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THE PROPOSED METHOD OF FILLING NAVIGATIONAL NET ON UNKNOWN LAND MARKS

ABSTRACT In this paper the author shows a proposal of expanding a set of coastal navigational aids by the navigator during the coastal voyage. The method implies that landmarks, the coordinates of which are unknown, are included into the net of optical aids. This proposal uses the sequence method of geodesic measurements adjustment. The application of sequence method can be use in solving some navigational problems on the warship and merchant ships during coastal voyage. This article presents the part of the geodesic method of work which deals with the problem of navigation calculations.

INTRODUCTION

The landmarks with characteristic colors and shape create optical navigational net. This kind of navigational aids now is used less than it used to be in the beginning of the last century. The development of the automated ship navigation control systems and satellite systems causes the integrated use of them. High accuracy and easiness of their use reduce the application frequency of terrestrial methods. Minimizing the time of fixing position at sea with its high accuracy is one of the most important task for the navigator while keeping the bridge watch. Fast development of navigational technology began the process of fitting the bridge watch. Fast development of navigational technology began the process of fitting the ship bridge with the device called “one-man-bridge”. This equipment uses only the radio-navigational systems for fixing position. Still the land navigational aids are very important for the navigator, especially during a coastal voyage. Therefore, the problem of modernizing the methods of fixing the position with the use of land navigational aids seem very important. The new methods should give the possibility of easy algorithmization and automatization of navigational calculations. Consequently, the time of fixing the position can be reduced. The time of fixing the position should be decreased and it should be comparable with the time of fixing performed by the satellite receiver.

This paper shows that it is possible to adapt some geodesic methods of measurement adjustment for the purposes of maritime navigation. Moreover, using the geodesic methods gives the possibility of using characteristic landmarks not marked on navigational charts while coastal voyage. These landmarks can be use in the process of maritime navigation when the geodesic method described below is applied.

DESCRIBING OF THE ADJUSTMENT PROBLEM

The proposal shown in this paper is based on the possibility of taking bearings and distances to land mark (called station ***M***) by navigator. The geometrical system of navigation marks which number is *k>2* and station ***M*** are giving the possibility writing the following equation of corrections:

$$\mathbf{V}_j = \mathbf{A}_{P_j} \cdot \hat{\mathbf{X}}_{P_j} + \mathbf{A}_M^j \cdot \hat{\mathbf{X}}_M^j + \mathbf{L}_j \tag{1}$$

where:

$\hat{\mathbf{X}}_{P_j} = \begin{bmatrix} \hat{d}x_{P_j} \\ \hat{d}y_{P_j} \end{bmatrix}$ - the vector of increments of coordinates of the approximate position P_j;

$\hat{\mathbf{X}}_M^j = \begin{bmatrix} \hat{d}x_M^j \\ \hat{d}y_M^j \end{bmatrix}$ - the vector of increments of coordinates of the approximate position station ***M***

Introduce the following symbols:

$$\mathbf{A}_j = \left[\mathbf{A}_{P_j} \ \vdots \ \mathbf{A}_M^j\right] \quad \text{and} \quad \hat{\mathbf{X}}_j = \begin{bmatrix} \hat{\mathbf{X}}_{P_j} \\ \hat{\mathbf{X}}_M^j \end{bmatrix}$$

where:

$$\mathbf{A}_{P_j} = \begin{bmatrix} \frac{\partial NR_1}{\partial x_{P_j}} & \frac{\partial NR_1}{\partial y_{P_j}} \\ \frac{\partial NR_2}{\partial x_{P_j}} & \frac{\partial NR_2}{\partial y_{P_j}} \\ \cdots & \cdots \\ \frac{\partial NR_k}{\partial x_{P_j}} & \frac{\partial NR_k}{\partial y_{P_j}} \\ \frac{\partial NR_M^j}{\partial x_{P_j}} & \frac{\partial NR_M^j}{\partial y_{P_j}} \\ \frac{\partial d_M^j}{\partial x_{P_j}} & \frac{\partial d_M^j}{\partial y_{P_j}} \end{bmatrix}; \qquad \mathbf{A}_M^j = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \cdots & \cdots \\ 0 & 0 \\ \frac{\partial NR_M^j}{\partial x_M^j} & \frac{\partial NR_M^j}{\partial y_M^j} \\ \frac{\partial d_M^j}{\partial x_M^j} & \frac{\partial d_M^j}{\partial x_M^j} \end{bmatrix}$$

