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THEORETICAL FOUNDATION FOR THE INTERACTIVE NAVIGATIONAL STRUCTURE, TAKING INTO CONSIDERATION THE DETERMINISTIC MODEL OF SURVEY ERRORS

ABSTRACT

The paper describes and solves the problem of optimization, conforming to the Interactive Navigational Structure, represented by the functional model and the model of deterministic errors. The objective functions of the optimization problem have been formulated basing on the M-estimation theory, extended with adjustment points (free M-estimation). In the estimation process there have been determined not only estimators of the Interactive Navigational Structure points' coordinates (proper fix's coordinates, new adjustment points coordinates etc.) but also estimators of deterministic survey errors models' parameters.

INTRODUCTION

The work [Czaplewski, 2004] presents the Interactive Navigational Structure (IANS). The Structure is composed of a set of the adjustment points Z (visual signs, radio-navigational stations, reference stations of DGPS system etc.) and the points W, which are obtained in course of determination. In the cited work the author has distinguished in the set W the following subsets: P – craft's proper fixes and R – points determined basing on the P set and complementing the set of adjustment points Z (when the settled adjustment criteria are fulfilled).

A peculiar feature of the Interactive Navigational Structure (apart from existence of R set) is the following representation:

$$\mathsf{F}:\quad [T_{\mathsf{W}}(\mathsf{W}), T_{\mathsf{Z}}(\mathsf{Z})] \rightarrow T_{\mathsf{O}}(\mathsf{O}) \tag{1}$$

where: O is the set of observations; T is the function of attenuation.

In the work [Czaplewski 2004], as well as in the work [Czaplewski, Wiśniewski 2006] it has been assumed that:

1. IANS is the object of d-degrees of freedom (the feature is a consequence of taking into consideration the adjustment points set Z in representation (1).
2. Observations which constitute set O are random error biased of probability distributions with gross errors.
3. Coordinates of points which constitute Z are pseudo-observations of the settled covariance matrix and probability distributions with the admixture (the admixture represents the coming off adjustment points).

In this work we have assumed moreover that surveys' results (set O elements) are biased with deterministic errors. Such errors, what had been indicated in the work [Wiśniewski, 2002], are functions of certain independent variables, describing environment of survey (i.e. atmospheric pressure, humidity, temperature, survey instruments condition etc.).

THEORETICAL ASSUMPTIONS

Deterministic Errors' Model

In the conceptions with IANS application presented till nowadays there has been assumed that the survey results $x_i,\ i=1,\dots,n\quad (\{x_i\}\equiv \mathsf{O})$ constitute the random vector $\mathbf{x}\in R^{n,1}$ of the expected value $E(\mathbf{x})=\overline{\mathbf{x}}$ and covariance matrix $\mathbf{C}_\mathbf{x}=E\{(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^T\}=E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\}$, where $\overline{\mathbf{x}}$ – a vector of true measured values, $\boldsymbol{\varepsilon}=\mathbf{x}-\overline{\mathbf{x}}$ – the survey random errors' vector (in general presenting probabilities distributions with the admixture). According to the suggested in the work development of IANS theory there has been assumed that not only:

$$\mathbf{x}=\overline{\mathbf{x}}+\boldsymbol{\varepsilon} \tag{2}$$

but also

$$\mathbf{x}=\overline{\mathbf{x}}+\mathbf{s}+\boldsymbol{\varepsilon} \tag{3}$$

where: **s** is the deterministic errors' vector, of general model as follows:

$$\mathbf{s}=\mathbf{G}(y_1,y_2,\dots,y_t)=\mathbf{G}(\mathbf{Y}),\quad \mathbf{Y}\in R^{t,1} \tag{4}$$

