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A UNIFORM CURRENT IN SHIP MANOEUVRING MATHEMATICAL MODEL

ABSTRACT: Some essential aspects of mathematical modelling the uniform current dynamic impact on ship manoeuvring are discussed and assessed. a simulation of a pure drift, and such a drift coupled with a turning motion is next performed for a small tanker. The one-knot current gives rise to the maximum yaw velocity of order fifteen degrees per minute associated however with a relatively long response time.

INTRODUCTION

The sea current (or stream) in the horizontal plane takes a lot of concern in the shiphandling practise, ship manoeuvring simulation (waterway design, development of safe and efficient shiphandling strategies for masters and pilots under given environmental circumstances), and finally in the mathematical model-based ship automatic control of ship manoeuvres.

In the most general case the current is really non-uniform and non-stationary. The latter is however get round by the practically adequate quasi-stationary assumption. An alteration in both magnitude and direction and in a vertical profile (for deep draft vessels in shallow waters, where the current vanishes near the bottom) is commonly regarded while dealing with the non-uniform sea current effect. Moreover, the same spatial distribution of the current affects different ships quite unlikely, dependent upon their size and the underwater hull shape- longer ships are usually more sensitive under normal horizontal gradients of the current. For simplicity purposes, an average current is often adopted being more or less adequate in particular applications.

In the aspect of ship hydrodynamics, the influence of local transverse cross-current velocity (in ship's body axes) on the sectional (elementary) sway force and yaw moment, and thus the effect of the lateral velocity total spreading upon the resulting force and moment, is rarely known in the reliable way. Such a research is presently rather a field of the computational fluid dynamics than the physical scale model platform.

Some promising attempts exist in the literature e.g. [Misiag, 1992], [Bavin et al., 1991], [Li/Wu, 1990], but much more investigations are still to be done. The current is usually introduced into the manoeuvring mathematical model using the so-called relative water concept- all terms concerned with the surrounding water action (including added masses) shall be referenced to a ship's velocities through the water, while all other items must be linked to the over-ground velocities.

Since the local lateral velocity of a manoeuvring ship in still water also changes (it vanishes in the so-called pivot point), the usual and widely published data on hull hydrodynamic excitations (hull derivatives) in ship manoeuvring can be extensively used in the early validation of the non-uniform current dynamic models. The manoeuvring hull forces and moment, despite the obvious linear velocity square relationship, depend to the largest extent on the drift angle (amidships) and non-dimensional yaw velocity.

From the standpoint of mathematical modelling and simulation, a difficulty of accounting for the non-uniform current relies on constructing the map (database) of current vectors, sufficiently discrete and easy to operate on in the real time, together with a fast interpolation algorithm.

Most of published experimental (or numerical as well) current force and moment data e.g. [OCIMF, 1994] relate to a uniform oblique current in deep and shallow water for a stationary ship. These are essentially the same charts as those applied among others in usual four-quadrant ship manoeuvring predictions in still water. Furthermore, because they are primarily purposed for mooring restraint calculations in an open-wharf condition, they lack for a yaw motion contribution to the hull sway force and yaw moment. If a ship is now free to move, this yaw influence must be therefore provided from other sources.

The current induced yaw motion of a ship is widely recognised by navigators in the shiphandling practise. However, this is exclusively attributed to the non-uniform current effects in rivers and canals (the fastest flow is in the centre-line and slowing down towards either bank), e.g. [Nowicki, 1999], [MacElrevey, 1998], [Armstrong, 1994], which are even sometimes actively utilised in performing sharp turns during routine passage and deberthing manoeuvres. Of course, if there is no flow at a ship's stern while a ship's bow gets a strong flow at the same time, a very high bow-out yaw moment (due to the pressure centre moved far ahead from amidships) occurs. The drift is comparatively smaller than the turning in this situation. Such a trend is opposite in the uniform current, but the produced heading change is still remarkable and shall not be neglected, as contrary to a lot of opinions in shiphandling manuals.

Reverting to the uniform current generated yaw moment, refer e.g. to [OCIMF, 1994], a loaded (even-keel) ship always experiences in case of bow or quarter currents the ship's bow or stern getting away from the current inflow accordingly. On the other hand, on a ballasted (trimmed by stern) ship the yaw moment is consistently in the same direction i.e. for a current from portside, the ship turns to starboard.

