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COMPARATIVE EXAMINATIONS OF THE NONLINEAR KALMAN FILTERS APPLIED TO POSITIONING SYSTEMS

ABSTRACT

In positioning systems Kalman filters are used for estimation and also for integration of data from navigation systems and sensors. The Kalman filter (KF) is an optimal linear estimator when the process noise and the measurement noise can be modeled by white Gaussian noise. In situations when the problems are nonlinear or the noise that distorts the signals is non-Gaussian, the Kalman filters provide a solution that may be far from optimal. Nonlinear problems can be solved with the extended Kalman filter (EKF). This filter is based upon the principle of linearizing the state transition matrix and the observation matrix with Taylor series expansions. Unscented Kalman filter with comparison to EKF does not linearize the model but operates on the statistical parameters of the measurement and state vectors that are subsequently nonlinearly transformed. The unscented Kalman filter is based on the unscented transformation (UT). This paper presents a comparison of the estimation quality for two nonlinear measurement models of the following Kalman filters: covariance filter (KF), extended filter (EKF) and unscented filter (UKF). There are descriptions of models and analysis of obtained results in this article. The comparison of filtration quality was done in MATLAB environment.

INTRODUCTION

Navigation is a science which deals with estimation of current position of an object and guidance of mobile objects according to determined route or trajectory. Navigation for the most part of its history was developed under the influence of needs in sea navigation. Development of aviation and air force, motor transport, motorization and individual tourism was the next cause of progress in domain of navigation. Many manners and navigation techniques were formed within the space of years. Today Global Navigation Satellite System (GNSS) is the most often used system for location and determination of position in space. Satellite signals may fade in heavily urbanized and forested areas or as results of interference and noises.

Because of this restriction autonomous navigation systems have operated for years. Only integration of navigation data from those two systems makes possible to realize precise position estimation.

Kalman filters in many applications are used for navigation and also for measurement information integration coming from GNSS and from Inertial Navigation System. The classical Kalman filter is used for linear dynamic systems [1] moreover extended Kalman filter EKF for nonlinear systems [1], [3] or unscented Kalman filter UKF [2], [4] – [9]. Unscented Kalman filter with comparison to EKF does not linearize the model but operates on the statistical parameters of the measurement and state vectors that are subsequently nonlinearly transformed. The unscented Kalman filter is based on the unscented transform (UT).

Kalman filtration is based on the following models of state and measurement vectors respectively:

$$\mathbf{x}(k+1)=\mathbf{f}\left[\mathbf{x}(k),\ \mathbf{u}(k),\ \mathbf{w}(k)\right] \quad \text{for} \quad \mathbf{w}(k)\sim N\left[\mathbf{0},\ \mathbf{Q}(k)\right],$$

$$\mathbf{z}(k+1)=\mathbf{h}\left[\mathbf{x}(k),\ \mathbf{v}(k)\right] \quad \text{for} \quad \mathbf{v}(k)\sim N\left[\mathbf{0},\ \mathbf{R}(k)\right]. \tag{1}$$

Vector $\mathbf{x}(k)$ is n-dimensional state vector in the moment k, $\mathbf{z}(k)$ is p-dimensional measurement vector in the moment k, $\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w})$ denotes nonlinear state function describing dynamic behavior of the system between k+1 and k moments, $\mathbf{u}$ is the input system vector, $\mathbf{w}$ is the noise state vector, $\mathbf{Q}$ is the covariance matrix of the noise state (denotes uncertainty in the dynamic model during transition from k+1 to k moments, $\mathbf{h}(\mathbf{x}, \mathbf{v})$ denotes nonlinear measurement function, $\mathbf{v}$ p-dimensional vector of measurement noise, $\mathbf{R}$ is covariance matrix of measurement errors with dimensions $p \times p$.

A SYSTEM

The system model has assumed as an object in space, moving accordingly to the Newton equation:

$$x = x_0 + vt + \frac{1}{2}at^2 . \tag{2}$$

When determining components of position x, we assumed, that our object moves with constant velocity (acceleration a = 0) and velocity components (v) are additive Gaussian noise. Fig. 1 presents analyzed model in the graphic form.

Process model is described by state vector in the following form:

$$\mathbf{x} = \begin{bmatrix} p_x & p_y & p_z & v_x & v_y & v_z \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

where: $p_{x,\,y,\,z}$ are components of the position in Cartesian coordinates, $v_{x,\,y,\,z}$ are components of the velocity. Speed vector components are Gaussian noise.

State matrix has the form:

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \Delta t \mathbf{I}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \end{bmatrix}, \tag{4}$$

where: Δt is the time step between moments k and k+1, $\mathbf{I}$ is identity matrix.

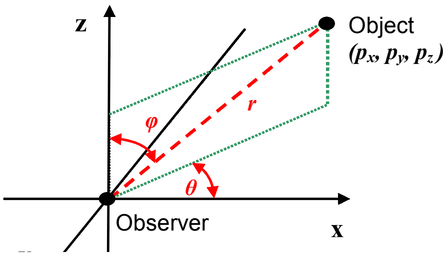

Fig. 1. Positioning principle

In this case the state vector (3) and state matrix (4) are identical for covariance filter (KF), extended filter (EKF) and unscented filter (UKF).