By $y_i, i = 1,...,t$ there have been determined the mentioned before independent variables, treated at this point as parameters of deterministic errors' model. We have assumed that the variables are known at a certain level y_i^o (in the cited work [Wiśniewski, 2002] there had been assumed that y_i are variables which explain wherefrom $\mathbf{s} = \mathbf{Yc}$ is there, where $\mathbf{Y} \in R^{n,t}$ is a matrix of the known variables y_i realizations, whereas $\mathbf{c}$ – the vector of unknown parameters; on contrary, the assumed in this work model $\mathbf{s} = \mathbf{G}(\mathbf{Y})$ is of more general character). Thus, if the vector $\mathbf{Y}^o = \left[y_1^o,...,y_t^o\right]^T$ is known and if $\mathbf{y} = \mathbf{y}^o + \mathbf{d}_y$, then with sufficient approximation one may write (after Taylor series expansion) as follows:

$$\mathbf{s} = \mathbf{G}\left(\mathbf{Y}^o\right) + \left[\partial_{\mathbf{Y}}\mathbf{G}(\mathbf{Y})\right]_{\mathbf{Y}=\mathbf{Y^o}} \mathbf{d}_{\mathbf{Y}} = \mathbf{G}\left(\mathbf{Y}^o\right) + \mathbf{Bd}_{\mathbf{Y}} \tag{5}$$

where: $B = \left[\partial_{\mathbf{Y}}\mathbf{G}(\mathbf{Y})\right]_{\mathbf{Y}=\mathbf{Y^o}} \in R^{n,t}$.

Functional IANS Model

In case we assume (the same as in the works [Czaplewski, 2004; Czaplewski, Wiśniewski, 2006]), that sets W, Z are replaced by the vector of coordinates of the points which form the mentioned sets, it means:

$$\left.\begin{aligned} \mathsf{W} &\rightarrow \mathbf{X}_{\mathsf{W}} \in R^{r_W,1} \\ \mathsf{Z} &\rightarrow \mathbf{X}_{\mathsf{Z}} \in R^{r_{\mathsf{Z}},1} \end{aligned}\right\} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{X}_{\mathsf{W}} \\ \mathbf{X}_{\mathsf{Z}} \end{bmatrix} = \mathbf{X} \in R^{r,1}$$

$\left(r = r_{\mathsf{W}} + r_{\mathsf{Z}}\right)$, then object's representation (1) can be replaced by the following formal representation:

$$\mathbf{F}: \mathbf{X} \rightarrow \overline{\mathbf{x}} \tag{6}$$

Taking into consideration the model (3) we achieve:

$$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{s} + \boldsymbol{\varepsilon} \Leftrightarrow \mathbf{x} = \mathbf{F}(\mathbf{X}) + \mathbf{s} + \boldsymbol{\varepsilon} \tag{7}$$

Let the vector of proper fixes' coordinates be known at a level $\mathbf{X}_{\mathsf{W}}^0$. Then $\hat{\mathbf{X}}_{\mathsf{W}} = \mathbf{X}_{\mathsf{W}}^0 + \mathbf{d}_{\mathbf{X}_{\mathsf{W}}}$. As IANS is treated as a free object, there for in a result of optimization of a position, the vector of increments $\mathbf{d}_{\mathbf{X}_Z}$ should also be added to the adjustment points' coordinates; in other words $\hat{\mathbf{X}}_Z = \mathbf{X}_Z^0 + \mathbf{d}_{\mathbf{X}_Z}$. Thus

$$\mathbf{X} = \begin{bmatrix} \hat{\mathbf{X}}_W \\ \hat{\mathbf{X}}_Z \end{bmatrix} = \begin{bmatrix} \mathbf{X}_W^0 \\ \mathbf{X}_Z^0 \end{bmatrix} + \begin{bmatrix} \mathbf{d}_{\mathbf{X}_W} \\ \mathbf{d}_{\mathbf{X}_Z} \end{bmatrix} = \mathbf{X}^0 + \mathbf{d}_{\mathbf{X}} \tag{8}$$

where: $\mathbf{X} = \left[\hat{\mathbf{X}}_W^T, \hat{\mathbf{X}}_Z^T\right]^T$, $\mathbf{X}^0 = \left[\left(\mathbf{X}_W^0\right)^T, \left(\mathbf{X}_Z^0\right)^T\right]^T$, $\mathbf{d}_{\mathbf{X}} = \left[\mathbf{d}_{\mathbf{X}_W}^T, \mathbf{d}_{\mathbf{X}_Z}^T\right]^T$.

