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Jacek Szymanowski, Jarosław Grzelak, Stanisław Popowski
Telecommunications Research Institute (PIT)

DYNAMIC INITIAL SETTINGS UPDATE METHOD IN INERTIAL NAVIGATION SYSTEMS

ABSTRACT. The paper presents a method for the ***initial settings*** selection in *Inertial Navigation Systems* (**INS**), applying the **DISUPT** (***Dynamic Initial Settings UpdaTe***) technique. This technique is understood as a correction of the *initial setting*s (*initial azimuth* and *gyroscope drifts*) when the *vehicle* is in the *navigation mode* in order to achieve the best accuracy of the navigation. The first part of the work describes the *initial settings quality criteria*; including their *computation algorithms*, examples of characteristics and *sensitivity analysis*. Further two types of the **DISUPT** methods have been presented. Finally, the optimization results and conclusions concerning further research have been formulated.

INTRODUCTION

The study is a continuation of the works on the accuracy improvement methods in *inertial navigation systems* **UNZ-50** conducted by the Telecommunications Research Institute. The study develops and describes practical implementation of the **DISUPT** method. The presented method updates the ***initial settings*** such as: *initial azimuth* ***azpocz*** and *gyroscope* ***drifts*** =[*driftx*, *drifty*, *driftz*], which have an essential influence on the *navigation accuracy*. We assume that the remaining *initial errors* influence *navigation accuracy* less, at least by one order of magnitude. The **DISUPT** technique is a step forward on the formerly examined **ZUPT** procedure (***Zero-velocity UpdaTe***) [3], [4], [6], [7] where solely the ***initial azimuth*** value has been corrected. The ZUPT procedure considerable disadvantage was the need to stop the vehicle, verify *stopping criteria* and take measurements while these *criteria* are fulfilled. The presented **DISUPT** technique eliminates mentioned disadvantages, and it allows for estimating *gyroscope drifts*. The logging *raw data* from **PIT UNZ-50** system– containing **LITTON IMU LN-200** with triad of **FOG** gyros of l deg/h bias and triad of silicon accelerometers of 1 mg bias - have been used for simulation and optimization.

INITIAL SETTINGS QUALITY CRITERIA

Criteria definitions

We assume that **INS** works properly, i.e., *navigation errors* are contained within allowed limits, if:

1. the ***Z ax deviation*** from a *local vertical position* will be *minimal* when a *vehicle* is in motion and
2. the ***position error*** will be *minimal* at the *stopping* point.

The first requirement can be formulated as an **Unconstrained Minimization** (**UM**) problem:

$$\min_x \left\{ \sum_{k=0}^{N} U(x,k) = \sum_{k=0}^{N} \sqrt{\Delta\Theta^2(x,k) + \Delta\Phi^2(x,k)} \right\} \tag{1}$$

where: $\Delta\Theta\,(x,k)$ is *pitch error* at ***k***th moment with respect to *initial settings vector* ***x*** defined as:

$$\Delta\Theta(x,k) = \Theta_N(x,k) - \Theta_P(x,k) \tag{2}$$

$\Theta_N(x,k)$ is *pitch* angle at ***k***th moment, calculated on the base of *gyroscopes measurements* during *navigation*

$\Theta_P(x,k)$ is *pitch* angle at ***k***th moment calculated on the base of *accelerometers* measurements when a *vehicle* is in motion

$\Delta\Phi\,(x,k)$ is *roll error* at ***k***th moment with respect to *initial settings vector* ***x*** defined as:

$$\Delta\Phi(x,k) = \Phi_N(x,k) - \Phi_P(x,k) \tag{3}$$

$\Phi_N(x,k)$ is *roll* angle at ***k***th moment, calculated on the base of *gyroscopes measurements* during *navigation*

$\Phi_P(x,k)$ is *roll* angle at ***k***th moment calculated on the base of *accelerometers* measurements when a *vehicle* is in motion