The measured parameters in the measurement model are: r – distance from object to observer (radar), θ – azimuth, φ – elevation. Thus the measurement vector is given by:

$$\mathbf{z} = \begin{bmatrix} r & \theta & \varphi \end{bmatrix}^{\mathrm{T}}. \tag{5}$$

Dependency between object position in the Cartesian coordinates and measurements in spherical coordinates is described by nonlinear function and it can be given by the equations:

$$r = \sqrt{p_x^2 + p_y^2 + p_z^2}, \tag{6}$$

$$\theta = \arctan\left(\frac{p_y}{p_x}\right), \tag{7}$$

$$\varphi = \arccos\left(\frac{p_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}\right). \tag{8}$$

The last equation after transformation gives Cartesian object coordinates:

$$p_x = r\cos\theta\sin\varphi\,, \tag{9}$$

$$p_y = r\sin\theta\cos\varphi\,, \tag{10}$$

$$p_z = r\cos\varphi\,. \tag{11}$$

Measurement matrix for discrete and unscented filters is given by:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

The vector function **h**(*) has the following form:

$$\mathbf{h} = \begin{bmatrix} \sqrt{p_x^2 + p_y^2 + p_z^2} \\ \arctan\left(\dfrac{p_y}{p_x}\right) \\ \arccos\left(\dfrac{p_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}\right) \end{bmatrix}. \tag{13}$$

In the case of extended filter, linearization of the measurement function **h** for each measurement step by the use of partial derivative relatively to all elements of state vector should be made. The final measurement matrix for extended Kalman filter is as follows:

$$\mathbf{H} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k)} = \begin{bmatrix} \dfrac{p_x}{r} & \dfrac{p_y}{r} & \dfrac{p_z}{r} & 0 & 0 & 0 \\ \dfrac{p_y}{r^2 - p_z^2} & \dfrac{p_x}{r^2 - p_z^2} & 0 & 0 & 0 & 0 \\ \dfrac{p_x p_z}{r^2\sqrt{r^2 - p_z^2}} & \dfrac{p_y p_z}{r^2\sqrt{r^2 - p_z^2}} & \dfrac{-\sqrt{r^2 - p_z^2}}{r^2} & 0 & 0 & 0 \end{bmatrix}. \tag{14}$$

B SYSTEM

Dynamics in B system is more complex, because the object moves around. This situation causes nonlinear relationships both in state and measurement matrix. Figure 2 in detail illustrates considered object.

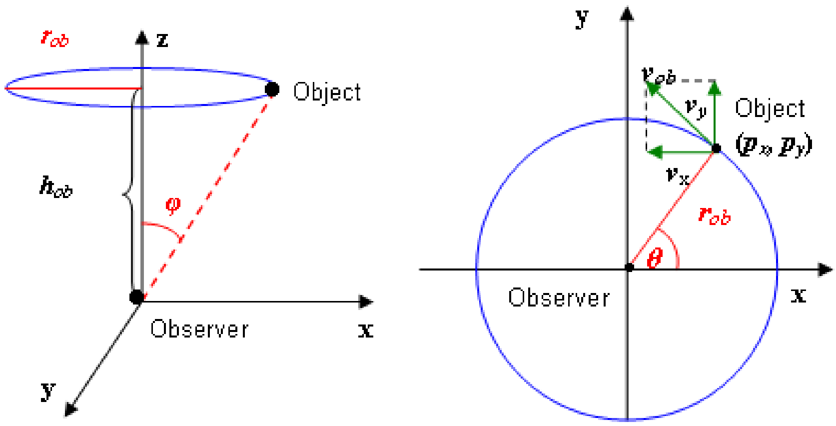

Fig. 2. Positioning principle

Can see that a nonlinear relation exists between measurements from the system and elements of the motion:

$$\tan\theta = \frac{v_x}{v_y} = \frac{p_y}{p_x}, \qquad v_x = v_{ob}\sin\theta, \qquad v_y = v_{ob}\cos\theta,$$

$$v_{ob} = \sqrt{v_x^2 + v_y^2}, \qquad r_{ob} = \sqrt{p_x^2 + p_y^2}. \tag{15}$$

Azimuth and angular velocity of the object can be calculated via the following formulas:

$$\theta(k+1) = \theta(k) + \omega(k)\Delta t, \qquad \omega(k) = \frac{v_{ob}(k)}{r_{ob}(k)}. \tag{16}$$

In the presented system initial values of vector state are described by the followings:

$$\mathbf{x}(0) = \begin{bmatrix} r_{ob} & 0 & h_{ob} & 0 & v_{ob} & 0 \end{bmatrix}^{\mathrm{T}}. \tag{17}$$

Nonlinear state functions (in the moment *k*) are defined as follows:

$$\mathbf{f}(k)=\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix}=\begin{bmatrix} r_{ob}\cos\left[\arctan\left(\frac{p_y}{p_x}\right)+\omega(k)\Delta t\right] \\ r_{ob}\sin\left[\arctan\left(\frac{p_y}{p_x}\right)+\omega(k)\Delta t\right] \\ p_z(k)+v_z(k)\Delta t \\ v_{ob}\sin\left[\arctan\left(\frac{p_y}{p_x}\right)+\omega(k)\Delta t\right] \\ v_{ob}\cos\left[\arctan\left(\frac{p_y}{p_x}\right)+\omega(k)\Delta t\right] \\ v_z(k) \end{bmatrix}. \tag{18}$$

This nonlinear equation requires linearization, which in extended Kalman filter is performed around the estimated object's trajectory. For the EKF state matrix has been calculated as a matrix of derivatives of nonlinear **f**(*) function with respect to the components of the state vector **x**.

