

ADAM WEINTRIT, PIOTR KOPACZ  
Gdynia Maritime University

## **ON COMPUTATIONAL ALGORITHMS IMPLEMENTED IN MARINE NAVIGATIONAL SOFTWARE USED IN MARINE NAVIGATION ELECTRONIC DEVICES AND SYSTEMS**

### **ABSTRACT**

In the paper the authors attempt to present the computational problem related to the navigational algorithm (meridian arc formula) implemented in the software applied in marine navigation electronic devices and systems, such as GNSS (GPS, GLONASS, Galileo), AIS, ECDIS/ECS, and other marine GIS.

From the early days of the development of the basic navigational software built into satellite navigational receivers, it has been noted that for the sake of simplicity and a number of other reasons, this navigational software is often based on the simple methods of limited accuracy. It is surprising that even nowadays the use of navigational software is still used in a loose manner, sometimes ignoring basic computational principles and adopting oversimplified assumptions and errors such as the wrong combination of spherical and ellipsoidal calculations in different steps of the solution of a particular sailing problem. The lack of official standardization on both the 'accuracy required' and the equivalent 'methods employed', in conjunction to the 'black box solutions' provided by GNSS navigational receivers and navigational systems (ECDIS and ECS) suggest the necessity of a thorough examination of the issue of sailing calculations for navigational systems and GNSS receivers.

### **Keywords:**

ECDIS, great circle, rhumb-line, sailing calculations.

### **INTRODUCTION**

These matters became especially important with the appearance of the satellite system TRANSIT and were discussed for example in: Holmstrom J. S. 'A new approach to the theory of geodesics on an Ellipsoid' (Navigation, 1976, Vol. 23, No. 3), Williams R. and Phytian J. E. 'Navigating along Geodesic Paths on the surface of

a spheroid' (Navigation, 1985, Vol. 42, No. 2) as well as in Polish literature (Śledzinski J., Felski A., Osada K. and other authors). The discrepancies between the results on the spherical and the ellipsoidal model of the Earth are in the order of 0.27% according to Tobler [12], and in the order of 0.5% according to Earle [7]. In reality these discrepancies can exceed 13 nautical miles (about 24 km) for a number of common navigational routes. An example of such a discrepancy is shown through the calculation of the shortest navigational distance from a departure location in the west coast of USA such as the entrance of San Francisco bay (37°45.047'N, 122°42.023'W) to a destination point in Japan such as the approaches to Yokohama harbour (34°26.178'N, 139°51.139'E). This calculation on the spherical earth model using spherical trigonometry and the classical assumption that 1 minute of a great circle arc is equal to the international nautical mile (1852 metres) yields a distance of 4489.9 nautical miles. The calculation of this distance on the WGS-84 ellipsoid, using very accurate methods for the calculation of long geodesics, as the method of Vincenty [15], yields 4502.9 nautical miles. For this example the difference in calculated distances on the spherical model from those on the ellipsoid is 13 nautical miles (~24 km).

Despite these discrepancies the use of the spherical model in traditional navigation for most practical purposes is considered satisfactory. Nevertheless for the case of sailing computations in GIS navigational systems such as ECDIS and other ECS systems the computations must be conducted on the ellipsoid in order to eliminate these errors but without seeking the submeter accuracies pursued in other geodetic applications. According to [9] seeking extremely high accuracy for marine navigation purposes does not offer any real benefit and requires more computing power and processing time. For these reasons and before proceeding with the adoption of any geodetic computational method on the ellipsoid for sailing calculations it is required to adopt realistic accuracy standards in order not only to eliminate the significant errors of the spherical model but also to avoid the exaggerated and unrealistic requirements of submeter accuracy [9].