equation (1) we can be written as follows:

$$\mathbf{V}_j = \left[\mathbf{A}_{P_j} \ \vdots \ \mathbf{A}_M^j\right] \cdot \begin{bmatrix} \hat{\mathbf{X}}_{P_j} \\ \hat{\mathbf{X}}_M^j \end{bmatrix} + \mathbf{L}_j \tag{2}$$

Therefore, in stage (1) the adjustment problem is rewritten in the following way:

$$\begin{cases} \mathbf{V}_j = \mathbf{A}_j \hat{\mathbf{X}}_j + \mathbf{L}_j \\ \mathbf{P}_j = \mathbf{Q}_j^{-1} \\ \mathbf{\Phi}(\hat{\mathbf{X}}_j) = \mathbf{V}_j^T \mathbf{P}_j \mathbf{V}_j = \min \end{cases} \tag{3}$$

and the cofactor matrix is:

$$\mathbf{Q}_j = \begin{bmatrix} m_{NR_1}^2 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & m_{NR_k}^2 & 0 & 0 \\ 0 & 0 & 0 & m_{NR_M^j}^2 & 0 \\ 0 & 0 & 0 & 0 & m_{d_M^j}^2 \end{bmatrix}$$

Solution of this problem is equation:

$$\hat{\mathbf{X}}_j = \mathbf{\Theta}_j \mathbf{l}_j \tag{4}$$

with cofactor matrix $\mathbf{Q}_{\hat{X}_j} = \mathbf{\Theta}_j$, where:

$$\mathbf{\Theta}_j = (\mathbf{A}_j^T \mathbf{P}_j \mathbf{A}_j)^{-1}$$

$$\mathbf{l}_j = -\mathbf{A}_j^T \mathbf{P}_j \mathbf{A}_j$$

writing the matrixes $\mathbf{\Theta}_j$ and $\mathbf{l}_j$ as follows:

$$\mathbf{\Theta}_j = \begin{bmatrix} \mathbf{\Theta}_{11}^j & \vdots & \mathbf{\Theta}_{12}^j \\ \cdots & \vdots & \cdots \\ (\mathbf{\Theta}_{12}^j)^T & \vdots & \mathbf{\Theta}_{22}^j \end{bmatrix};$$

$$\mathbf{l}_j = -\begin{bmatrix} \mathbf{A}_{p_j}^T \\ (\mathbf{A}_M^j)^T \end{bmatrix} \mathbf{P}_j \mathbf{L}_j = \begin{bmatrix} -\mathbf{A}_{p_j}^T \mathbf{P}_j \mathbf{L}_j \\ -(\mathbf{A}_M^j)^T \mathbf{P}_j \mathbf{L}_j \end{bmatrix} = \begin{bmatrix} \mathbf{l}_1^j \\ \mathbf{l}_2^j \end{bmatrix}$$

we obtain:

$$\hat{\mathbf{X}}_j = \begin{bmatrix} \hat{\mathbf{X}}_{P_j} \\ \hat{\mathbf{X}}_M^j \end{bmatrix} = \begin{bmatrix} \mathbf{\Theta}_{11}^j & \vdots & \mathbf{\Theta}_{12}^j \\ \cdots & \vdots & \cdots \\ (\mathbf{\Theta}_{12}^j)^T & \vdots & \mathbf{\Theta}_{22}^j \end{bmatrix} \cdot \begin{bmatrix} \mathbf{l}_1^j \\ \mathbf{l}_2^j \end{bmatrix} \tag{5}$$

However the cofactor matrix of vector $\hat{\mathbf{X}}_j$ has the structure of the matrix $\mathbf{\Theta}_j$.