The present paper concentrates on the manoeuvring behaviour analysis of an unsteered ship in the uniform current (the propulsion and helm are set to zero) as to explore basic dynamic properties of such current type, which are rarely realised by navigators. This would also enable working out some reference data for shiphandling tasks, and of course for similar studies enhancing the knowledge of the stationary current effect- there is a lack of appropriate results in the literature.

MANOEUVRING EQUATIONS IN THE UNIFORM CURRENT

There are two forms of ship motions equations, written always in moving body axes- vector (general motions, 6DOF) and scalar (manoeuvring specific motions, 3DOF) ones, which are widely used in ship manoeuvring analyses as dependent upon actual objectives and computation power.

The former are studied e.g. in [Artyszuk, 1998]. And after incorporating the uniform current effect, they read:

$$\mathbf{M}_{6x6}\cdot\begin{bmatrix}\left[\dfrac{d\vec{v}^{g}}{dt}\right]\\ \left[\dfrac{d\vec{\omega}}{dt}\right]\end{bmatrix}=\begin{bmatrix}\vec{F}\left(\vec{v}^{w},\vec{\omega}\right)-\vec{\omega}\times\left(\vec{L}_{O}+\vec{Q}_{i}\right)\\ \vec{M}\left(\vec{v}^{w},\vec{\omega}\right)-\vec{\omega}\times\left(\vec{H}+\vec{K}_{i}\right)-\vec{v}^{g}\times\vec{L}_{O}-\vec{v}^{w}\times\vec{Q}_{i}\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\left[\vec{Q}_{i}\right]\\ \left[\vec{K}_{i}\right]\end{bmatrix}=\left[m_{ij}\right]_{6x6}\cdot\begin{bmatrix}\left[\vec{v}^{w}\right]\\ \left[\vec{\omega}\right]\end{bmatrix} \tag{2}$$

$$\vec{v}^{w}=\vec{v}^{g}-\vec{v}^{c},\quad \vec{v}^{c}=\begin{bmatrix}v_{x}^{c}\\ v_{y}^{c}\\ v_{z}^{c}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{bmatrix}\cdot\begin{bmatrix}\left|\vec{v}^{c}\right|\cos\gamma_{c}\\ \left|\vec{v}^{c}\right|\sin\gamma_{c}\end{bmatrix}\\ 0\end{bmatrix} \tag{3}$$

where: $\mathbf{M}_{6x6}$ - generalised mass matrix,

$\left[m_{ij}\right]_{6x6}$ - added mass matrix,

$\vec{v}$, $\vec{\omega}$ - linear and angular velocity vectors,

$\vec{F}$, $\vec{M}$ - external force and moment (excluding impact of added masses),

$\vec{L}_O$, $\vec{H}$ - ship's linear and angular momentum (ground velocity $\vec{v}^g$ associated),

$\vec{Q}_i$, $\vec{K}_i$ - added linear and angular momentum,

$\vec{v}^c$ - current velocity vector,

$\left|\vec{v}^c\right|, \gamma_c$ - current magnitude and direction (in earth coordinates),

'g', 'w' - superscripts denoting ground and water related terms.

For further explanations and details of eqs. (1) to (3), the mentioned [Artyszuk, 1998] shall be consulted. All vectors inside matrices in the applied above matrix notation shall be regarded as column vectors.

The practical 3DOF equations of manoeuvring in the uniform current yield as follows:

$$\begin{cases} \frac{dv_x^g}{dt}\left(m+m_{11}\right) = mv_y^g\omega_z + \underline{c_m}m_{22}v_y^w\omega_z + F_x\left(v_x^w, v_y^w, \omega_z\right) \\ \frac{dv_y^g}{dt}\left(m+m_{22}\right) = -\left(mv_x^g\omega_z + m_{11}v_x^w\omega_z\right) + F_y\left(v_x^w, v_y^w, \omega_z\right) \\ \frac{d\omega_z}{dt}\left(J_z+m_{66}\right) = -\left(m_{22}-m_{11}\right)v_x^w v_x^w + M_z\left(v_x^w, v_y^w, \omega_z\right) \end{cases} \tag{4}$$

where: v_x, v_y, ω_z - surge, sway, and yaw velocity,

m, J_z - ship's mass and moment of inertia,

m_{11}, m_{22}, m_{66} - added masses and inertia,

c_m - empirical viscous factor (sometimes equal to zero).