### **ACCURACY REQUIREMENTS FOR SAILING CALCULATIONS IN GIS**

The IMO performance standards for ECDIS [16] do not provide specific accuracy standards for sailing calculations, except for the following general requirements:

*It should be possible to carry out route planning and route monitoring in a simple and reliable way.*

*The accuracy of all calculations performed by ECDIS should be independent of the characteristics of the output device and should be consistent with the SENC accuracy.*

Setting accuracy requirements in relation to SENC, depends directly on the category of the Electronic Navigational Charts (ENCs) installed in the SENC. This is a reasonable requirement for calculations relating to real time positions that affect the safety of navigation when using ECDIS. This safety is assured through the installation of the proper ENCs in the SENC. Nevertheless these standards, when applied to set the accuracy of sailing calculations for route planning may result in vague, ambiguous and sometimes unreasonable standards due to their direct dependency on the installed ENCs. This deficiency is illustrated in the attempt to apply this general ECDIS accuracy requirement for consistency with SENC accuracy in sailing calculations. Taking into consideration that the SENC contains ENCs of various categories, the average compilation scale of each category and considering SENC accuracy equivalent to 0.5 mm at the compilation scale of the contained ENCs, we obtain accuracy requirements ranging from 5 metres to more than 1250 metres (even to 5.000 metres for 'category 1' ENCs compiled from 1/10.000.000 paper charts).

Table 1. Accuracy requirements for sailing calculations [9]

Calculated Distance	Maximum Acceptable Error
up to 250 nautical miles	0,1 nautical miles
between 250 and 500 nautical miles	0,2 nautical miles
between 500 and 2000 nautical miles	0,3 nautical miles
greater than 2000 nautical miles	0,5 nautical miles

For the above mentioned reasons the study for the development of more realistic formulas for the computation of the length of the arc of the meridian has been based on the requirements of table 1 rather than on the IMO general ECDIS accuracy requirements.

## THE LENGTH OF THE MERIDIAN ARC IN SAILING CALCULATIONS

The calculation of the length of the arc of the meridian is a basic prerequisite for many accurate sailing calculation methods on the ellipsoid concerning both Rhumb-Line Sailing (RLS) and shortest sailings on the ellipsoid such as Great Elliptic Sailing (GES). A lot of specific papers present in detail the advantages and benefits of these methods [2, 6, 9].

It is noted though that in certain sailing calculation methods it is not necessary to calculate the length of the meridian arc. Typical examples of these methods concern:

- RLS calculations by the employment of the general formulas of the Mercator projection [11] and isometric latitude [5];
- calculation of shortest sailings paths on the ellipsoid by a geodetic inverse method such as the Andoyer-Lambert method proposed by the Admiralty Manual of Navigation [1].

RLS calculations employing direct formulas on the ellipsoid, which require the calculation of the length of the arc of the meridian [1, 2] are simpler than those employing the Mercator projection formulas and isometric latitude. In addition the formulas on the ellipsoid provide more flexibility for the solution of the direct problem for the calculation of the geodetic coordinates of an unlimited number of intermediate points for the purpose of the display of RLS routes on the electronic chart of the ECDIS and ECS systems.

If we consider the great ellipse as an inclined version of the meridian ellipse, it is possible to calculate the great elliptic arc (sailing distance) in a similar way to that used for the calculation of the meridian arc. Various numerical tests and comparisons show that discrepancies in the computed distances between the ‘geodesic’ and the ‘great elliptic arc’ are practically negligible for marine navigation [6, 8, 18]. Moreover GES calculations are much simpler and straightforward and can be easily implemented in navigational software. They provide the same and in some cases, higher accuracy than other methods and formulas for sailing calculations on the ellipsoid. An example is that GES calculations provide more accurate results than the geodesic inverse solutions with the Lambert method. GES calculations can be also used for the precise calculation of the geodetic coordinates of an unlimited number of intermediate points along the great elliptic arc, and thus be implemented in GIS navigational systems (ECDIS and ECS) for the display of navigational paths on the electronic chart. The purpose of this paper is to present new simpler and faster formulas for meridian arc computations that can be immediately implemented

in various sailing calculation methods that require the calculation of the meridian arc. The detailed presentation of these sailing calculations can be found in the relevant bibliographic references, in particular [9].