$$\mathbf{F}=\frac{\partial \mathbf{f}}{\partial \mathbf{x}}=\begin{bmatrix} \frac{\partial f_1}{\partial p_x} & \frac{\partial f_1}{\partial p_y} & \frac{\partial f_1}{\partial p_z} & \frac{\partial f_1}{\partial v_x} & \frac{\partial f_1}{\partial v_y} & \frac{\partial f_1}{\partial v_z} \\ \frac{\partial f_2}{\partial p_x} & \frac{\partial f_2}{\partial p_y} & \frac{\partial f_2}{\partial p_z} & \frac{\partial f_2}{\partial v_x} & \frac{\partial f_2}{\partial v_y} & \frac{\partial f_2}{\partial v_z} \\ \frac{\partial f_3}{\partial p_x} & \frac{\partial f_3}{\partial p_y} & \frac{\partial f_3}{\partial p_z} & \frac{\partial f_3}{\partial v_x} & \frac{\partial f_3}{\partial v_y} & \frac{\partial f_3}{\partial v_z} \\ \frac{\partial f_4}{\partial p_x} & \frac{\partial f_4}{\partial p_y} & \frac{\partial f_4}{\partial p_z} & \frac{\partial f_4}{\partial v_x} & \frac{\partial f_4}{\partial v_y} & \frac{\partial f_4}{\partial v_z} \\ \frac{\partial f_5}{\partial p_x} & \frac{\partial f_5}{\partial p_y} & \frac{\partial f_5}{\partial p_z} & \frac{\partial f_5}{\partial v_x} & \frac{\partial f_5}{\partial v_y} & \frac{\partial f_5}{\partial v_z} \\ \frac{\partial f_6}{\partial p_x} & \frac{\partial f_6}{\partial p_y} & \frac{\partial f_6}{\partial p_z} & \frac{\partial f_6}{\partial v_x} & \frac{\partial f_6}{\partial v_y} & \frac{\partial f_6}{\partial v_z} \end{bmatrix}, \tag{19}$$

where:

$$\frac{\partial f_1}{\partial p_x} = \frac{p_x}{r_{ob}}\cos\gamma + r_{ob}\left(\frac{v_{ob}\Delta t p_x}{r_{ob}^3} + \frac{p_y}{p_x^2\left(1+\frac{p_y^2}{p_x^2}\right)}\right)\sin\gamma \, ,$$

$$\frac{\partial f_1}{\partial p_y} = \frac{p_y}{r_{ob}}\cos\gamma + r_{ob}\left(\frac{v_{ob}\Delta t p_y}{r_{ob}^3} - \frac{1}{p_x\left(1+\frac{p_y^2}{p_x^2}\right)}\right)\sin\gamma \, ,$$

$$\frac{\partial f_1}{\partial p_z} = 0 \, , \quad \frac{\partial f_1}{\partial v_x} = \frac{\Delta t v_x}{v_{ob}}\sin\gamma \, , \quad \frac{\partial f_1}{\partial v_y} = \frac{\Delta t v_y}{v_{ob}}\sin\gamma \, , \quad \frac{\partial f_1}{\partial v_z} = 0 \, ,$$

$$\frac{\partial f_2}{\partial p_x} = \frac{p_x}{r_{ob}}\sin\gamma - r_{ob}\left(\frac{p_y}{p_x^2\left(1+\frac{p_y^2}{p_x^2}\right)} + \frac{v_{ob}\Delta t p_x}{r_{ob}^3}\right)\cos\gamma \, ,$$

$$\frac{\partial f_2}{\partial p_y} = \frac{p_y}{r_{ob}}\sin\gamma + r_{ob}\left(\frac{1}{p_x\left(1+\frac{p_y^2}{p_x^2}\right)} - \frac{v_{ob}\Delta t p_y}{r_{ob}^3}\right)\cos\gamma \, ,$$

$$\frac{\partial f_2}{\partial p_z} = 0 \, , \quad \frac{\partial f_2}{\partial v_x} = \frac{\Delta t v_x}{v_{ob}}\cos\gamma \, , \quad \frac{\partial f_2}{\partial v_y} = \frac{\Delta t v_y}{v_{ob}}\cos\gamma \, , \quad \frac{\partial f_2}{\partial v_z} = 0 \, ,$$

$$\frac{\partial f_3}{\partial p_x} = 0 \, , \quad \frac{\partial f_3}{\partial p_y} = 0 \, , \quad \frac{\partial f_3}{\partial p_z} = 1 \, , \quad \frac{\partial f_3}{\partial v_x} = 0 \, , \quad \frac{\partial f_3}{\partial v_y} = 0 \, , \quad \frac{\partial f_3}{\partial v_z} = \Delta t \, ,$$

$$\frac{\partial f_4}{\partial p_x} = v_{ob}\left(-\frac{p_y}{p_x^2\left(1+\frac{p_y^2}{p_x^2}\right)} - \frac{v_{ob}\Delta t p_x}{r_{ob}^3}\right)\cos\gamma \, ,$$

$$\frac{\partial f_4}{\partial p_y} = v_{ob}\left(\frac{1}{p_x\left(1+\frac{p_y^2}{p_x^2}\right)} - \frac{v_{ob}\Delta t p_y}{r_{ob}^3}\right)\cos\gamma \, ,$$

$$\frac{\partial f_4}{\partial p_z} = 0 \, , \quad \frac{\partial f_4}{\partial v_x} = \frac{\left(r_{ob}\sin\gamma + v_{ob}\Delta t\cos\gamma\right)}{r_{ob}v_{ob}}v_x \, ,$$

$$\frac{\partial f_4}{\partial v_y} = \frac{\left(r_{ob}\sin\gamma + v_{ob}\Delta t\cos\gamma\right)}{r_{ob}v_{ob}}v_y \, , \quad \frac{\partial f_4}{\partial v_z} = 0 \, ,$$