By expansion of the function $\mathbf{F}(\mathbf{X}) = \mathbf{F}\left(\hat{\mathbf{X}}_W, \hat{\mathbf{X}}_Z\right)$ into Taylor series limited to the first terms, we obtain as follows:

$$\begin{aligned} \mathbf{F}(\mathbf{X}) &= \mathbf{F}\left(\mathbf{X}^0\right) + \left[\partial_{\mathbf{X}_W} \mathbf{F}(\mathbf{X})\right]_{\mathbf{X}_W = \mathbf{X}_W^0} \mathbf{d}_{\mathbf{X}_W} + \left[\partial_{\hat{\mathbf{X}}_Z} \mathbf{F}(\mathbf{X})\right]_{\hat{\mathbf{X}}_Z = \mathbf{X}_Z^0} \mathbf{d}_{\mathbf{X}_Z} = \\ &= \mathbf{F}\left(\mathbf{X}^0\right) + \mathbf{A}_W \mathbf{d}_{\mathbf{X}_W} + \mathbf{A}_Z \mathbf{d}_{\mathbf{X}_Z} = \\ &= \mathbf{F}\left(\mathbf{X}^0\right) + \mathbf{A}\mathbf{d}_{\mathbf{X}} \end{aligned} \tag{9}$$

where: $\mathbf{A}_W = \left[\partial_{\mathbf{X}_W} \mathbf{F}(\mathbf{X})\right]_{\mathbf{X}_W = \mathbf{X}_W^0} \in R^{n,r_W}$, $\mathbf{A}_Z = \left[\partial_{\hat{\mathbf{X}}_Z} \mathbf{F}(\mathbf{X})\right]_{\hat{\mathbf{X}}_Z = \mathbf{X}_Z^0} \in R^{n,r_Z}$,
$\mathbf{A} = \left[\mathbf{A}_W, \mathbf{A}_Z\right] \in R^{n,r}$.

Having the above expansions carried out, also taking into account that $\mathbf{s} = \mathbf{G}\left(\mathbf{Y}^o\right) + \mathbf{B}\mathbf{d}_{\mathbf{Y}}$, the functional IANS model can be presented in the following formula:

$$\begin{aligned} &\mathbf{x} = \mathbf{F}(\mathbf{X}) + \mathbf{s} + \boldsymbol{\varepsilon} \quad \Leftrightarrow \\ &\mathbf{x} = \mathbf{A}_W \mathbf{d}_{\mathbf{X}_W} + \mathbf{A}_Z \mathbf{d}_{\mathbf{X}_Z} + \mathbf{B}\mathbf{d}_{\mathbf{Y}} + \mathbf{F}\left(\mathbf{X}_W^0, \mathbf{X}_Z^0\right) + \mathbf{G}\left(\mathbf{Y}^0\right) + \boldsymbol{\varepsilon} = \\ &= \mathbf{A}\mathbf{d}_{\mathbf{X}} + \mathbf{B}\mathbf{d}_{\mathbf{Y}} + \mathbf{F}\left(\mathbf{X}_W^0, \mathbf{X}_Z^0\right) + \mathbf{G}\left(\mathbf{Y}^0\right) + \boldsymbol{\varepsilon} \end{aligned} \tag{10}$$

Moreover, let $\mathbf{L} = \mathbf{F}\left(\mathbf{X}_W^0, \mathbf{X}_Z^0\right) + \mathbf{G}\left(\mathbf{Y}^0\right) - \mathbf{x}$ be the free terms vector, whereas $\mathbf{v} = -\boldsymbol{\varepsilon}$ the vector of corrections. Then

$$\mathbf{x} = \mathbf{F}(\mathbf{X}) + \mathbf{s} + \boldsymbol{\varepsilon} \quad \Leftrightarrow \quad \mathbf{v} = \mathbf{A}\mathbf{d}_{\mathbf{X}} + \mathbf{B}\mathbf{d}_{\mathbf{Y}} + \mathbf{L} \tag{11}$$

or

$$\mathbf{v} = \widetilde{\mathbf{A}}\widetilde{\mathbf{d}}_{\mathbf{X}} + \mathbf{L} \tag{12}$$

where: $\widetilde{\mathbf{A}} = [\mathbf{A}, \mathbf{B}]$, $\widetilde{\mathbf{d}}_{\mathbf{X}} = \left[\mathbf{d}_{\mathbf{X}}^T, \mathbf{d}_{\mathbf{Y}}^T\right]^T$.