### GEODETIC FORMULAS FOR THE MERIDIAN ARC LENGTH

The methods and formulas used to calculate the length of the arc of the meridian for precise sailing calculations on the ellipsoid, such as ‘rhumb-line sailing’, ‘great elliptic sailing’ and ‘geodesic sailing’ are simplified forms of general geodetic formulas used in geodetic applications. In this section an overview of the most important geodetic formulas along with general comments and remarks on their use is carried out. For consistency purposes and in order to avoid confusion in certain formulas the symbolization has been changed from that of the original sources. The fundamental equation for the calculation of the length of the arc of the meridian on the ellipsoid  $M_0^\varphi$  (fig. 1), is:

$$M_0^\varphi = \int_0^\varphi R_M d\varphi \tag{1}$$

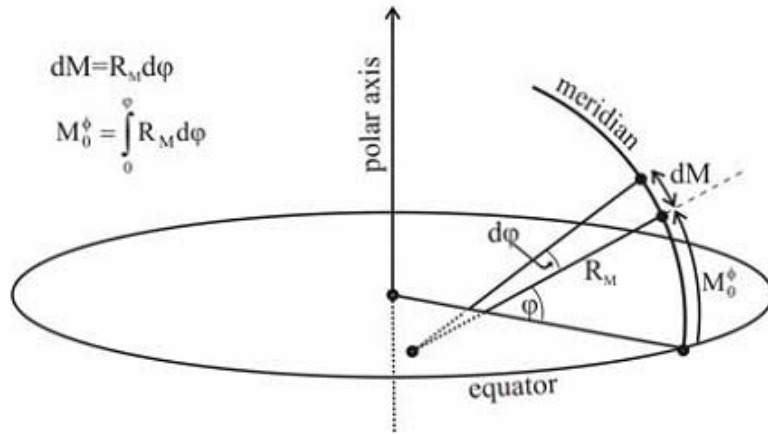


Fig. 1. The length of the arc of the meridian [9]

In (1),  $R_M$  is the radius of curvature of the meridian given by (2).

$$R_M = \frac{a(1-e^2)^{\frac{3}{2}}}{(1-e^2 \sin^2 \varphi)^{\frac{3}{2}}} \tag{2}$$

In (2),  $a$  and  $e$  are the semi-major axis and the eccentricity of the ellipsoid, respectively.

Replacing the value of  $R_M$  from (2) in (1), we obtain:

$$M_0^\varphi = \int_0^\varphi \frac{a(1-e^2)}{(1-e^2 \sin \varphi)^{\frac{3}{2}}} d\varphi \quad (3)$$

Equation (3) can be transformed to an elliptic integral of the second type, which cannot be evaluated in a 'closed' form. The calculation can be performed either by numerical integration methods, such as Simpson's rule, or by the binomial expansion of the denominator to rapidly converging series, retention of a few terms of these series and further integration by parts. According to Snyder [11] and Torge [13], Simpson's numerical integration does not provide satisfactory results and consequently the standard computation methods are based on the use of series expansion formulas. Expanding the denominator of (3) by the binomial theorem yields:

$$M_0^\varphi = a \cdot (1 - e^2) \int_0^\varphi \left( 1 + \frac{3}{2} e^2 \sin^2 \varphi + \frac{15}{8} e^4 \sin^4 \varphi + \frac{35}{16} e^6 \sin^6 \varphi \right) d\varphi \quad (4)$$

Since the values of powers of  $e$  are very small, equation (4) is a rapidly converging series. Integrating (4) by parts we obtain:

$$M_0^\varphi = a(1 - e^2) \left( \left( 1 + \frac{3}{4} e^2 + \dots \right) \varphi - \left( \frac{3}{8} e^2 + \frac{15}{32} e^4 \dots \right) \sin 2\varphi + \left( \frac{15}{256} e^4 + \frac{105}{1024} e^6 + \dots \right) \sin 4\varphi + \dots \right) \quad (5)$$

Equation (5) is the standard geodetic formula for the accurate calculation of the meridian arc length, which is proposed in a number of textbooks such as in Torge's 'Geodesy' using up to  $\sin(2\varphi)$  terms, [13] and in Veis' 'Higher Geodesy' using up to  $\sin(8\varphi)$  terms [14]. A rigorous derivation of (5) for terms up to  $\sin(6\varphi)$ , is presented in [10].