In both types of the above equations the time derivative of current velocity vector equals zero:

$$\frac{d\vec{v}^c}{dt} = 0, \text{ hence } \frac{d\vec{v}^w}{dt} = \frac{d\vec{v}^g}{dt} \tag{5}$$

Therefore the added masses (to be always linked with water relative velocities) on the left side of eqs. (4) may be combined with the ship's mass.

However, according to some authors e.g. [Wichers, 1987], [Li/Wu, 1990], [Misiag, 1992], the relationship (5) may not be accepted and additional contributions must be embedded as arising from $m_{11} \cdot dv_x^c / dt$ in the surge equation and $m_{22} \cdot dv_y^c / dt$ in the sway equation, which lead to (see e.g. (3)):

$$m_{11}\frac{dv_x^c}{dt}=+m_{11}v_y^c\omega_z\,,\quad m_{22}\frac{dv_y^c}{dt}=-m_{22}v_x^c\omega_z \tag{6}$$

Surprisingly, the validation of such a pure mathematical derivation (6) is only apparent, in fact it violates the hydrodynamic rule of inverting the flow and shall be disregarded. For example, the same forces shall be exerted both on a ship fixed over ground and exposed to the uniform current, and on a ship moving at the same speed but in still water. If the ships are free to turn in such situations, very different (in magnitude and sign) centrifugal forces appear under conditions of (6).

UNIFORM CURRENT IMPACT UPON SHIP DYNAMIC DRIFTING

In the following, no yaw motion ($\omega_z = 0$) of a ship is assumed i.e. the yaw moment exerted by the uniform current is believed to be compensated by other control forces. Also a ship is initially fixed over ground on the north heading (e.g. by mooring ropes or anchor) and then made free to move by inertia - no propulsion is operated. Under such conditions, the water related velocity is non-zero and equal in absolute units to the current velocity.

The main particulars of an exemplary small chemical tanker serving as a base of the computations are depicted in Tab. 1.

Table 1. Ship particulars

type:	chemical tanker	MAIN ENGINE:	
DWT	6000[t]	type:	diesel
HULL:		P_{En} (power)	3600[kW]
m	8950[t]	n_n (revs)	146[rpm]
J_z	$5.2\cdot10^6$[tm^2]	PROPELLER:	
m_{11}	6%m	type:	CPP
m_{22}	100%m	D (diameter)	4.1[m]
m_{66}	83%J_z	$(P/D)_n$ (pitch ratio)	0.8719
L (length)	97.4[m]	RUDDER:	
B (beam)	16.6[m]	type:	Schilling
T (draft)	7.1[m]	A_R (area)	12.3[m^2]
c_B (block coeff.)	0.76[-]	λ (aspect)	1.5[-]

The motion of the tanker will be thus governed by the hull hydrodynamic surge (longitudinal resistance) and sway (lateral resistance) forces.

The below relationships are considered next and called 'simplified':

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} F_{xH} \\ F_{yH} \\ M_{zH} \end{bmatrix} = \begin{bmatrix} 0.5\rho L T v_x^2 \cdot c_{fxh0} \\ 0.5\rho L T v_y^2 \cdot c_{fyh90} \\ 0.5\rho L^2 T \left(v_x v_y \cdot c_{mzh-\beta} + \omega_z^2 L^2 \cdot c_{mzh-\omega} \right) + (m_{22} - m_{11}) v_x v_y \end{bmatrix} \tag{7}$$

where for the chemical tanker the subsequent parameters are adopted:

$$c_{fxh0} = -0.01436\,,\ c_{fyh90} = +0.5\,,\ c_{mzh-\beta} = -0.2\,,\ c_{mzh-\omega} = -0.07$$

In [Artyszuk, 2003] quite similar expressions have been optimised but only in the range of drift angle and nondimensional yaw velocity as experienced in the full scale sea manoeuvring trials. That hybrid model is named 'refined' in the present investigations.

Under a head current ($\gamma_c = 180°$), the ship starts to move with the current after a long time (when the velocity through the water tends to zero), since a relatively low resistance to the forward motion exists - see Fig. 1 (left) for the simplified model of hull hydrodynamics. In the case of a side current (270°), the ship responds much quicker- the lateral resistance is almost 35 times higher than the longitudinal one, though the total mass involved in the pure sway motion is doubled. The time constant in this direction is of order 10[min]. In the shallow water, a ship's response to the current will be essentially faster because of the higher resistance in both axes, though the resistance increase in each direction is quite different.