After solving this problem we obtain the increments to the approximate coordinates of ship position at sea with suitable cofactor matrix $(\hat{\mathbf{X}}_{P_j}, \mathbf{Q}_{P_j})$ the increments to the approximate coordinates of station ***M*** with own the cofactor matrix $(\hat{\mathbf{X}}_M^j, \mathbf{Q}_M^j)$.

The ship is sailing farther and the navigator is fixing new position P_{j+1} and taking the new measurement to station ***M***. For new ship position we can write new system of corrections:

$$\mathbf{V}_{j+1} = \mathbf{A}_{p_{J+1}} \hat{\mathbf{X}}_{P_{j+1}} + \mathbf{A}_M^{j+1} \hat{\mathbf{X}}_M^{j+1} + \mathbf{L}_{j+1} \tag{6}$$

and according to the sequence adjustment, we assume that the estimator $\hat{\mathbf{X}}_M^{j+1}$ constitutes the sum of previous estimator of vector $\hat{\mathbf{X}}_M^j$ and corrections $\mathbf{V}_M^{j+1}$ i.e.:

$$\hat{\mathbf{X}}_M^{j+1} = \hat{\mathbf{X}}_M^j + \mathbf{V}_M^{j+1} \tag{7}$$

from this it can be obtained:

$$\mathbf{V}_M^{j+1} = \hat{\mathbf{X}}_M^{j+1} - \hat{\mathbf{X}}_M^j \tag{8}$$

where $\hat{\mathbf{X}}_M^j$ constitutes the known absolute terms of this equation. And now we have system of two correction equations in moment *j + 1*:

$$\begin{cases} \mathbf{V}_{j+1} = \mathbf{A}_{p_{J+1}} \hat{\mathbf{X}}_{P_{j+1}} + \mathbf{A}_M^{j+1} \hat{\mathbf{X}}_M^{j+1} + \mathbf{L}_{j+1} \\ \mathbf{v}_M^{j+1} = \hat{\mathbf{X}}_M^{j+1} - \hat{\mathbf{X}}_M^j \end{cases} \tag{9}$$

and now we can introduce the following symbols:

$$\widetilde{\mathbf{V}}_{j+1} = \begin{bmatrix} \mathbf{V}_{j+1} \\ \mathbf{v}_M^{j+1} \end{bmatrix};\ \mathbf{A}_{j+1} = \begin{bmatrix} \mathbf{A}_{P_{j+1}} & \mathbf{A}_M^{j+1} \\ \underline{0} & \underline{\mathbf{E}} \end{bmatrix};\ \widetilde{\mathbf{L}}_{j+1} = \begin{bmatrix} \mathbf{L}_{j+1} \\ -\hat{\mathbf{X}}_M^j \end{bmatrix};\ \hat{\mathbf{X}}_{j+1} = \begin{bmatrix} \hat{\mathbf{X}}_{P_{j+1}} \\ \hat{\mathbf{X}}_M^{j+1} \end{bmatrix}$$

can be written as follows:

$$\widetilde{\mathbf{V}}_{j+1} = \mathbf{A}_{j+1} \hat{\mathbf{X}}_{j+1} + \hat{\mathbf{L}}_{j+1} \tag{10}$$

Therefore the adjustment problem changes into:

$$\begin{cases} \widetilde{\mathbf{V}}_{j+1} = \mathbf{A}_{j+1} \hat{\mathbf{X}}_{j+1} + \hat{\mathbf{L}}_{j+1} \\ \widetilde{\mathbf{P}}_{j+1} = \widetilde{\mathbf{Q}}_{j+1}^{-1} \\ \Phi(\hat{\mathbf{X}}_{j+1}) = (\widetilde{\mathbf{V}}_{j+1})^T \widetilde{\mathbf{P}}_{j+1} \widetilde{\mathbf{V}}_{j+1} = \min \end{cases} \tag{11}$$

where the cofactor matrix equals:

$$\tilde{\mathbf{Q}}_{j+1}=\begin{bmatrix}\mathbf{Q}_{j+1} & \vdots & \underline{0}\\ \cdots & \vdots & \cdots\\ \underline{0} & \vdots & \mathbf{Q}_M^j\end{bmatrix}=\begin{bmatrix} m^2_{NR'_1} & 0 & 0 & 0 & 0 & 0 & \vdots & 0\\ 0 & m^2_{NR'_2} & 0 & 0 & 0 & 0 & \vdots & 0\\ 0 & 0 & \ddots & 0 & 0 & 0 & \vdots & 0\\ 0 & 0 & 0 & m^2_{NR'_k} & 0 & 0 & \vdots & 0\\ 0 & 0 & 0 & 0 & m^2_{NR_M^{j+1}} & 0 & \vdots & 0\\ 0 & 0 & 0 & 0 & 0 & m^2_{d_M^{j+1}} & \vdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots\\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \mathbf{Q}_M^j\end{bmatrix}$$

Solution of this problem is system of equation:

$$\hat{\mathbf{X}}_{j+1}=\mathbf{\Theta}_{j+1}\mathbf{l}_{j+1}\leftarrow\tilde{\mathbf{Q}}_{X_{j+1}}=\mathbf{\Theta}_{j+1} \tag{12}$$

where:

$$\mathbf{\Theta}_{j+1}=(\mathbf{A}^T_{j+1}\tilde{\mathbf{P}}_{j+1}\mathbf{A}_{j+1})^{-1}=\begin{bmatrix}\mathbf{\Theta}_{11}^{j+1} & \vdots & \mathbf{\Theta}_{12}^{j+1}\\ \cdots & \vdots & \cdots\\ (\mathbf{\Theta}_{12}^{j+1})^T & \vdots & \mathbf{\Theta}_{22}^{j+1}\end{bmatrix}$$

$$\mathbf{l}_j=-\mathbf{A}^T_{j+1}\tilde{\mathbf{P}}_{j+1}\tilde{\mathbf{L}}_{j+1}=-\begin{bmatrix}\mathbf{A}^T_{P_{j+1}}\mathbf{P}_{j+1}\mathbf{L}_{j+1}\\ (\mathbf{A}_M^{j+1})^T\mathbf{P}_{j+1}\mathbf{L}_{j+1}-\mathbf{P}_M^j\hat{\mathbf{X}}_M^j\end{bmatrix}$$

$$\tilde{\mathbf{P}}_{j+1}=\begin{bmatrix}\mathbf{P}_{j+1} & \vdots & \underline{0}\\ \cdots & \vdots & \cdots\\ \underline{0} & \vdots & \mathbf{P}_M^j\end{bmatrix}=\tilde{\mathbf{Q}}^{-1}_{j+1}=\begin{bmatrix}\mathbf{Q}^{-1}_{j+1} & \vdots & \underline{0}\\ \cdots & \vdots & \cdots\\ \underline{0} & \vdots & (\mathbf{Q}_M^j)^{-1}\end{bmatrix}$$

Again, we can introduce the following symbols:

$$\mathbf{l}_1^{j+1}=-\mathbf{A}^T_{P_{j+1}}\mathbf{P}_{j+1}\mathbf{L}_{j+1} \tag{13}$$

$$\mathbf{l}_2^{j+1}=-\mathbf{A}_M^{j+1}\mathbf{P}_{j+1}\mathbf{L}_{j+1} \tag{14}$$

obtain then:

$$\mathbf{l}_{j+1}=\begin{bmatrix}\mathbf{l}_1^{j+1}\\ \mathbf{l}_2^{j+1}+\mathbf{P}_M^j\hat{\mathbf{X}}_M^j\end{bmatrix} \tag{15}$$

and next

$$\hat{\mathbf{X}}_{j+1}=\begin{bmatrix}\hat{\mathbf{X}}_{P_{j+1}}\\ \hat{\mathbf{X}}_M^{j+1}\end{bmatrix}=\mathbf{\Theta}_{j+1}\mathbf{l}_{j+1}=\begin{bmatrix}\mathbf{\Theta}_{11}^{j+1} & \vdots & \mathbf{\Theta}_{12}^{j+1}\\ \cdots & \vdots & \cdots\\ (\mathbf{\Theta}_{12}^{j+1})^T & \vdots & \mathbf{\Theta}_{22}^{j+1}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{l}_1^{j+1}\\ \mathbf{l}_2^{j+1}+\mathbf{P}_M^j\hat{\mathbf{X}}_M^j\end{bmatrix} \quad (16)$$