Equation (5) can be written in the form of equation (6) provided by Veis [14]

$$M_0^\varphi = a(1 - e^2)(M_0\varphi - M_2\sin 2\varphi + M_4\sin 4\varphi - M_6\sin 6\varphi + M_8\sin 8\varphi + \dots) \quad (6)$$

$$M_0 = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 + \frac{11025}{16384}e^8 + \dots$$

$$M_2 = \frac{3}{8}e^2 + \frac{15}{32}e^4 + \frac{525}{1024}e^6 + \frac{2205}{4096}e^8$$

$$M_4 = \frac{15}{256}e^4 + \frac{105}{1024}e^6 + \frac{2205}{8820}e^8 + \dots$$

$$M_6 = \frac{35}{3072}e^6 + \frac{315}{12288}e^8 + \dots$$

$$M_8 = \frac{315}{130784}e^8 + \dots$$

Equation (7) is derived directly from equation (6) for the direct calculation of the length of the meridian arc between two points (A and B) with latitudes  $\varphi_A$  and  $\varphi_B$ . In the numerical tests for the assessment of the relevant errors of selected alternative formulas, we will refer to equations (6) and (7) as the ‘Veis-Torge’ formulas [9].

$$M_{\varphi_A}^{\varphi_B} = a(1 - e^2)M_0[(\varphi_A - \varphi_B) - M_2(\sin 2\varphi_B - \sin 2\varphi_A) + M_4(\sin 4\varphi_B - \sin 4\varphi_A) - M_6(\sin 6\varphi_B - \sin 6\varphi_A) + M_8(\sin 8\varphi_B - \sin 8\varphi_A)] \quad (7)$$

Equations (6) and (7) are the basic series expansion formulas used for the calculation of the meridian arc. They are rapidly converging since the value of the powers of  $e$  is very small. In most applications, very accurate results are obtained by formula (6) and the retention of terms up to  $\sin(6\varphi)$  or  $\sin(4\varphi)$  and 8<sup>th</sup> or 10<sup>th</sup> powers of  $e$ . For sailing calculations on the ellipsoid it is adequate to retain only up to  $\sin(2\varphi)$  terms, whereas for other geodetic applications it is adequate to retain up to  $\sin(4\varphi)$  or  $\sin(6\varphi)$  terms. The basic formulas (6) and (7) can be further manipulated and transformed to other forms. The most common of these forms is formula (8). Simplified versions of (8) (retaining up to  $A_6$  and  $e^6$  terms only) are proposed in textbooks such as in Bomford’s ‘Geodesy’ [3] and in the ‘Admiralty Manual of Navigation’ [1].

$$M_0^\varphi = a(A_0\varphi - A_22\varphi + A_44\varphi - A_66\varphi + A_88\varphi \dots) \quad (8)$$

$$M_0 = 1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 - \frac{175}{16384}e^8 \dots$$

$$A_2 = \frac{3}{8}\left(e^2 + \frac{1}{4}e^4 + \frac{1}{15}e^6 + \frac{35}{512}e^8 \dots\right)$$

$$A_4 = \frac{15}{256}\left(e^4 + \frac{3}{4}e^6 + \frac{35}{64}e^8 \dots\right)$$

$$A_6 = \frac{35}{3072}e^6 + \frac{175}{12228}e^8 \dots$$

$$A_8 = \frac{315}{131072}e^8 \dots$$

It should be noted that an important step in the solution included in the above-cited work is simplification by the omission of the expansion part into power series of mathematical solutions (see formula (6)), previously known from the literature, i.e., [13, 14], and reliance in the explanatory memorandum of application, in particular,

on the amount of the available processing power of modern calculating machine (processor). The vertical (green) and horizontal (red) line in (6) shows applied limitation of the computational formula. In our opinion this criterion is relevant from a practical point of view, but temporary, given the growth and availability of computing power, including GIS. Calculating speed measured by the parameter CPU (Central Processing Unit called Speed Measurement) should not restrict the available accuracy, and this takes place in the proposed algorithm.