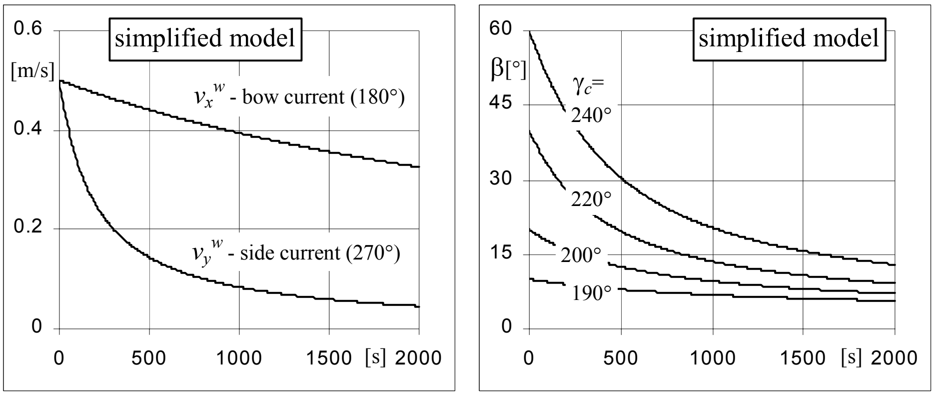

Fig. 1. Drop of water-related velocities (left) and drift angle (right) in current 0.5m/s

The right part of Fig. 1 shows a change of the water-related drift angle for an oblique current from starboard side. In all the cases of the current set, the drift angle diminishes due to the sway velocity going down. It means that the ship does not follow the current direction.

The slope angles of curves in both charts of Fig. 1 depend not only on the hull hydrodynamic parameters but also on the current velocity. Since the very simple instance of motion, the calculations of Fig. 1 can be completed analytically as well.

CURRENT IMPACT UPON COMBINED DRIFTING AND TURNING

To include the yaw effect of the uniform current upon a drifting ship, the full manoeuvring equations (4) with the chemical tanker have been solved- both simplified and refined hull hydrodynamics models are engaged. The former is best suited for preliminary examinations.

Fig. 2 illustrates three manoeuvring motions for the tanker subject to one knot current (0.5m/s) 30° from the starboard bow. The attention shall be paid here to rather long time response of the yaw motion and a relatively low peak in the yaw velocity as compared with parameters of the classical turning circle test. Nevertheless, the yaw impact of the uniform current is significant. The maximum heading change is about 100° (see the right bottom chart)- the ship gets at this point nearly an equilibrium steady motion. The trajectories with ship's contours are demonstrated in Fig. 3.

Other directions of the uniform current have been also simulated with the refined model, which resulted in yaw motions as presented in Fig. 4. The maximum of yaw velocity rapidly changes while commencing to deviate the current inflow from the abeam direction. After that, the magnitude of such extreme is kept constant, though its position along the time axis moves to the right.