$$\frac{\partial f_5}{\partial p_x} = v_{ob}\left(\frac{p_y}{p_x^2\left(1+\frac{p_y^2}{p_x^2}\right)} + \frac{v_{ob}\Delta t p_x}{r_{ob}^3}\right)\sin\gamma ,$$

$$\frac{\partial f_5}{\partial p_y} = v_{ob}\left(\frac{v_{ob}\Delta t p_y}{r_{ob}^3} - \frac{1}{p_x\left(1+\frac{p_y^2}{p_x^2}\right)}\right)\cos\gamma ,$$

$$\frac{\partial f_5}{\partial p_z} = 0 , \quad \frac{\partial f_5}{\partial v_x} = \frac{\left(r_{ob}\cos\gamma - v_{ob}\Delta t \sin\gamma\right)}{r_{ob}v_{ob}} v_x ,$$

$$\frac{\partial f_5}{\partial v_y} = \frac{\left(r_{ob}\cos\gamma - v_{ob}\Delta t \sin\gamma\right)}{r_{ob}v_{ob}} v_y , \quad \frac{\partial f_5}{\partial v_z} = 0 ,$$

$$\frac{\partial f_6}{\partial p_x} = 0 , \quad \frac{\partial f_6}{\partial p_y} = 0 , \quad \frac{\partial f_6}{\partial p_z} = 0 , \quad \frac{\partial f_6}{\partial v_x} = 0 , \quad \frac{\partial f_6}{\partial v_y} = 0 , \quad \frac{\partial f_6}{\partial v_z} = 1 ,$$

where: $\gamma = \arctan\left(\frac{p_y}{p_x}\right) + \frac{v_{ob}}{r_{ob}}\Delta t$.

For discrete and unscented Kalman filter state matrix has a form given by (4). An observation matrix **H** in the measurement model is identical as in the first model (11).

SIMULATION RESULTS

The accuracy comparisons have been examined by the use of simulation in the MATLAB environment. In order to ensure the same conditions, research of filters were realized with identical form of state vector covariance matrix **Q**, measurement matrix **R** and initial state vector covariance matrix **P**(0) in both systems. Similarly to Julier [3] the following parameters of unscented transform have been assumed: $\lambda = 3$, $\beta = 1$, $\kappa = 3$.

Furthermore the following values of noise covariance matrix have been applied:

$$\mathbf{Q} = diag\left[0.0225\ m^2 \quad 0.0225\ m^2 \quad 0.0225\ m^2 \quad 0.49\ m^2 s^{-2} \quad 0.49\ m^2 s^{-2} \quad 0.49\ m^2 s^{-2}\right], \tag{20}$$

covariance matrix of measurement noise:

$$\mathbf{R} = diag\left[0.7225\ m^2 \quad 0.16\ deg^2 \quad 0.16\ deg^2\right], \tag{21}$$

initial covariance matrix of vector state errors:

$$\mathbf{P}(0) = diag\left[1\ m^2 \quad 1\ m^2 \quad 1\ m^2 \quad 1\ m^2 s^{-2} \quad 1\ m^2 s^{-2} \quad 1\ m^2 s^{-2}\right]. \tag{22}$$

Research of A System

Results are presented it the form of object position in the Cartesian coordinates estimated by the use of covariance filter (DKF – green line), extended filter (EKF – red line) and unscented filter (UKF – blue line). Real position is denoted by solid, black line (fig. 3 – 5). The examinations results include values of mean square error (**mse**) of estimated state vector:

$$\mathbf{mse}\ {*}\mathrm{KF} = \sqrt{\left(\hat{\mathbf{x}} - \mathbf{x}_{real}\right)^{\mathrm{T}} \left(\hat{\mathbf{x}} - \mathbf{x}_{real}\right) / n}\ , \tag{23}$$

and covariance error **P**, according to Kalman filtering theory, estimated component of state vector **x** (fig. 6 – 8):

$$\mathrm{cov}\ {*}\mathrm{KF} = \mathbf{P}(k+1) = \mathbf{F}(k)\mathbf{P}(k)\mathbf{F}(k)^{\mathrm{T}} + \mathbf{Q}(k). \tag{24}$$

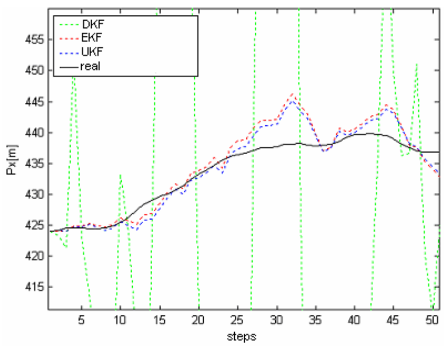

Fig. 3. Estimated component p_x of the position

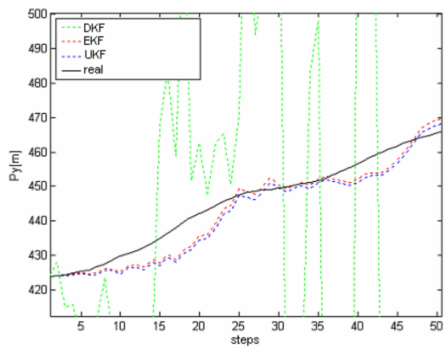

Fig. 4. Estimated component p_y of the position

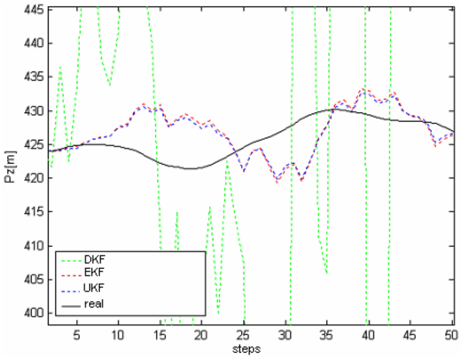

Fig. 5. Estimated component p_z of the position

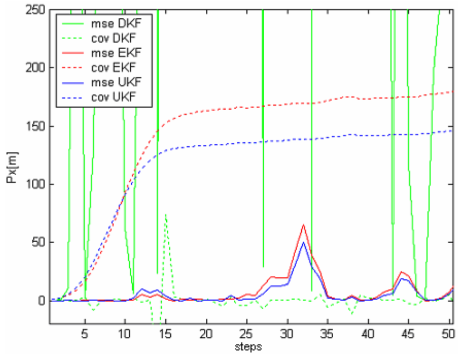

Fig. 6. Mean square error of the estimated component p_x and covariance error P_{px} of the position