#### THE PROPOSED NEW FORMULAS BY PALLIKARIS, TSOULOS AND PARADISSIS

The proposed new formulas implying from section 4 for the calculation of the length of the meridian in sailing calculations on the WGS-84 ellipsoid in meters and international nautical miles are (9) and (10), respectively. We observe the following formulas are based on (6) after ‘horizontal’ and ‘vertical’ approximations in (6) indicated by green and red lines above and then applied in (7). This operation yields.

$$M_{\varphi_A}^{\varphi_B} = 111132.95251 \cdot \Delta\varphi - 16038.50861 \cdot \left( \sin\left(\frac{\varphi_B \cdot \pi}{90}\right) - \sin\left(\frac{\varphi_A \cdot \pi}{90}\right) \right) \quad (9)$$

$$M_{\varphi_A}^{\varphi_B} = 60.006994 - 8.660102 \cdot \left( \sin\left(\frac{\varphi_B \cdot \pi}{90}\right) - \sin\left(\frac{\varphi_A \cdot \pi}{90}\right) \right) \quad (10)$$

In both formulas (9) and (10) the values of geodetic latitudes  $\varphi_A$  and  $\varphi_B$  are in degrees and the calculated meridian arc length in meters and international nautical miles, respectively. Formulas (9) and (10) have been derived from (7) for the WGS-84, since the geodetic datum employed in Electronic Chart Display and Information Systems is WGS-84. The derivation of the proposed formulas is based on the calculation of the  $M_0$  and  $M_2$  terms of (7) using up to the 8th power of  $e$ . This is equivalent to the accuracy provided by (8) using  $A_0$  and  $A_2$  terms with subsequent  $e$  terms extended up to the 10<sup>th</sup> power since in formula (7) the terms  $M_0, M_2, M_4 \dots$  are multiplied by  $(1-e^2)$ . According to the numerical tests carried out, which are presented in the next section, the proposed formulas have the following advantages [9]:

- they are much simpler than and more than twice as fast as traditional geodetic methods of the same accuracy;
- they provide extremely high accuracies for the requirements of sailing calculations on the ellipsoid.

#### NUMERICAL TESTS AND COMPARISONS

Errors have been calculated as discrepancies from the complete formula (6) with up to  $\sin(8\varphi)$  terms (calculation of the 18 meridian arcs presented in [9]).



Table 2. Error assessment of formulas providing submeter accuracies [9]

	Formula (6) with up to $\sin(6\varphi)$ terms	Formula (6) with up to $\sin(4\varphi)$ terms
average error	0,22 mm	-4,4 mm
maximum error	4 mm	21,98 mm
minimum error	0,03	-21,95

For the accuracy assessment of the evaluated formulas, the ‘Veis-Torge’ formulas with up to  $\sin(8\varphi)$  terms (formula (6)) were adopted as the most accurate standard. Formula (6) provides slightly higher accuracy because it contains more complete terms than all the other formulas. For instance, comparing formulas (6) and (8) it is noted that in formula (6) the  $e$  power terms are computed up to the 10<sup>th</sup> power, instead of the 8<sup>th</sup> power in formula (8), since the terms  $M_0, M_2, M_4, \dots$  in formula (6) are multiplied by  $(1 - e^2)$ , whereas the terms  $A_0, A_2, A_4, \dots$  in formula (8) contain up to  $e^8$  terms. Calculations of the meridian arc distance performed with formula (6) matches perfectly with geodesic distances (between corresponding points on the meridian) calculated with Vincenty’s algorithm [15]. The latter is considered as one of the most precise methods for the calculation of long geodesics however not the only one e.g. Pittman’s method.

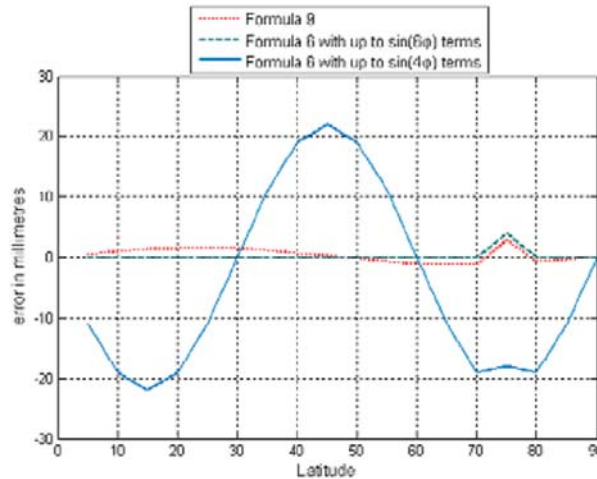


Fig. 2. Error assessment of formulas providing submeter accuracies [9]

According to the above mentioned results of the accuracy assessment and the CPU time required, the proposed formulas can considerably simplify existing calculation methods of comparable accuracy on the ellipsoid such as in rhumb-line sailing [2, 5] and great elliptic sailing [4, 6, 8, 18]. This simplification does not reduce the accuracy of existing methods and algorithms. Referring to numerical tests in [9] the exemplary positions are as in table 3.