we obtain the approximate equations:

$$\hat{\mathbf{X}}_{P_{j+1}}=\mathbf{\Theta}_{11}^{j+1}\mathbf{l}_1^{j+1}+\mathbf{\Theta}_{12}^{j+1}(\mathbf{l}_2^{j+1}+\mathbf{P}_M^j\hat{\mathbf{X}}_M^j)\leftarrow\mathbf{Q}_{x_{j+1}}=\mathbf{\Theta}_{11}^{j+1} \quad (17)$$

$$\hat{\mathbf{X}}_M^{j+1}=(\mathbf{\Theta}_{12}^{j+1})^T\mathbf{l}_1^{j+1}+\mathbf{\Theta}_{22}^{j+1}(\mathbf{l}_2^{j+1}+\mathbf{P}_M^j\hat{\mathbf{X}}_M^j)\leftarrow\mathbf{Q}_M^{j+1}=\mathbf{\Theta}_{22}^{j+1} \quad (18)$$

Assuming that the navigator fixes his position in P_{j+2}, we can carry out the last stage of fixing coordinates station ***M***. Additionally, we can improve the accuracy of station ***M*** fix-position. We can do it according to equations (6) – (18). In this stage we should calculate the covariance matrix. It is necessary, because we should know the accuracy of the calculation carried out and it helps estimating of the usefulness proposed method of navigation. Therefore, the adjustment problem will be as follows:

$$\begin{cases}\tilde{\mathbf{V}}_{j+2}=\mathbf{A}_{j+2}\hat{\mathbf{X}}_{j+2}+\hat{\mathbf{L}}_{j+2}\\ \tilde{\mathbf{C}}_{j+2}=m_{0_{j+2}}^2\tilde{\mathbf{Q}}_{j+2}\\ \Phi(\hat{\mathbf{X}}_{j+2})=(\tilde{\mathbf{V}}_{j+2})^T\tilde{\mathbf{P}}_{j+2}\tilde{\mathbf{V}}_{j+2}=\min\end{cases} \quad (19)$$

where:

$$\tilde{\mathbf{P}}_{j+2}=\tilde{\mathbf{Q}}_{j+2}^{-1}$$

solution this problem, as in previous stage, is two vectors of increments with the appropriate covariance matrices:

$$\hat{\mathbf{X}}_{P_{j+2}}=\mathbf{\Theta}_{11}^{j+2}\mathbf{l}_1^{j+2}+\mathbf{\Theta}_{12}^{j+2}(\mathbf{l}_2^{j+2}+\mathbf{P}_M^{j+1}\hat{\mathbf{X}}v_M^{j+1})\leftarrow\mathbf{C}_{x_{j+2}}=m_{0_{j+2}}^2\mathbf{\Theta}_{11}^{j+2} \quad (20)$$

$$\hat{\mathbf{X}}_M^{j+2}=(\mathbf{\Theta}_{12}^{j+2})^T\mathbf{l}_1^{j+2}+\mathbf{\Theta}_{22}^{j+2}(\mathbf{l}_2^{j+2}+\mathbf{P}_M^{j+1}\hat{\mathbf{X}}_M^{j+1})\leftarrow\mathbf{C}_M^{j+2}=m_{0_{j+2}}^2\mathbf{\Theta}_{22}^{j+2} \quad (21)$$

where

$$m_{0_{j+2}}^2=\frac{\tilde{\mathbf{V}}_{j+2}^T\tilde{\mathbf{Q}}_{j+2}\tilde{\mathbf{V}}_{j+2}}{k+2-a} \quad (22)$$

and:
k – number of new measurements of navigational landmarks;
a – number of unknown coordinates increments.

EXAMPLE

Let's assume that the ship sails as in the figure 1. The navigator is using five navigational marks all the time. They coordinates are given in Table 1.