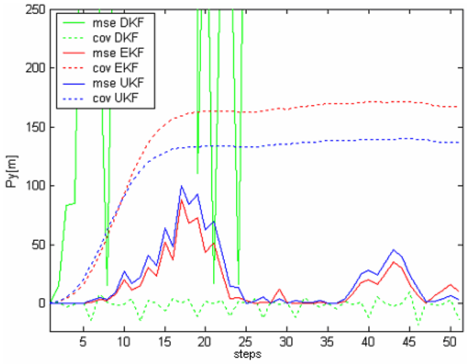

Fig. 7. Mean square error of the estimated component p_y and covariance error P_{py} of the position

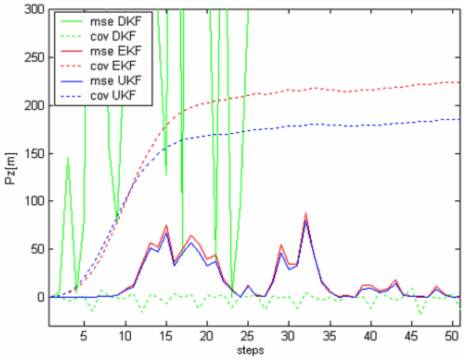

Fig. 8. Mean square error of the estimated component p_z and covariance error P_{pz} of the position

The estimation of object speed in the Cartesian coordinates (fig. 9 – 11) and determination of mean square error and covariance error of speed components (fig. 12 – 14) has also been done.

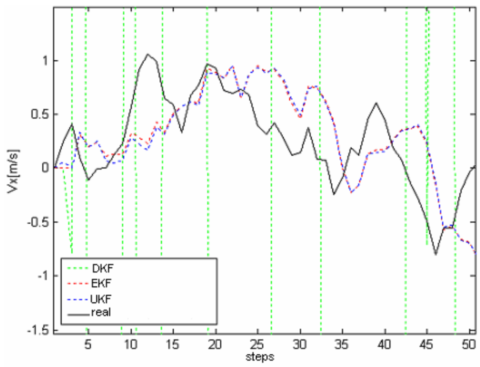

Fig. 9. Estimated v_x speed component

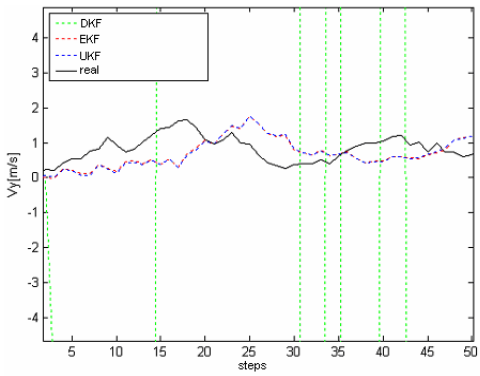

Fig. 10. Estimated v_y speed component

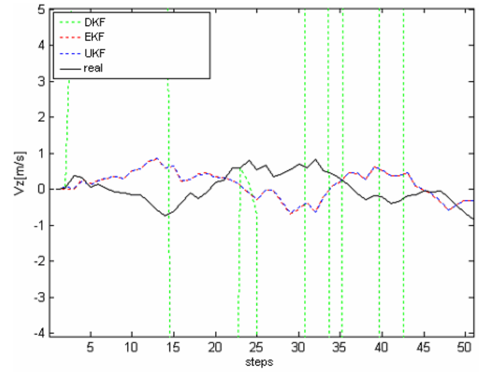

Fig. 11. Estimated v_z speed component

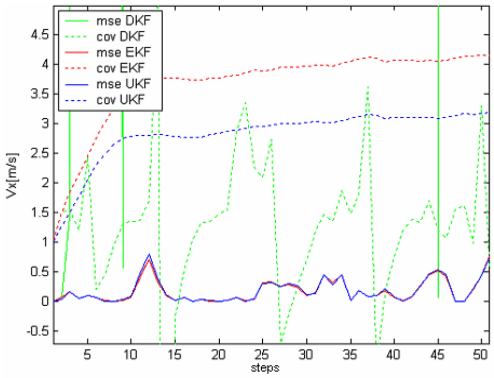

Fig. 12. Mean square error of estimated v_x and covariance error P_{vx} of speed component

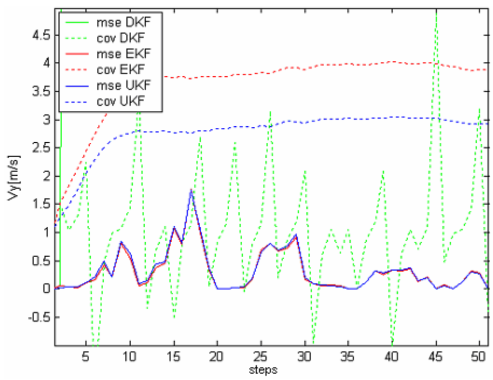

Fig. 13. Mean square error of estimated v_y and covariance error P_{vy} of speed component