Table 3. Geographical positions' coordinates applied in numerical tests [own study]

	Harbour	Position			
		Latitude		Longitude	
		[°]	[']	[°]	[']
A	New Port-Boston	41	06,336	-071	23,460
B	Leath Harbour	-54	07,902	-036	32,280
C	Buenos Aires	-36	03,762	-055	30,240
D	Dakar	14	22,146	-017	27,240
E	Pearth	-32	06,216	115	34,080
F	Mombasa	-04	05,922	039	42,660
G	San Francisco	37	45,048	-122	42,000
H	Yokohama	34	26,178	139	51,120
I	Valparaiso	-33	00,000	-071	36,660
J	Sydney	-33	46,212	151	31,980
K	Cape of Good Hope	-34	25,608	018	25,920
L	Rio de Janeiro	-23	01,914	-043	07,200
M	Beirut	33	56,016	035	30,300
N	Tobruk	32	03,846	024	00,660
O	Lisbon	38	37,206	-009	13,380

In addition to obtained results in [9] we compare in table 4 the distance and angle measurements we had observed in the commercial ECDIS widely used in world-wide shipping. The figures show the relative error keeping the calculations according to Vincenty's algorithm as the reference (0-level).

Table 4. Comparison of numerical tests on distance and angle calculations [9, 17]

From	To	ECDIS		Vincety's	ECDIS [#]		GES algorithm [Pallikaris, Tsoulos 2009]		Navpack [Hohenkerk 2004]	
		D [Nm]	Az [°]	D [Nm]	D [Nm]	Az [°]	D [Nm]	Az [°]	D [Nm]	Az [°]
F	E	4557,19000	122,0	4556,10000	4557,23000	122,0	4557,19000	122,0	4551,20000	122,1
I	H	9242,79000	281,8	9241,92000	9242,78000	281,8	9242,80000	281,9	9233,70000	282,0
G	H	4502,89000	302,1	4502,34927	4502,86000	302,0	4502,89000	302,0	4489,70000	302,0
A	K	6699,26000	117,2	6698,49520	6699,26000	117,2	6699,26000	117,3	6702,10000	117,4
D	B	4213,53000	191,8	4213,04600	4213,54000	191,8	4213,53000	191,8	4224,50000	191,7
C	K	3541,10000	112,2	3540,69170	3541,08000	112,2	3541,10000	112,1	3530,80000	112,1
A	O	2813,75000	71,5	2813,42570	2813,74000	71,5	2813,75000	71,6	2804,90000	71,6
L	K	3268,79000	116,8	3269,20270	3268,78000	116,8	3268,79000	116,7	3261,10000	116,8
M	N	590,37000	262,2	590,32440	590,39000	262,2	590,37000	262,2	589,00000	262,1
J	I	6129,11000	144,2	6128,41000	6128,95000	144,2	6129,12000	144,0	6113,00000	144,1

As the Navpack states the reason for the greatest discrepancies the following figures present the numerical results including and excluding the Navpack calculations, i.e fig. 3 and 4, respectively.

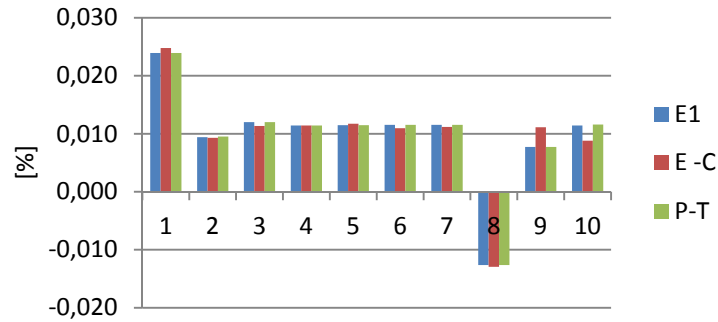


Fig. 3. Numerical tests comparison on distance and angle referred to table 4 excluding Navpack (N) [own study]