Also, the second requirement can be presented as an **Unconstrained Minimization** (**UM**) problem:

$$\min_x \left\{ D(x,k_S) = \sqrt{(Xt - Xi)^2 + (Yt - Yi)^2} \right\} \tag{4}$$

where:

(Xt, Yt) is true position

(Xi, Yi) is position received from **INS**

ks the vehicle stoppage moment

Let us denote *objective functions* of the (**UM**) problems (1) and (4) by ***Kryt1*** and ***Kryt2***, respectively. Note that:

- for calculating ***Kryt2*** (4), *Xt* and *Yt* values can be received from **GPS** or any other optional available *source data* on the *control point*
- for the (**UM**) problems (1) and (4), the *components* x_i of *initial settings vector* ***x*** have not been defined yet. These decision variable selections, as well as their solving order, have a vital influence on the *navigation accuracy*. These issues will be discussed further in subchapter 3.1

Let us describe a *computation algorithm* of ***Kryt1*** when a *vehicle* is in motion. ***Kryt2*** calculation manner is obvious, thus no more attention will be paid to it.

Computation Algorithm

In order to compute the *function value* ***Kryt1*** at ***k***th moment, one should determine ***errors*** (2) and (3) calculations manner. Note that **UNZ** *algorithm* gives $\Theta_N(x,k)$ and $\Phi_N(x,k)$ values at ***k***th moment. Thus a question arises how to calculate, on the grounds of *accelerometers* measurements, *pitch* and *roll* angles.

Let us consider *cinematic equations* [2], as follows:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix} + \begin{bmatrix} g\sin\Theta_p \\ -g\cos\Theta_p\sin\Phi_p \\ -g\cos\Theta_p\cos\Phi_p \end{bmatrix} \tag{5}$$

where: a_x, a_y, a_z are linear acceleration components in the *vehicle related frame*

p, q, r are angular rate components in the *vehicle related frame*

u, v, w are linear velocity components along the x, y, z *vehicle axes*

$\dot{u}$, $\dot{v}$, $\dot{w}$ are acceleration components along the x, y, z *vehicle axes*

g is gravitational acceleration

We assume that:

- when a *vehicle* is in ***rectilinear motion,*** angular rates *p*, *q*, *r* can be neglected, while the *linear velocity u* and the *acceleration* $\dot{u}$ acting along the ***x*** ax are solely taken into account
- when a *vehicle* is in ***curvilinear motion***, all angular rates *p*, *q*, *r* can occur but only *r* and *u* values have a predominant significance. Thus, the *centripetal acceleration* value *ru* is essential for this motion.

Taking into consideration above *assumptions*, the equation (5) can be simplified as follows:

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \dot{u} \\ ru \end{bmatrix} + \begin{bmatrix} g\sin\Theta_p \\ -g\cos\Theta_p \sin\Phi_p \end{bmatrix} \tag{6}$$

thus:

$$\Theta_P = \operatorname{asin}\left(\frac{a_x - \dot{u}}{g}\right) \tag{7}$$

$$\Phi_P = -\operatorname{asin}\left(\frac{a_y - ru}{g\cos\Theta_P}\right) \tag{8}$$

Using an ***odometer*** to measure the *displacement of the vehicle Dr*, the ***Computation Algorithm*** of the *function value* ***Kryt1*** can be formulated as follows:

1. Calculate *linear velocity and acceleration* at ***k***th moment:

$$u(k) = \frac{Dr(k) - Dr(k-1)}{\Delta T} \tag{9}$$

$$\dot{u}(k) = \frac{u(k) - u(k-1)}{\Delta T} \tag{10}$$

where: ΔT is a *step size* equal 10 ms

2. Calculate *centripetal acceleration* at ***k***th moment:

$$ru(k) = r(k) * u(k) \tag{11}$$

3. Calculate *low-pass filtered averaged values* of: a_x, a_y, $\dot{u}$, ru
4. Calculate *pitch* and *roll* angles from (7) and (8) at ΔTg =100*ΔT
5. Calculate *pitch* and *roll* errors from (2) and (3), then compute their *estimates* using the *Kalman filter algorithm*
6. Calculate ***Kryt1*** from (1)

Remark. The selections of the *averaging period* and the applied *filter parameters* have been found as results of the simulation tests. The *data* collected during *terrain tests* of **UNZ-50** system prototype mounted on a cross - country light vehicle has been used for this purpose.