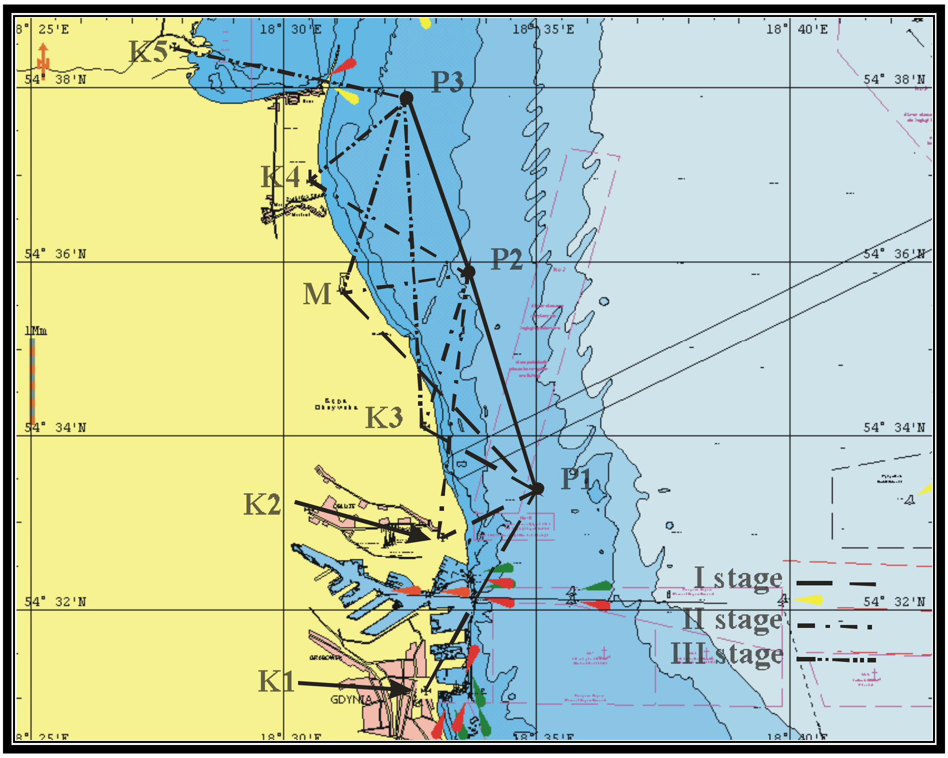

Fig. 1. The graphical interpretation of example

Table 1. The coordinates of navigational landmarks and ship's position in the presented example

Navigational landmarks	The plane coordinates		Ship's Positions	The plane coordinates	
	X	Y		X	Y
K1	6046534,77	341177,47	P1	6051014,67	343671,36
K2	6049663,62	341608,34	P2	6055303,74	342449,57
K3	6052094,59	341496,63	P3	6059187,86	341457,54
K4	6057279,65	339008,93			
K5	6060324,49	336282,02			

At first let's assume that the navigator, in position P1, took bearings to three navigational marks (K1, K2, K3). These bearings are $NR_1 = 208{,}5°$; $NR_2 = 236{,}0°$; $NR_3 = 296{,}0°$. And next the approximate coordinates of station ***M*** are calculated with giroskope bearing ($NR_{M1} = 316{,}5°$) and distance ($d_M = 5450$m).

According to equations (1) - (5) increments of ship – coordinates and incerements of station M coordinates were estimated:

$$\hat{\mathbf{X}}_{P_j} = \begin{bmatrix} dx_{p_1} \\ dy_{p_j} \end{bmatrix} = \begin{bmatrix} -29,37 \\ 25,11 \end{bmatrix}; \qquad \hat{\mathbf{X}}_M^j = \begin{bmatrix} dx_M^j \\ dy_M^j \end{bmatrix} = \begin{bmatrix} 12,04 \\ 53,20 \end{bmatrix}$$

giving the improved ship and station ***M*** position:

$$\hat{\mathbf{X}}_j = \begin{bmatrix} x_{p_j} \\ y_{p_1} \\ x_M^j \\ y_M^j \end{bmatrix} = \begin{bmatrix} 6050985,30 \\ 343696,47 \\ 6055005,34 \\ 340016,23 \end{bmatrix}$$