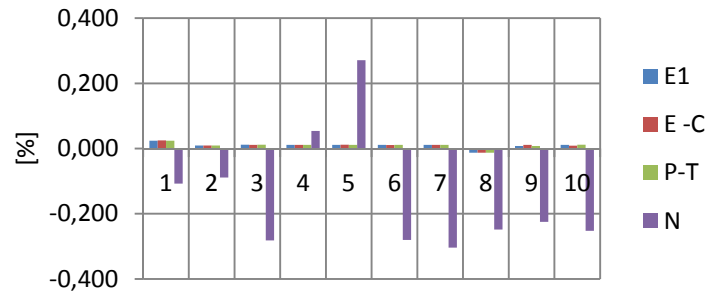


Fig. 4. Numerical tests comparison on distance and angle referred to table 4 including Navpack (N) [own study]

### CONCLUSIONS

The proposed new formulas by Pallikaris, Tsoulos and Paradissis [9] for the calculation of the meridian arc are sufficiently precise for sailing calculations on the ellipsoid, because the maximum error for the calculation of the length of the meridian arc for very long distances is less than 17 meters. It is pointed out that they are about 235% faster than the alternative geodetic methods and formulas of the same accuracy. Higher sub metre accuracies can be obtained by the use of more complete equations with additional higher order terms. Seeking this higher accuracy for sailing calculations does not have any practical value for marine navigation and simply adds more complexity to the calculations. In other than navigation applications, where higher sub metre accuracy is required, the Bowring formulas showed to be approximately two times faster than alternative geodetic formulas of similar accuracy. Despite the fact that contemporary computers are fast enough to handle more complete

geodetic formulas of sub meter accuracy, a basic principle for the design of navigational systems is the avoidance of unnecessary consumption of computing power. Saving and reserving computer resources is always beneficial for the improvement of the systems effectiveness on the evolving new navigational functions and applications such as the handling of greater amounts of cartographic and navigational information, the capability for 3-D presentation etc. The proposed formulas provide a more realistic balance between accuracy and computing power required for the sailing calculations in a GIS environment and particularly in ECDIS, in compliance with the performance standards of the International Maritime Organization (IMO). These formulas can be immediately used not only for the development of new algorithms for sailing calculations, but also for the simplification of existing algorithms without degrading the accuracies required for precise navigation. The simplicity of the proposed method allows for its easy implementation on pocket calculators for the execution of accurate sailing calculations on the ellipsoid [9].

Original contribution affects and verifies established views based on approximated computational procedures (mainly power series) used in the software of marine navigational systems and devices. The current stage of knowledge enables to implement geodesics based computations which present higher accuracy. It also lets to assess the quality of contemporary algorithms used in practical marine applications. Currently, on the pages of a reputable scientific 'Journal of Navigation', there is a discussion on the problem of calculation procedures for marine navigation (great ellipse sailing (GES), a rhumb-line sailing (RLS), great circle (GC), geodetic lines), as evidenced by works published by researchers from different countries and institutions.

Vincenty's algorithm allowing calculation of geodetic distances along lines of rotational ellipsoid is taken as a reference point for verification and determining the accuracy of the calculations. Numerical analysis associated with the power series development involves giving the solution to elliptic integrals of the second kind  $\int \sqrt{1 - k^2 \sin^2 t} dt$ , which occurs when calculating the length of the arc of a great ellipse of a spheroid in sailing. The second of the arguments supporting the use of the above algorithm says about the threshold accuracy established in practice of 1 Nm in the global modelling and hence no need, according to the authors of the above cited paper, to obtain greater accuracy than the proposed computational algorithm. We claim this argument which is, in principle, questionable but acceptable for many applications. The authors in the journal 'Coordinates' and a research paper presented at the International Conference TransNav in 2011 [17] drew attention to the merits of navigational calculations based on geodesics, both locally and globally, stressing the importance of the function of curvature of the modelling surface. The validity of