The ***Kryt1*** criterion examples with respect to ***k***, for the two values of *initial settings vector*: ***xopt*** *– optimal*, ***xpocz*** *– initial before optimization*, are illustrated in Fig.1. As we can remark in this figure, the *inclination* of ***Kryt1*** decreases for ***xopt***. Thus, it seems to be a *good criterion* for searching this value with a *minimization algorithm*.

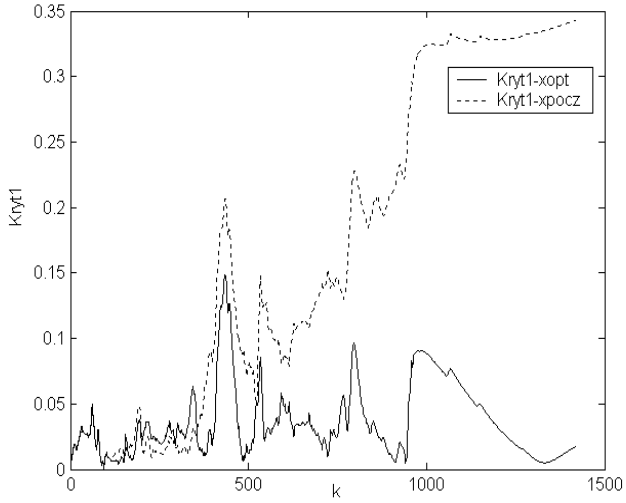

Fig.1. *Kryt1* with respect to *x* and *k*

Sensitivity Analysis

In order to examine *initial settings vector components* ***x***=[*azpocz****,*** *driftx*, *drifty*, *driftz*] influence on ***Kryt1*** and ***Kryt2*** criteria, a series of simulations in MATLAB 6.0 environment have been carried out on the *data* collected during *terrain tests* of **UNZ-50** system prototype mounted on a cross - country light vehicle. Figs.2 - 6 illustrate the examples of simulation results.

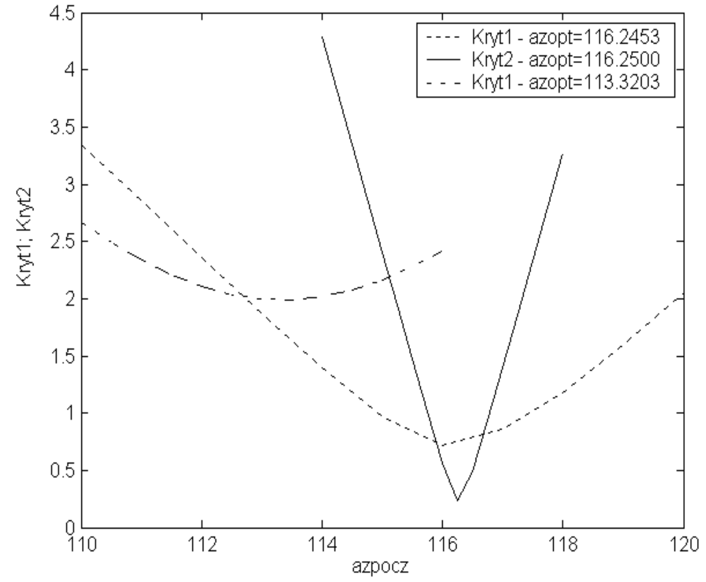

Fig.2. *Kryt1* and *Kryt2* with respect to *azpocz* and fixed values of ***drifts***
dash-dot line – driftx=0, *drifty*=0, *driftz*=0;
*solid and dotted lines – optimal **drifts*** values