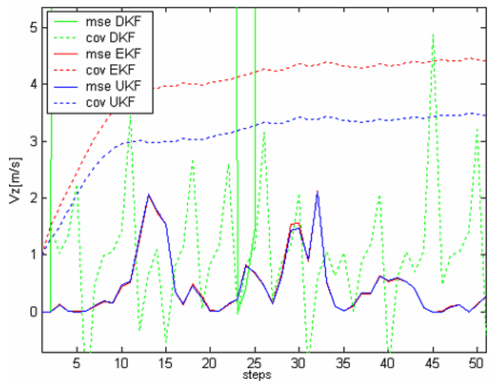

Fig. 14. Mean square error of estimated v_z and covariance error P_{vz} of speed component

Additionally, estimated distance from object to observer and azimuth and elevation of the object are presented in fig. 15 – 17.

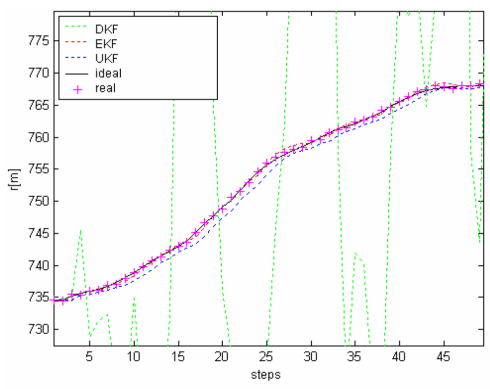

Fig. 15. Estimated distance from object to observer

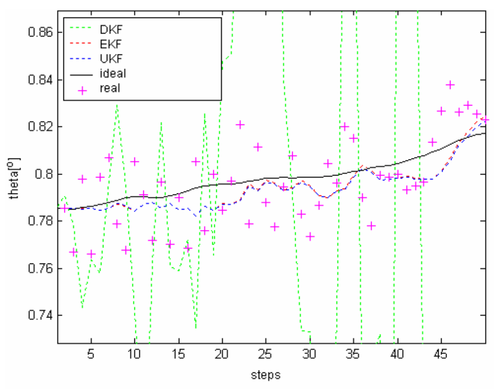

Fig. 16. Estimated azimuth

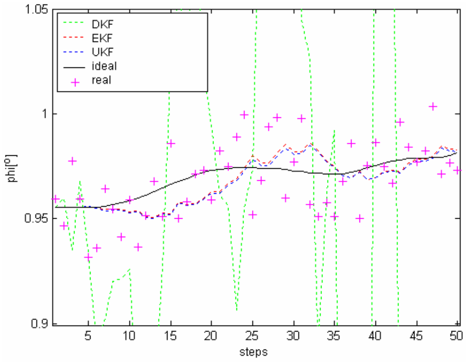

Fig. 17. Estimated elevation

Research of B System

This system was tested in the same conditions as A system. Results are presented it the form of object position in the Cartesian coordinates estimated by the use of covariance filter DKF, extended filter EKF and unscented filter UKF (fig. 18 – 20). The examinations results include values of mean square error of estimated state vector and covariance error estimated components of the position (fig. 21 – 23).

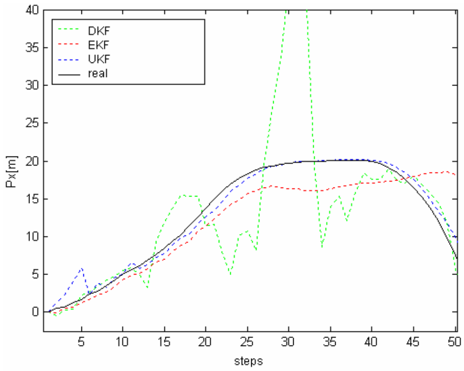

Fig. 18. Estimated component p_x of the position

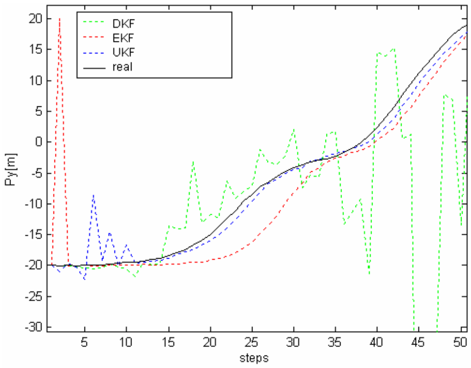

Fig. 19. Estimated component p_y of the position

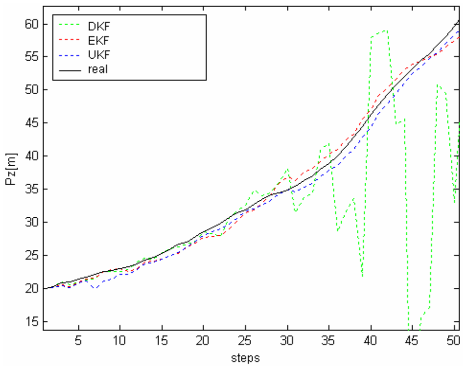

Fig. 20. Estimated component p_z of the position

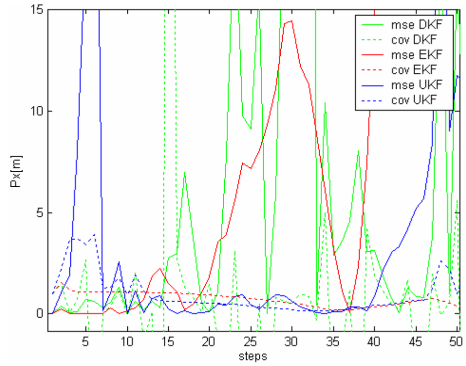

Fig. 21. Mean square error of the estimated component p_x and covariance error P_{px} of the position