the discussed problem is also related to the fact that at present navigation algorithms based on the lines being arcs of great ellipse or a geodetic instead of a rhumb-line used in practice for hundreds of years and cartographically associated primarily with conformal Mercator's projection is used more and more frequently. In addition, it should be noted that there are no standards on the use map projections in created present marine Geographic Information System. Today, the same system ECDIS/ECS can potentially use about 20 different projections in which graphical presentation of the above mentioned lines used in navigation is differ significantly. This affects the correct interpretation of the information generated in GIS, particularly in navigation, by the more and more mass user of the system. Scientific workshop employed to solve the problem makes use of various tools, i.e. of differential geometry, marine geodesy (marine navigation), analysis of measurement error, approximation theory and problems of modelling and computational complexity, mathematical and descriptive statistics, mathematical cartography. Geometric problems are important aspect of the tested models which are used as the basis of calculations and solutions implemented in contemporary navigational devices and modern electronic chart systems.

### **Acknowledgments**

This paper was supported in part by the Polish National Science Centre under research grant No. NN 526 205540 — Contract No. 2055/B/T02/2011/40.

### **REFERENCES**

- [1] Admiralty Manual of Navigation, 1987, Vol. 1, General navigation, Coastal Navigation and Pilotage, Ministry of Defence (Navy), London, The Stationery Office.
- [2] Bennett G. G., Practical rhumb-line calculations on the spheroid, *The Journal of Navigation*, 1996, Vol. 49, No. 1, pp. 112–119.
- [3] Bomford G., *Geodesy*, Oxford University Press, 1985.
- [4] Bowring B. R., New Equations for Meridional Distance, *Bulletin Geodesique* 1983, 57, pp. 374–381.
- [5] Bowring B. R., The geometry of Loxodrome on the Ellipsoid, *The Canadian Surveyor*, 1985, Vol. 39, No. 3.
- [6] Earle M. A., Vector Solution for Navigation on a Great Ellipse, *The Journal of Navigation* 2000, Vol. 53, No. 3, pp. 473–481.

- [7] Earle M. A., Sphere to Spheroid Comparisons, *The Journal of Navigation*, 2006, Vol. 59, No. 3, pp. 491–496.
- [8] Pallikaris A., Latsas G., New algorithm for great elliptic sailing (GES), *The Journal of Navigation*, 2009, Vol. 62, pp. 493–507.
- [9] Pallikaris A., Tsoulos L., Paradissis D., New Meridian Arc Formulas for Sailing Calculations in GIS, *International Hydrographic Review*, May 2009, pp. 24–34.
- [10] Pearson F., *Map Projection: Theory and Applications*, CRC Press, Boca Raton, Ann Arbor, London — Tokyo 1990.
- [11] Snyder J. P., *Map Projections: A Working Manual*, U.S. Geological Survey, Professional Paper 1395, Washington 1987.
- [12] Tobler W., A comparison of spherical and ellipsoidal measures, *The Professional Geographer*, 1964, Vol. XVI, No. 4, pp. 9–12.
- [13] Torge W., *Geodesy*, 3<sup>rd</sup> ed., Walter de Gruyter, Berlin — New York 2001.
- [14] Veis G., *Advanced Geodesy*, chap. 3, National Technical University of Athens (NTUA), 1992.
- [15] Vincenty T., Direct and Inverse Solutions of Geodesics on the Ellipsoid with Application of Nested Equations, *Survey Review*, 1975, Vol. XXII, No. 176, pp. 88–93.
- [16] Weintrit A., *The Electronic Chart Display and Information System (ECDIS), An Operational Handbook*, A Balkema Book, CRC Press, Taylor & Francis Group, Boca Raton — London — New York — Leiden, 2009, p. 1101.
- [17] Weintrit A., Kopacz P., A Novel Approach to Loxodrome (Rhumb-Line), Orthodrome (Great Circle) and Geodesic Line in ECDIS and Navigation in General, [in:] ed. A. Weintrit, T. Neumann, *Methods and Algorithms in Navigation, Marine Navigation and Safety of Sea Transportation*, A Balkema Book, CRC Press, Taylor & Francis Grup, Leiden 2011, pp. 123–132.
- [18] Williams R., The Great Ellipse on the Surface of the Spheroid, *The Journal of Navigation*, 1996, Vol. 49, No. 2, pp. 229–234.

Received May 2012

Reviewed October 2012