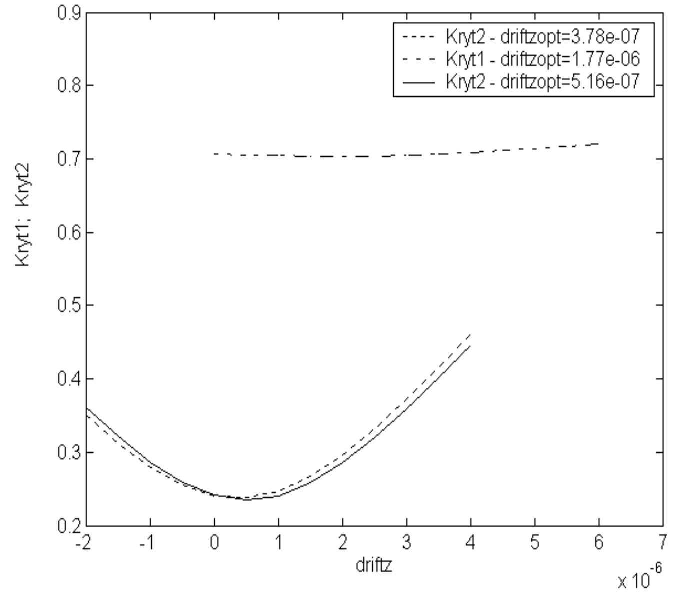

Fig.3. *Kryt1* and *Kryt2* with respect to *driftz* and fixed values of *azpocz*, *driftx*, *drifty*; *dotted lines – driftx*=0, *drifty*=0, *driftz*=0; *solid and dash-dot lines – azpocz = azpopt*, *driftx = driftxopt*, *drifty = driftyopt*;

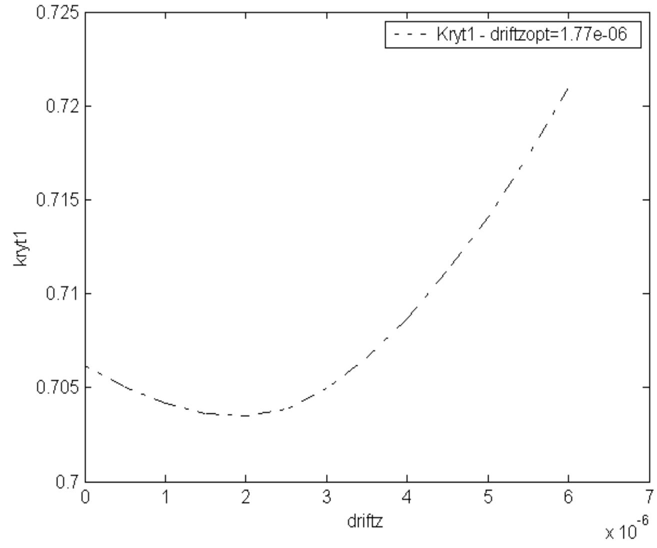

Fig. 4. *Kryt1* with respect to *driftz* and fixed values of *azpocz*, *driftx*, *drifty*; *dash-dot line – azpocz = azpopt*, *driftx = driftxopt*, *drifty = driftyopt*; This *curve* is the same as in Fig.3.