The analysed Interactive Navigation Structure is, what has already been pointed out before, a free object with defect d (defect is identified with a number of object's freedom degrees). For this reason the matrix **A** and thereby the matrix $\widetilde{\mathbf{A}}$ are not matrices with full column rank. The row of matrices is as follows:

$$rank(\mathbf{A}) = rank(\widetilde{\mathbf{A}}) = r - d = u$$

(if $d > 0$ then $u < r$). According to the free adjustment theory (e.g. [Wiśniewski 2005]) we may put the matrix **A** and vector $\mathbf{d_X}$ in the following block forms:

$$\mathbf{A} = \left[\mathbf{A}_1 \in R^{n,u}, \mathbf{A}_2^{n,d}\right], \quad \mathbf{d_X} = \left[\mathbf{d}_{\mathbf{X}_1}^T \in R^{1,u}, \ \mathbf{d}_{\mathbf{X}_2}^T \in R^{1,d}\right]^T \quad (rank(\mathbf{A}_1) = u)$$

The functional IANS model, accommodated to free adjustment (with regard to deterministic errors) may therefore be presented by the following formula:

$$\mathbf{v} = \mathbf{A}\mathbf{d_X} + \mathbf{B}\mathbf{d_Y} + \mathbf{L} \quad \Leftrightarrow \quad \mathbf{v} = \mathbf{A}_1\mathbf{d}_{\mathbf{X}_1} + \mathbf{A}_2\mathbf{d}_{\mathbf{X}_2} + \mathbf{B}\mathbf{d_Y} + \mathbf{L} \tag{13}$$

or

$$\mathbf{v} = \mathbf{A}_1\mathbf{d}_{\mathbf{X}_1} + \mathbf{B}\mathbf{d_Y} + \mathbf{A}_2\mathbf{d}_{\mathbf{X}_2} + \mathbf{L} = \mathbf{D}\mathbf{d} + \mathbf{A}_2\mathbf{d}_{\mathbf{X}_2} + \mathbf{L} \tag{14}$$

where: $\mathbf{D} = [\mathbf{A}_1, \ \mathbf{B}] \in R^{n,u+t}$, $\mathbf{d} = \left[\mathbf{d}_{\mathbf{X}_1}^T, \mathbf{d}_{\mathbf{Y}}^T\right]^T \in R^{1,u+t}$ whereat $rank(\mathbf{D}) = u + t$ (matrix with full column rank).

OPTIMIZATION PROBLEM AND SOLUTION

Optimization problem (adjustment problem) is formulated basing on the following optimization criteria:

$\min\left\{\Phi(\widetilde{\mathbf{d}}_\mathbf{X}) = \mathbf{v}^T\widehat{\mathbf{P}}\mathbf{v}\right\} = \hat{\mathbf{v}}^T\widehat{\mathbf{P}}\hat{\mathbf{v}}$ – method of least squares in M-estimation;

$\min\left\{\Phi_\mathbf{X}(\widetilde{\mathbf{d}}_\mathbf{X}) = \widetilde{\mathbf{d}}_\mathbf{X}^T\widehat{\mathbf{P}}_\mathbf{X}\widetilde{\mathbf{d}}_\mathbf{X}\right\} = \hat{\widetilde{\mathbf{d}}}_\mathbf{X}^T\widehat{\mathbf{P}}_\mathbf{X}\hat{\widetilde{\mathbf{d}}}_\mathbf{X}$ – free adjustment.

Matrices $\widehat{\mathbf{P}}$ and $\widehat{\mathbf{P}}_\mathbf{X}$ are equivalent matrices of weights, respectively of the observations' vector **x** and coordinates' vector **X**. These matrices, determined applying the diagonal matrices of attenuation $\mathbf{T}(\mathbf{v})$ and $\mathbf{T}(\widetilde{\mathbf{d}}_\mathbf{X})$, i.e. the matrices

$$\widehat{\mathbf{P}} = \mathbf{T}(\mathbf{v})\mathbf{P}, \qquad \widehat{\mathbf{P}}_\mathbf{X} = \mathbf{T}_\mathbf{X}(\widetilde{\mathbf{d}}_\mathbf{X})\mathbf{P}_\mathbf{X}$$

turn out from the assumption about existence of the coming off values ($\mathbf{P}$ and $\mathbf{P}_\mathbf{X}$ are original, diagonal matrices of weights, resulting from the assumed accuracies and accuracy of points W and Z coordinates) within the both – the set of observations and the set of coordinates. Taking into consideration the accepted optimization criteria and functional IANS model having a form (14) one may suggest the following problem of optimization:

$$\left.\begin{array}{l} \mathbf{v} = \mathbf{A}\mathbf{d}_\mathbf{X} + \mathbf{B}\mathbf{d}_\mathbf{Y} + \mathbf{L} = \widetilde{\mathbf{A}}\widetilde{\mathbf{d}}_\mathbf{X} + \mathbf{L} = \mathbf{D}\mathbf{d} + \mathbf{A}_2\mathbf{d}_{\mathbf{X}_2} + \mathbf{L} \\ \min \Phi\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) = \hat{\mathbf{v}}^T \widehat{\mathbf{P}} \hat{\mathbf{v}} \\ \min \Phi_\mathbf{X}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) = \hat{\widetilde{\mathbf{d}}}_\mathbf{X}^T \widehat{\mathbf{P}}_\mathbf{X} \hat{\widetilde{\mathbf{d}}}_\mathbf{X} \end{array}\right\} \tag{15}$$

The result of seeking $\min \Phi\left(\widetilde{\mathbf{d}}_\mathbf{X}\right)$ is a system of normal equations

$$\widetilde{\mathbf{A}}^T \widehat{\mathbf{P}} \widetilde{\mathbf{A}} \widetilde{\mathbf{d}}_\mathbf{X} + \widetilde{\mathbf{A}}^T \widehat{\mathbf{P}} \mathbf{L} = \mathbf{0} \quad \Leftrightarrow \quad \widetilde{\mathbf{D}}^T \mathbf{P} \widetilde{\mathbf{D}} \widetilde{\mathbf{d}} + \widetilde{\mathbf{D}}^T \mathbf{P} \mathbf{L} = \mathbf{0}$$

where $\widetilde{\mathbf{D}} = [\mathbf{D},\ \mathbf{A}_2]$, $\mathbf{d} = \left[\mathbf{d}^T,\ \mathbf{d}_{\mathbf{X}_2}^T\right]^T$. The system $\widetilde{\mathbf{D}}^T \mathbf{P} \widetilde{\mathbf{D}} \widetilde{\mathbf{d}} + \widetilde{\mathbf{D}}^T \mathbf{P} \mathbf{L} = \mathbf{0}$ may also be presented in the following form:

$$\left.\begin{array}{l} \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{D}\mathbf{d} + \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{A}_2 \mathbf{d}_{\mathbf{X}_2} + \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{L} = \mathbf{0} \\ \mathbf{A}_2^T \widehat{\mathbf{P}} \mathbf{D}\mathbf{d} + \mathbf{A}_2^T \widehat{\mathbf{P}} \mathbf{A}_2 \mathbf{d}_{\mathbf{X}_2} + \mathbf{A}_2^T \widehat{\mathbf{P}} \mathbf{L} = \mathbf{0} \end{array}\right\} \tag{16}$$

Taking into account that $rank(\mathbf{D}) = rank\left(\mathbf{D}^T \widehat{\mathbf{P}} \mathbf{D}\right) = u + t$ and $\mathbf{d} \in R^{u+1,1}$, so seeking of $\min \Phi_\mathbf{X}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right)$ may be participated not only by the first equation of the system (16) (see also [Wiśniewski, 2005]). Thus a form of the additional optimization problem is as follows (for $\mathbf{\Omega} = [\mathbf{D}^T \widehat{\mathbf{P}} \mathbf{D},\ \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{A}_2]$):

$$\left.\begin{array}{l} \mathbf{\Omega} \widetilde{\mathbf{d}} + \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{L} = 0 \\ \min \Phi_\mathbf{X}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) \Leftrightarrow \min \Phi_{\mathbf{X}\kappa}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) \end{array}\right\} \tag{17}$$

where

$$\Phi_{\mathbf{X}\kappa}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) = \Phi_\mathbf{X}\left(\widetilde{\mathbf{d}}_\mathbf{X}\right) - 2\boldsymbol{\kappa}^T \left(\mathbf{\Omega}\mathbf{d} + \mathbf{D}^T \widehat{\mathbf{P}} \mathbf{L}\right) \tag{18}$$

is Lagrange's function with the vector of multipliers (correlates) $\boldsymbol{\kappa} \in R^{u+t}$. The problem's (17) solution is the vector