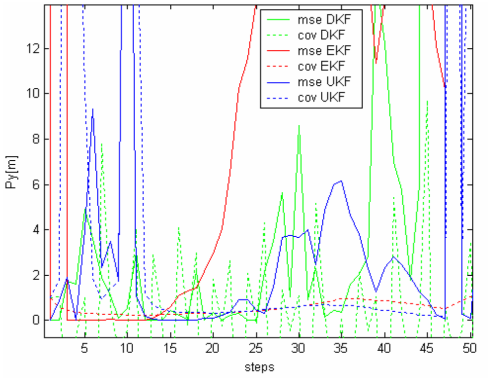

Fig. 22. Mean square error of the estimated component p_y and covariance error P_{py} of the position

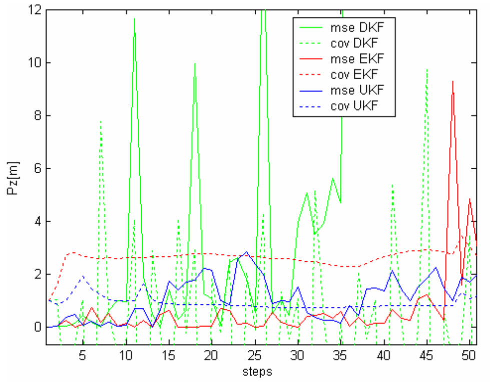

Fig. 23. Mean square error of the estimated component p_z and covariance error P_{pz} of the position

The estimation of object speed in the Cartesian coordinates (fig. 24 – 26) and determination of mean square error and covariance error of speed components (fig. 27 – 29) has also been done.

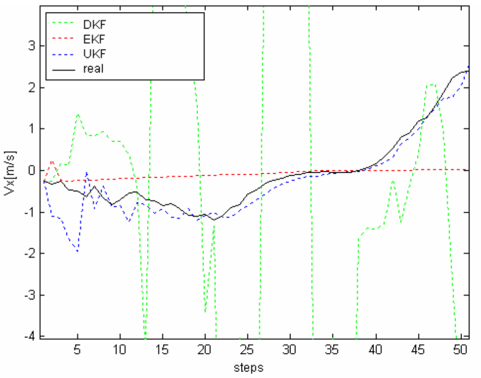

Fig. 24. Estimated v_x speed component

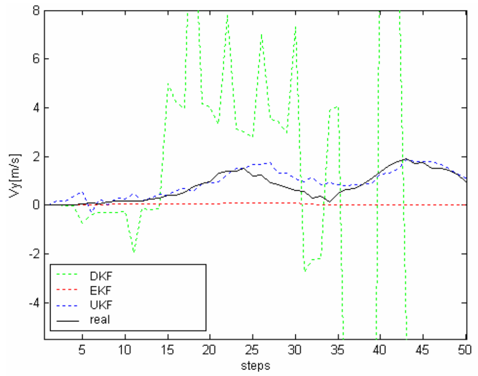

Fig. 25. Estimated v_y speed component

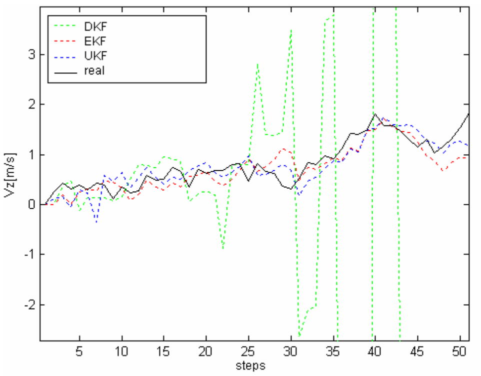

Fig. 26. Estimated v_z speed component

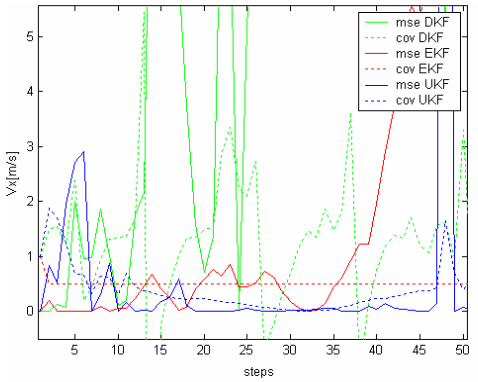

Fig. 27. Mean square error of estimated v_x and covariance error P_{vx} of speed component

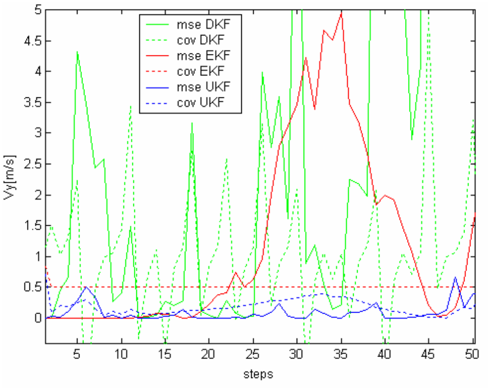

Fig. 28. Mean square error of estimated v_y and covariance error P_{vy} of speed component

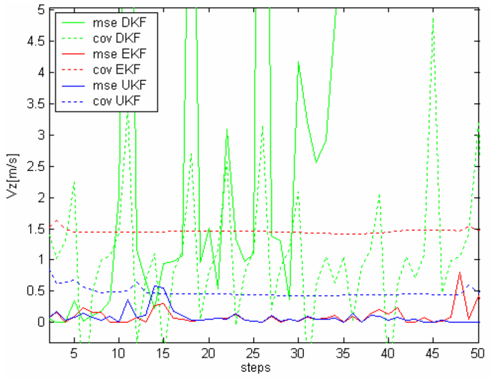

Fig. 29. Mean square error of estimated v_z and covariance error P_{vz} of speed component

Additionally, estimated distance from object to observer, azimuth and elevation of the object are presented in fig. 30 – 32.