The uniqueness of sequence method of measurement adjustment obligates us to calculate the cofactor matrixes in the indirect stages. These matrixes in i stage of calculate amount properly:

$$\mathbf{Q}_{p_j} = \begin{bmatrix} 693,95 & 63,93 \\ 63,93 & 298,63 \end{bmatrix}; \qquad \mathbf{Q}_{M_j} = \begin{bmatrix} 1945,93 & 1137,63 \\ 1137,63 & 1399,39 \end{bmatrix}$$

in P2 – position (moment *j+1*) the navigator took new bearings to navigational landmarks (K2, K3, K4): NR_4 = 188,0°; NR_5 = 196,0°; NR_6 = 300,5° and new bearing to station ***M***: NR_{M2} = 265,5°. And next according to equations (6) - (17) new increments of ship – coordinates and incerements of station ***M*** coordinates were estimated and they amount appropriately:

$$\hat{\mathbf{X}}_{P_{j+1}} = \begin{bmatrix} dx_{p_{j+1}} \\ dy_{p_j+1} \end{bmatrix} = \begin{bmatrix} 28,83 \\ 43,49 \end{bmatrix}; \qquad \hat{\mathbf{X}}_M^{j+1} = \begin{bmatrix} dx_M^{j+1} \\ dy_M^{j+1} \end{bmatrix} = \begin{bmatrix} 9,54 \\ 51,42 \end{bmatrix}$$

and they give the estimated ship position and station ***M*** – position in *j+1* moment:

$$\hat{\mathbf{X}}_{j+1} = \begin{bmatrix} x_{p_{j+1}} \\ y_{p_{j+1}} \\ x_R^{j+1} \\ y_R^{j+1} \end{bmatrix} = \begin{bmatrix} 6055332,57 \\ 3424493,06 \\ 6055014,88 \\ 340067,66 \end{bmatrix}$$

The cofactor matrixes in stage II of the calculations, amount:

$$\mathbf{Q}_{p_{j+1}} = \begin{bmatrix} 594,75 & 41,57 \\ 41,57 & 743,38 \end{bmatrix}; \qquad \mathbf{Q}_{M_{j+1}} = \begin{bmatrix} 1675,43 & 758,56 \\ 758,56 & 868,14 \end{bmatrix}$$

The third stage of calculations consists of measurements to three bearings to navigational marks (K3, K4, K5) and one bearing to station ***M***, which amount appropriately: NR_7 = 180,0°; NR_8 = 181,0°; NR_9 = 283,0°; NR_{R3} = 199,0°. According to equations (6) - (10) and (19) – (22) we can calculate increments of ship – coordinates and incerements of station *M* coordinates which amount suitably:

$$\hat{\mathbf{X}}_{P_{j+2}} = \begin{bmatrix} dx_{p_{j+2}} \\ dy_{p_{j+2}} \end{bmatrix} = \begin{bmatrix} 53,73 \\ 19,23 \end{bmatrix}; \qquad \hat{\mathbf{X}}_{M}^{j+2} = \begin{bmatrix} dx_M^{j+2} \\ dy_M^{j+2} \end{bmatrix} = \begin{bmatrix} 16,12 \\ 71,36 \end{bmatrix}$$

giving corrected ship - position in j+2 moment and finally station ***M*** – position:

$$\hat{\mathbf{X}}_{j+2} = \begin{bmatrix} x_{p_{j+2}} \\ y_{p_{j+2}} \\ x_M^{j+2} \\ y_M^{j+2} \end{bmatrix} = \begin{bmatrix} 6059241,59 \\ 341476,78 \\ 6059187,86 \\ 341457,54 \end{bmatrix}$$

in the last stage of sequence adjustment the covariance matrices were calculated. In the last stage of sequence adjustment the covariance matrixes were calculated:

$$\mathbf{C}_{P_{j+2}} = \begin{bmatrix} 366,29 & 9,84 \\ 9,84 & 583,48 \end{bmatrix}; \qquad \mathbf{C}_{M}^{j+2} = \begin{bmatrix} 473,34 & 214,30 \\ 214,30 & 245,26 \end{bmatrix}$$

the analysis of covariance matrices gave the possibility to describe root-mean-square error (M_M) of landmark ***M*** position fixing:

$$M_M = \pm\sqrt{473,34 + 245,26} = 26,80m$$

and root-mean-square error (M_P) of ship positin fixing at sea:

$$M_P = \pm\sqrt{366,29 + 583,48} = 30,81m$$

THE METHOD ACCURACY

The accuracy of the proposed method is based on the accuracy of navigational parameters. In navigation, we assume that:

- the root-mean-square error of course and bearing for the gyro-compass is of the range $\sigma_{K\dot{Z}} = \sigma_{N\dot{Z}} = 0,5^\circ \div 1,5^\circ$. In the presented example we assume: $\sigma_{K\dot{Z}} = \sigma_{N\dot{Z}} = 0,5$;

- the root-mean-square error of ship distance depends on the kind of distance recorder being at ship's disposal. So far these devices give the possibility of measurement of the distance with the accuracy $\sigma_{\Delta\log} = 0{,}1 \div 1{,}0\%$ of run ship distance. In our example this error amounts: $\sigma_d = 10{,}0m$

When we put into the equations of calculation the average values of fix - position errors in given conditions we can say that for the maximum values of ship's device errors we are able to determine the average values of position line errors, ship's position and station ***M*** position (for ships sailing along the sea shore no farther then 1 nautical mile from coast):

- the average values of position line error, for loxodromic bearing, will amount:

$$M_{lp_{NR}} = \sigma^{\circ}_{NR} arc1^{\circ} d = 1{,}5^{\circ} \cdot 0{,}01745 \cdot 9260 = 242m$$

- the average value of error of fixing the estimated ship position at sea, amounts:

$$M_Z = \sqrt{\sigma_S^2 + \sigma_P^2} = 171{,}5m$$

where:

σ_S - the average value of the transverse component error of fixing the estimated ship position:

$$\sigma_S = \frac{d}{60} \cdot \frac{\sigma^{\circ}_{K\dot{Z}}}{57{,}3^{\circ}}$$

σ_P - the average value of longitudinal component of error of estimated ship position fix amounts:

$$\sigma_P = d \cdot \frac{\sigma_{\Delta\log}}{100}$$

- the average errors of the approximate coordinates of station ***M*** amount:

$$M_M^O = \pm\sqrt{(1{,}5 \cdot 0{,}01745 \cdot 9260)^2 + 10^2} = 242{,}2m$$

Therefore, the average errors of fixing ship's position, in the third stage of example, which sails along the sea shore no farther then 1 nautical mile from coast amounts to:

$$M_O = \pm\sqrt{M_{lp_1}^2 + M_{lp_2}^2 + M_{lp_3}^2 + M_Z^2} = 484{,}1m$$

The full answer the question: whether this method is worth being used in practice? can be given only when we take into consideration the international recommendations. The IMO requirements say that at this distance from the coast the ship position accuracy of the ships in the Baltic Sea areas whose draught is no more than 15 metres, should be not less than 0,5 nautical mile (about926 m). So we can try to say that the proposed method could be used in some navigational calculation.

CONCLUSIONS

The presented method may be used it the following conditions:

- the navigational parameters should be determined with the best accuracy by the most précised devices which are on ship disposal.
- the influence of the hydro-meteorological conditions should be favorable.

When the conditions described in point 1 are fulfilled then it would be possible to use the described method. It offers the navigator new possibilities of using the landmarks to fixing ships position with enhanced accuracy.

The method being described in this paper, should be considered as the back – up - method. We can use the new object when we have changed ships route and we do not have any nautical charts except the general ones. It is worth to perform the research using full information of ship moving during fixing the land object position.

This research can help to improve this method by the automation of automatic the calculations of all the terrestrial navigation methods. It will give the new possibility because it can be considered as the new device of the „one – man – bridge" equipment.

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