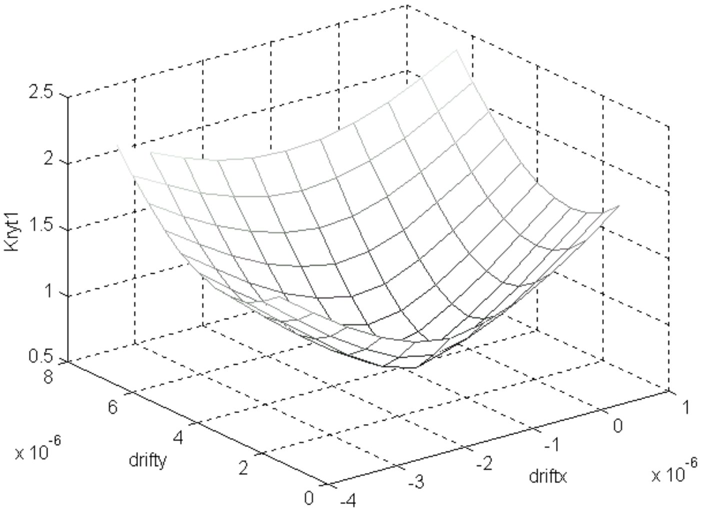

Fig. 5. *Kryt1* with respect to (*driftx*, *drifty*) and fixed values of *azpocz = azpopt*, *driftz = driftzopt*;

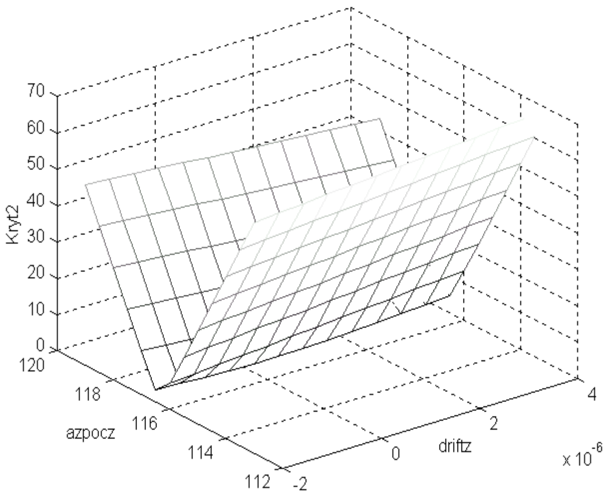

Fig.6. *Kryt2* with respect to (*driftz*, *azpocz*) and fixed values of *driftx = driftxopt*, *drifty = driftyopt*;

Analyzing these simulation results, let us note:

1. Both criteria ***Kryt1*** and ***Kryt2*** are *convex functions* with respect to *initial settings vector* ***components***. Thus, we can expect that *optimum values* of these ***components*** will be found successfully with a *minimization algorithm*.

2. The sensitivity of ***Kryt1*** to *azpocz*, *driftx*, *drifty*, *driftz* changes is considerably larger than ***Kryt2***. If the information on an *exact position* is available at the *vehicle stoppage moment*, then ***Kryt2*** should be applied for searching *optimum values* of *azpocz* and *driftz*. The *values* of *driftx* and *drifty* have less influence on this *optimum*.

DISUPT METHODS

Let us consider two types of the **DISUPT** method, namely:

1. ***Optimization method with the Exact Position Information*** (**OEPI**) available at the *vehicle stoppage moment*.

   As the simulation results show (for example in fig.3), the *fourth dimensional* **UM** problem (4) can be transformed to *fourth steps one-dimensional constrained minimization problems* solved sequentially. The diagram of this method is represented in Fig.7.

