit can be at any point of this section (with probability *p*) or can be out of this section (with probability *q*). Similarly, when the next successive vessel arrives to the fairway section with passing ban, opposite vessel can be at any point of this section (with probability *p*) or can be out of this section (with probability *q*). Hence we have four different cases of the covering the fairway section with passing ban by two successive units (fig.2).

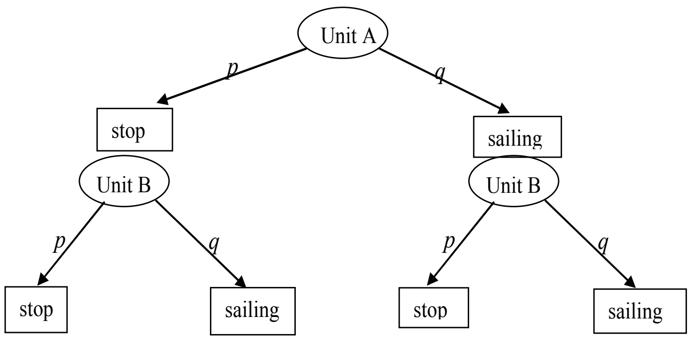

Fig.2. An events tree of speed changes during passing the fairway section with passing ban

DENSITY FUNCTION OF VARIABLE T

The probability density function of time between successive vessels leaving the fairway section with the passing ban is a convolution of two density functions [4, 6, 9]:

$$f(x) = \mathrm{conv}\left(f_X(x), f_{YA-YB}(x)\right) \tag{2}$$

A very important thing in this problem is the selection of random distributions describing particular random variables. In this paper the following assumptions have been made: the random variable X has an exponential distribution [1, 3, 5, 7, 8], the random variable Y has an uniform distribution [6]. Applying the operation of convolution, the density function of time between vessels A and B sailing in the section P_1P_2 was determined in four cases.

Case 1

With probability p^2 both units stop before point P_1 (fig. 2) and p.d.f of time between successive vessels sailing in the fairway section with the passing ban, has the form:

$$f_{T1}(u)=\begin{cases}\dfrac{(u+b-a)\cdot\lambda-1+e^{(a-u-b)\lambda}}{(b-a)^2\cdot\lambda} & \text{for } a-b\le u<0\\ \dfrac{(b-u-a)\cdot\lambda+1-2e^{-u\lambda}+e^{(a-u-b)\lambda}}{(b-a)^2\cdot\lambda} & \text{for } 0\le u<b-a\\ \dfrac{\left(e^{a\lambda}-e^{b\lambda}\right)^2\cdot e^{(-a-u-b)\lambda}}{(b-a)^2\cdot\lambda} & \text{for } b-a\le u<\infty\end{cases} \tag{3}$$

where: *a, b* – parameters of uniform distribution;
λ – parameter of exponential distribution.

Case 2

With probability pq first unit stops before point P_1 and second unit doesn't stop before section P_1P_2 (Y_B in formula (1) is equal to 0). Then p.d.f of variable T assumes the following form:

$$f_{T2}(u)=\begin{cases}\dfrac{1-e^{(-u-b)\lambda}}{b-a} & \text{for } -b\le u<-a\\ \dfrac{\left(e^{a\lambda}-e^{b\lambda}\right)\cdot e^{(-a-u-b)\lambda}}{b-a} & \text{for } -a\le u<\infty\end{cases} \tag{4}$$

where: *a, b* – parameters of uniform distribution;
λ – parameter of exponential distribution.

C a s e 3

With probability pq first unit sails without any limitation and the successive unit stops before the point P_1 (Y_A in formula (1) is equal to 0). Than p.d.f of time between successive vessels sailing in the fairway section with the passing ban has the form:

$$f_{T3}(u) = \begin{cases} \dfrac{-1 + e^{(a-u)\lambda}}{a-b} & \text{for } -b \le u < -a \\ \dfrac{\left(e^{a\lambda} - e^{b\lambda}\right) \cdot e^{-\lambda u}}{a-b} & \text{for } -a \le u < \infty \end{cases} \tag{5}$$

where: a, b – parameters of uniform distribution;
λ – parameter of exponential distribution.

C a s e 4

With probability q^2 both units don't stop before the point P_1 and p.d.f of the time between successive vessels sailing in the fairway section with the passing ban is equal to p.d.f. of the variable X:

$$f_{T4}(u) = f_X(u) = \begin{cases} 0 \; for \; u < 0 \\ \lambda \cdot e^{-\lambda u} \; for \, u \ge 0 \end{cases} \tag{6}$$

where: λ – parameter of exponential distribution.

CALCULATIONS

The probability density function of the time between successive vessels sailing in the fairway section with the passing ban is expressed by four formulae with appropriate probabilities (fig.3).

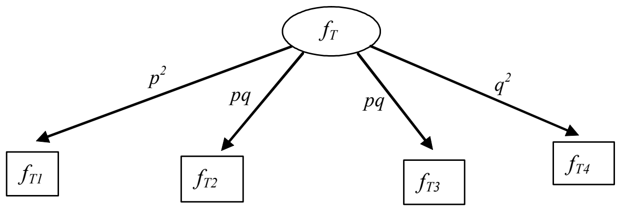

Fig.3. Four forms of p.d.f. of variable T

To determine probability that time between successive vessels sailing in the fairway section with the passing ban will be in the interval [c, d], we must use total probability rule and count four integrals of function $f_{Ti}(x)$ in appropriate limits:

$$P(c < T < d) = p^2 \cdot \int_c^d f_{T1}(x)dx + pq \cdot \int_c^d f_{T2}(x)dx + pq \cdot \int_c^d f_{T3}(x)dx + q^2 \cdot \int_c^d f_{T4}(x)dx \quad (7)$$

CONCLUSIONS

The convolution of particular random variables enables the distribution of a given phenomenon into a number of smaller elements, which can be examined more easily. Singling out four cases of vessel movement and total probability rule permitted the determination of a general density function form of the waiting time for the successive vessel's reporting after covering the fairway section with passing ban.

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