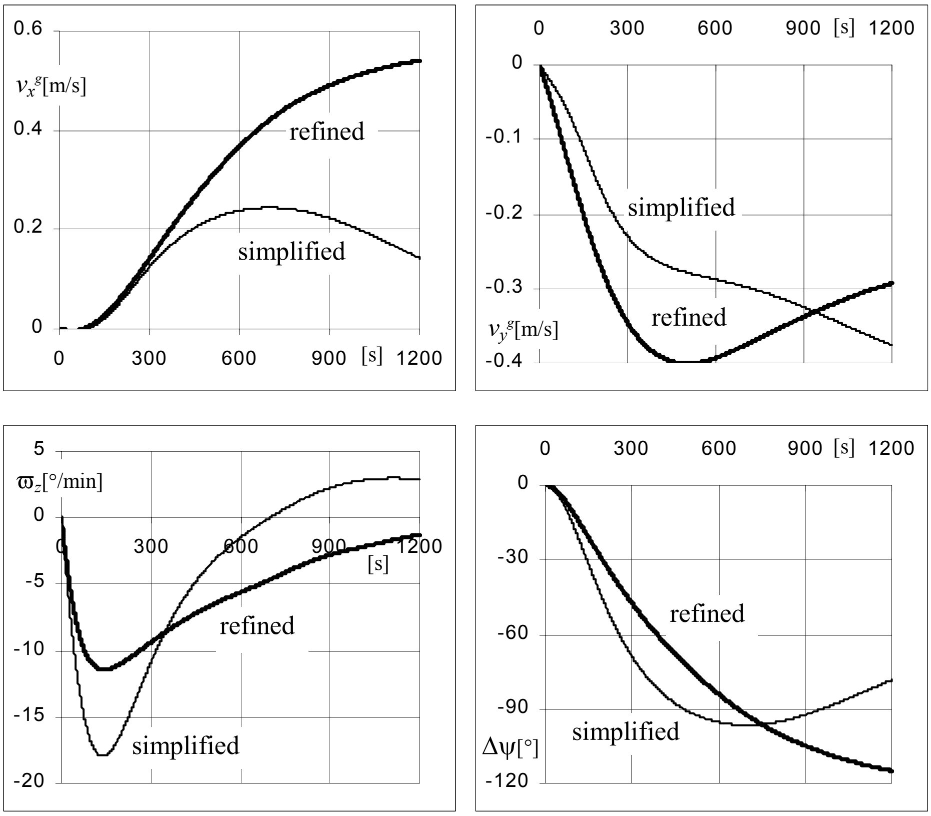

Fig. 2. Time histories of motion components - yaw included, current 210°/0.5m/s

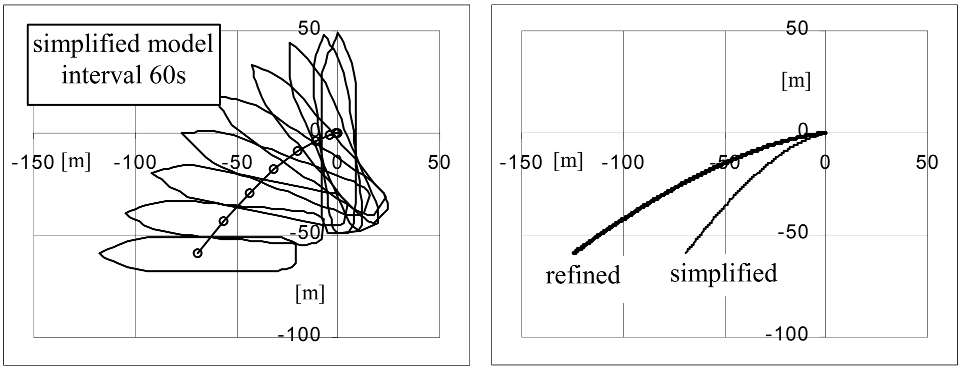

Fig. 3. Tracks in current 210°/0.5m/s

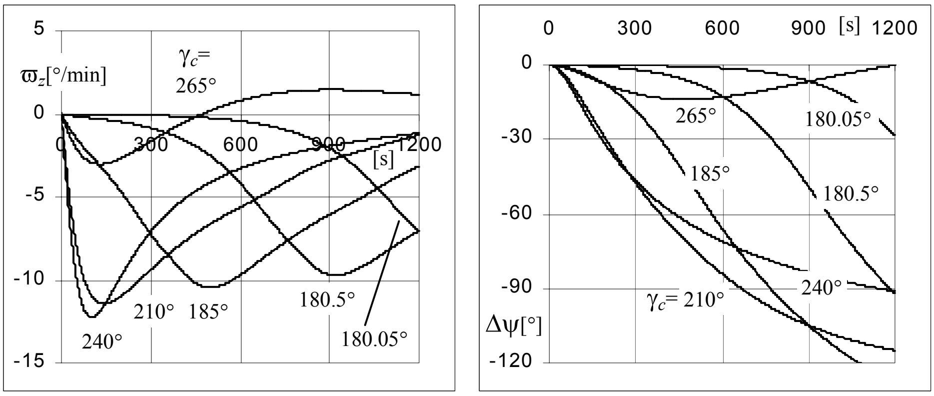

Fig. 4. Yaw velocity and heading vs. direction of current 0.5m/s - refined model.

The maximum heading change, as per Fig. 4, approximately reaches the same value 100° in the most of analysed conditions. This shall be attributed to a tendency of natural seeking a balance in the abeam direction to the current inflow.

FINAL REMARKS

It is advisable to perform analogical computations to the above for shallow water conditions. In order to arrive at reliable and firm conclusions, such numerical experiment requires a possession of corrective multipliers of the added masses and hull hydrodynamic characteristics at finite water depths.

Further research shall be scheduled in the nearest future on a non-uniform nature of the sea current in ship manoeuvring as dominating in a lot of circumstances. Also the impact of both uniform and non-uniform current upon rudder manoeuvres is of particular interest i.e. when the rudder and propeller forces are involved.

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