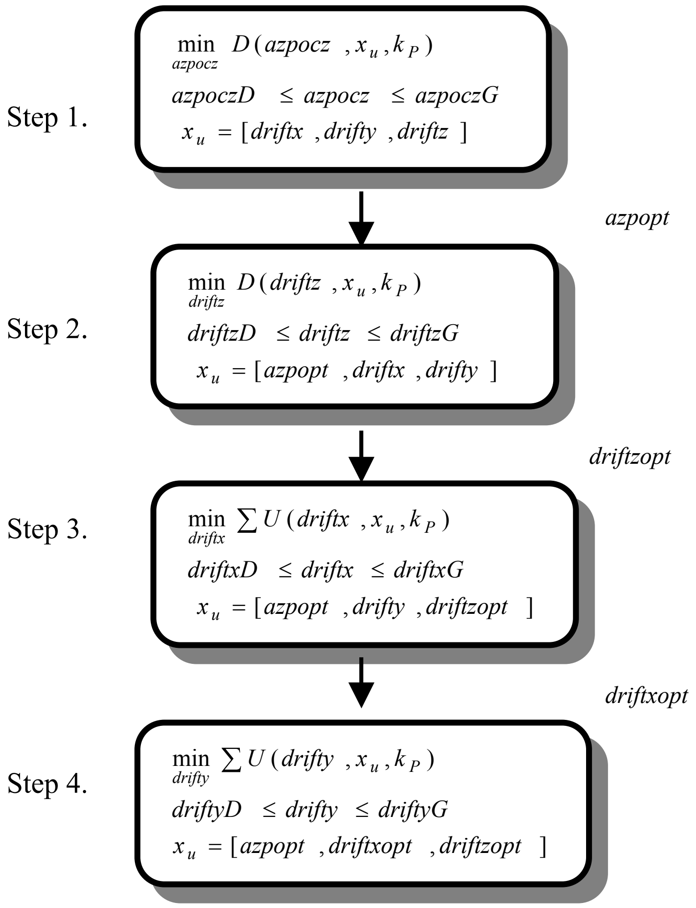

Fig. 7. The **OEPI** method diagram x_u is fixed values vector

2. ***Optimization*** method ***Without*** the ***Position Information*** (**OWPI**) available at the *vehicle stoppage moment.*

   In this case, in each step only the function values of ***Kryt1*** are available. Since ***Kryt1*** is more sensitive to changes of the *initial settings vector components*, the solving scheme presented in Fig.7 should be modified. The **OWPI** method diagram is illustrated in Fig. 8.

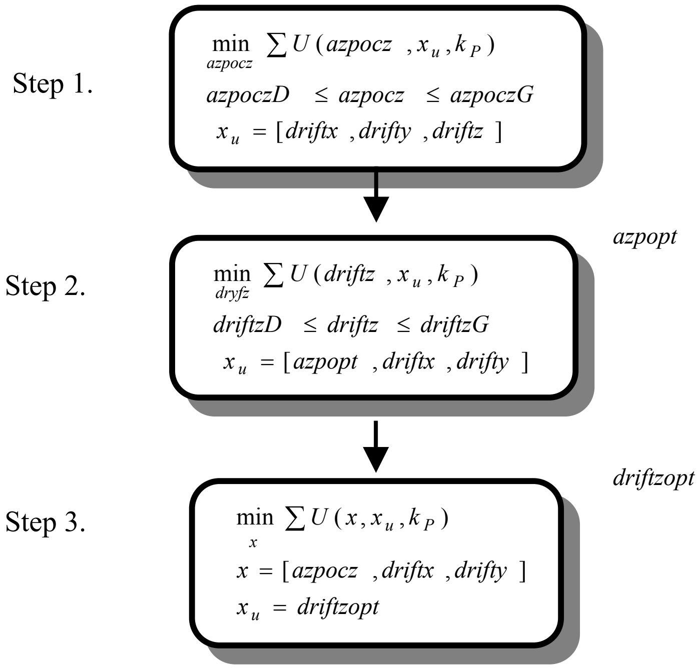

Fig. 8. The **OWPI** method diagram
x_u is fixed values vector

Comparing the both methods, we can remark that the **OEPI** method is distinctly faster than the **OWPI** method. It results from the fact that *fourth steps one-dimensional constrained minimization problems* in the **OEPI** method are solved jointly more rapidly than one *three-dimensional* **UM** problem present in the **OWPI** method (step 3).

In order to check proper functioning of two types of the **DISUP** method – **OEPI** and **OWPI**, a series of simulations in MATLAB 6.0 environment have been carried out on the *data* collected during *terrain tests* of **UNZ-50** system prototype mounted on a cross - country light vehicle. The ***simulation*** and ***optimization results*** for the selected data sets are presented in **Table 1** and **Table 2**.