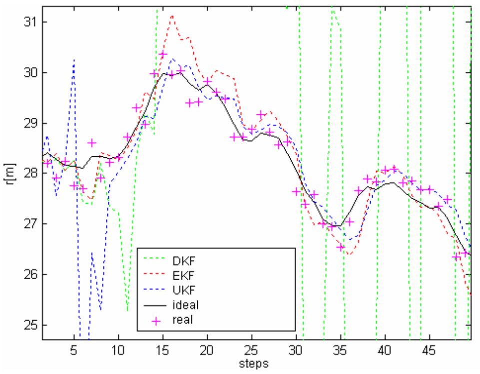

Fig. 30. Estimated distance from object to observer

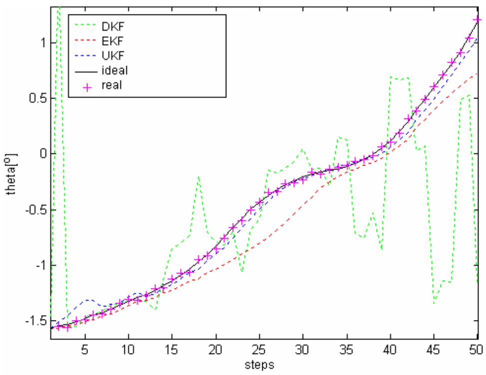

Fig. 31. Estimated azimuth

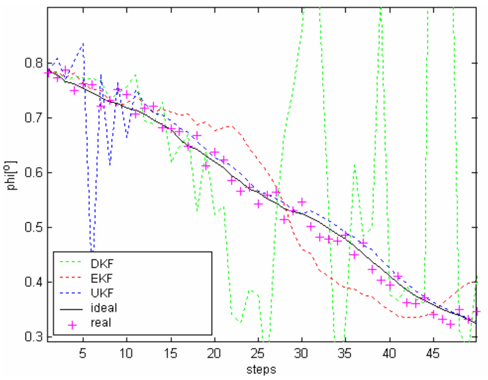

Fig. 32. Estimated elevation

For nonlinear measurement model extended and unscented filters estimate the object position nearly identically. Discrete Kalman filter is becoming non-optimal filter in the sense of minimizing the mean square error. When the mean square error is included within the area determined by the covariance error of the estimated vector state then filter is performing correctly. Last principle is satisfied for extended and unscented Kalman filters but not for DKF. During speed estimation one can see, that difference between extended and unscented filters is minimal. UKF gives smaller errors what results from nature of speed components, which are width-band Gaussian process. The bigger are the jumps of noise values the worse of extended Kalman filter performance is.

CONCLUSIONS

Results of estimation using Discrete and Extended and Unscented Kalman Filter for A and B system show that Unscented Kalman Filter operating as algorithm of data processing in system with nonlinear dynamics guarantees the best quality. Furthermore:

- any nonlinear transform makes Discrete Kalman Filter to stop being optimal in sense of minimum mean square error;
- stability loss in EKF is possible for long measurement steps;
- decrease of measurement steps enlarges computational costs as a result of complicated calculations of Jacobians;
- UKF algorithm does not require to calculate Jacobians and it simplifies its complexity;
- the Unscented Kalman Filter provides effective estimation in case of strongly nonlinear models.

REFERENCES

[1] Brown R. G., Hwang P. Y. C., Introduction to Random Signals and Applied Kalman Filtering with MATLAB Exercises and Solutions, John Wiley & Sons, Canada, 1997.

[2] Gordon N. J., Ristic B., Arulampalam S., Beyond the Kalman Filter – Particle Filters for Tracking Applications, Artech House, London 2004.

[3] Grewal M. S., Andrews A. P, Kalman filtering Theory and Practice Using MATLAB, John Wiley & Sons, Canada, 2001.

[4] Julier S. J., Uhlmann J. K., Durrant-Whyte H. F., A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators, IEEE Transactions on Automatic Control, March 2000, vol. 45, no. 3, pp. 477 – 482.

[5] Konatowski S., Sipa T., Position estimation using Unscented Kalman Filter, Annual of Navigation, 2004, no. 8, pp. 97 – 110.

[6] van der Merwe R., Wan E. A., The Square-root Unscented Kalman Filter for State and Parameter-estimation, Proceedings of International Conference on Acoustics, Speech, and Signal Processing, Salt Lake City, May 2001, vol. 6, pp. 3461 – 3464.

[7] van der Merwe R., Wan E. A., The Unscented Kalman Filter. Department of Electrical and Computer Engineering, Oregon Graduate Institute of Science and Technology, Beaverton, Oregon 2001.

[8] Wan E. A., van der Merwe R., The Unscented Kalman Filter to appear in Kalman Filtering and Neural Networks, Chap. 7, ed by Simon Haykin, John Wiley & Sons, USA, 2001.

[9] Wan E. A., van der Merwe R., The Unscented Kalman Filter for Nonlinear Estimation. Proc., IEEE Symp. Adaptive Systems for Signal Proc., Communication and Control (AS-SPCC), Lake Louise, Alberta, Canada, October 2000, pp. 153 – 158.

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