$$\hat{\hat{\mathbf{d}}} = \widehat{\mathbf{P}}_{\mathbf{X}}^{-1}\boldsymbol{\Omega}^T\hat{\boldsymbol{\kappa}} = -\widehat{\mathbf{P}}_{\mathbf{X}}^{-1}\boldsymbol{\Omega}^T\left(\boldsymbol{\Omega}\widehat{\mathbf{P}}_{\mathbf{X}}^{-1}\boldsymbol{\Omega}^T\right)^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{L} \tag{19}$$

whereat

$$\hat{\boldsymbol{\kappa}} = -\left(\boldsymbol{\Omega}\widehat{\mathbf{P}}_{\mathbf{X}}^{-1}\boldsymbol{\Omega}^T\right)^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{L} \tag{20}$$

The following structure of the equivalent weights' matrix $\widehat{\mathbf{P}}_{\mathbf{X}}$ (with the assumption of mutual independence of respective coordinates) conforms to the assumed structure of the functional optimization problem's model go (15).

$$\widehat{\mathbf{P}}_{\mathbf{X}} = Diag\left(\widehat{\mathbf{P}}_{\mathbf{d}}, \widehat{\mathbf{P}}_{\mathbf{X}_2}\right) = Diag\left(\widehat{\mathbf{P}}_{\mathbf{X}_1}, \widehat{\mathbf{P}}_{\mathbf{Y}}, \widehat{\mathbf{P}}_{\mathbf{X}_2}\right)$$

Then

$$\boldsymbol{\Omega}\widehat{\mathbf{P}}_{\mathbf{X}}^{-1}\boldsymbol{\Omega}^T = \mathbf{D}^T\widehat{\mathbf{P}}\mathbf{D}^T\widehat{\mathbf{P}}_{\mathbf{d}}^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{D} + \mathbf{D}^T\widehat{\mathbf{P}}\mathbf{A}_2\widehat{\mathbf{P}}_{\mathbf{X}_2}^{-1}\mathbf{A}_2^T\widehat{\mathbf{P}}\mathbf{D} = \boldsymbol{\Theta}$$

and next

$$\hat{\hat{\mathbf{d}}} = \begin{bmatrix} \hat{\mathbf{d}} \\ \ldots \\ \hat{\mathbf{d}}_{\mathbf{X}_2} \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{P}}_{\mathbf{d}}^{-1} & \\ & \widehat{\mathbf{P}}_{\mathbf{X}_2}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{D}^T\widehat{\mathbf{P}}\mathbf{D} \\ \mathbf{A}_2^T\widehat{\mathbf{P}}\mathbf{D} \end{bmatrix} \boldsymbol{\Theta}^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{L}$$

wherefrom

$$\hat{\mathbf{d}} = \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_1} \\ \hat{\mathbf{d}}_{\mathbf{Y}} \end{bmatrix} = \widehat{\mathbf{P}}_{\mathbf{d}}^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{D}\boldsymbol{\Theta}^{-1}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{L}$$

$$\hat{\mathbf{d}}_{\mathbf{X}_2} = \widehat{\mathbf{P}}_{\mathbf{X}_2}^{-1}\mathbf{A}_2^T\widehat{\mathbf{P}}\mathbf{D}\mathbf{D}^T\widehat{\mathbf{P}}\mathbf{L}$$

SUMMARY

The presented solution has not only been of theoretical significance but it may also be applied in designing and operating the Interactive Navigational Structures. The implemented model of deterministic errors is of general character. However in practice there may take place special cases thereof, corresponding with a specific navigational situation and accommodated to a level of reconnaissance of the survey environment properties. In the simplest case the deterministic errors can be reduced to constant nevertheless unknown systematic errors. Then $\mathbf{s} = \mathbf{B}\mathbf{y}$, where $\mathbf{B}$ is a matrix, assigning systematic errors to respective observations. Such a case in estimation, carried out with the least squares method, had been analyzed in the work [Wiśniewski, 1983].

The theory presented in this work may still be subject to further development, primarily as regards M-estimation. First of all it refers to the way of determining weights' matrixes of deterministic survey errors and the way of attenuation thereof (selecting elements of adequate attenuation matrix). Another important problem is algorithmization of the suggested solutions especially for extensive navigational structures (IANS of developed geometrical structure).

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