Table 1. OEPI Method

Method step	*Input data*		*Optimization results*		Position errors [m]	
					ΔX	ΔY
1	***azpocz***	109.503	***azpopt***	1.165084227e+02	1.3	25.8
	driftx	0	***driftxopt***			
	drifty	0	***driftyopt***			
	driftz	0	***driftzopt***			
2	***azpocz***	1.165084227e+02	***azpopt***	1.165084227e+02	2.8	25.3
	driftx	0	***driftxopt***			
	drifty	0	***driftyopt***			
	driftz	0	***driftzopt***	3.788255567e-07		
3	***azpocz***	1.165084227e+02	***azpopt***	1.165084227e+02	2.3	25.4
	driftx	0	***driftxopt***	-2.477250320e-06		
	drifty	0	***driftyopt***			
	driftz	3.788255567e-07	***driftzopt***	3.788255567e-07		
4	***azpocz***	1.165084227e+02	***azpopt***	1.165084227e+02	2.6	25.1
	driftx	-2.477250320e-06	***driftxopt***	-2.477250320e-06		
	drifty	0	***driftyopt***	3.940879436e-06		
	driftz	3.788255567e-07	***driftzopt***	3.788255567e-07		

Table 2. OWPI Method

Method step	*Input data*		*Optimization results*		Position errors [m]	
					ΔX	ΔY
1	***azpocz***	109.503	***azpopt***	1.133203997e+02	596.8	25.7
	driftx	0	***driftxopt***			
	drifty	0	***driftyopt***			
	driftz	0	***driftzopt***			
2	***azpocz***	1.133203997e+02	***azpopt***	1.133203997e+02	557.6	23.3
	driftx	0	***driftxopt***			
	drifty	0	***driftyopt***			
	driftz	0	***driftzopt***	-3.611973832e-06		
3	***azpocz***	1.133203997e+02	***azpopt***	1.162453601e+02	7.4	24.6
	driftx	0	***driftxopt***	-2.523063367e-06		
	drifty	0	***driftyopt***	3.807824797e-06		
	driftz	3.611973832e-06	***driftzopt***	-3.611973832e-06		

Remark. Before the **DISUPT** methods application, the *position errors* were as follows:
$\Delta X = 1315.4$ [m], $\Delta Y = 43.5$ [m] on the *trajectory length of the vehicle* =12080 [m].

CONCLUSIONS

The study contains an analysis of the *dynamic initial settings update* method by the **DISUP** technique. The assumed settings include: the *initial azimuth* value and the *three gyroscopes drift* values. As compared to the earlier considered **ZUPT** [3], [4], [7] technique, considerable navigation improvement has been achieved. The presented results are promising enough, so as it is worth considering to include the **DISUP** method in the **PIT-UNZ** system.

BIBLIOGRAPHY

1. MATLAB, *Optimization Toolbox*, User's Guide, Math Works, 2000.
2. Popowski S., Halama K., *UNZ Systems Accuracy Improvement Concept for the Moving Vehicles*, PIT files, 20071/2002.
3. Popowski S., Halama K., *ZUPT Technique Application Concept to Improve UNZ Systems*, PIT files 19845/2002.
4. Szymanowski J., Grzelak J., Popowski S., *Optimization of Alignment Process in Inertial Navigation System*, NATO Sensors&Electronics Technology Panel (SET) Symposium, Istanbul, Turkey, 2002.
5. Szymanowski J., Grzelak J., *Analysis and Optimization of the Initial Orientation Algorithms in the Inertial Navigation System*, PIT Studies, 128, 2001.
6. Szymanowski J., Grzelak J., Popowski S., *ZUPT Optimization in MATLAB 6.0. Environment*, PIT files, 19970/2002.
7. Szymanowski J., Grzelak J., Popowski S., *Static Initial Azimuth Update Method in Land Navigation Systems*, Annual of Navigation, N°6, 2003.

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