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**JAROSŁAW ARTYSZUK**
Maritime University of Szczecin, Poland

# CONVERSION OF A SECOND- TO FIRST-ORDER LINEAR NOMOTO MODEL IN THE LIGHT OF ZIGZAG MANOEUVRE PERFORMANCE

**ABSTRACT**

The problem of considerable difference between the first- and second-order linear Nomoto models is undertaken, not well covered in literature so far. If the former approximates the latter (better one, of a sound hydrodynamic interpretation) for some reasons, its parameters can not be easily derived from the other one, except for some specific rare cases. For such an identification purpose, we can use a simulated zigzag response and the classic procedure proposed by Nomoto in 1960. However, the first-order model thus developed yields somehow redefined constants against the original model, which lose their normal hydrodynamic (or kinematic) sense. In other words, it is very sensitive to the manoeuvre type on input, being therein the zigzag test. Therefore, the model is allowed to be only used for simulating motions essentially similar to the input zigzag. In other words, the identification procedure works like a blind curve-fitting and the first-order model (in contrast to second-order one) is inadequate for reflecting arbitrary manoeuvres, even for mild rudder as to be within 'linear' assumptions.

This study examines systematically and in detail such an incompatibility of the first order model in that it presents the conversion charts from the standpoint of 10°/10° zigzag test matching. One can receive higher or lower values for the parameters of first-order model, versus the second-order one, depending on the $T_3/T_2$ ratio of the latter model.

**Keywords:**
zigzag test, Kempf, ship, manoeuvring, steering, Nomoto

## 1. INTRODUCTION

In the author's former research [Artyszuk, 2016a-b, 2017] some intriguing properties of the second-order ship steering (or yaw) dynamics were discovered and explored, especially concerning their reduction to first-order dynamics, sometimes referred to as *K*-*T* model. They were almost completely ignored or underestimated in the original works of [Nomoto et al., 1957, [Nomoto, 1960] and further follow-ups by other researchers, where in the latter a sort of support or confirmation, more or less deliberate, could have been seen. The problem does not lie in inadequacy of the first-order model for simulating the zigzag manoeuvre, that is rather correct but if being looked at in a semi-empirical way, i.e. more like curve-fitting. Many theoretical approximations, as existing in the literature, to the first-order model, independent of manoeuvre and based on various more or less disputable physical assumptions, are really confusing and call for caution.

The controversy seems generally twofold. Firstly, it relates to the method of model reduction, i.e. how to transfer coefficients of the second-order model ($K^*$-$T_1$-$T_2$-$T_3$, mark '*' is here added to distinguish the second-order gain constant) to those of first-order. Secondly, the very specific properties of second-order model are not well appraised so far in the literature to enhance their wider usage. Generally, the gain constant of first-order model is being made equal to the second-order equivalent ($K=K^*$) with some exceptions as reported below.

The aim of the present paper is to examine in detail the effect of various instances of the second-order model, especially in extreme domains of their four parameters, on the determined (through zigzag analysis) *K* and *T* parameters of the first-order approximation. The mild and standard zigzag 10°/10° is selected to keep the linear hydrodynamics valid enough and to ensure a wider application or comparison/verification of the results. The investigations serve multiple-purpose: a) to provide at glance the right first-order model from hydrodynamic derivatives (through the second-order model), b) to explore the inherent relationships of such a conversion, c) to test the influence of the often neglected factor – rudder speed – on the results.

## 2. PAST RESEARCH AND PRACTICE

Below, from a huge amount of specific literature, is reviewed a selective yet fundamental/representative set of references for the subject.

[Fossen, 1994] shows through simulation the amplitude-phase frequency response of first- and second-order model for the benchmark Mariner class vessel if the [Nomoto et al., 1957] 'equivalency' formula $T = T_1+T_2-T_3$ (and $K=K^*$) is adopted. This was further reproduced in [Fossen, 2011], and such a good example of a ship was also commented in [Artyszuk, 2016b]. However, the cited book does not discuss the importance of the found differences, especially in the plot of phase lag, when applied to the zigzag simulation. One can even get an impression that the mentioned formula is general and applicable for arbitrary motion, although it is originally derived for the steady state phase of turning. According to experiences of the present author, based on multiple simulations and analytical studies, this phase is barely reached in the zig-zag manoeuvre, so such reduction formulas should be more firmly revised in the engineering practice and textbooks.

[Dudziak, 1988] assumes $T = T_1$ according to the actual relative magnitude of the three second-order time constants. This renders almost the same practical accuracy as in the previous, more 'sophisticated' case. However, the author does not seem to appreciate enough the second-order model in transforming it to the above simpler case just by straightforward omitting the second derivative of yaw and disregarding the effect of rudder speed (as also leading to vanishing $T_3$-component).

On the other hand, [Lewis (ed.), 1989] presents a reduction approach to the first-order model, with some vital consequences, that is rather wrongly assigned to Nomoto original works. It renders the model parameters absolutely different from [Nomoto et al., 1957], which is rather missed by the contributing authors, and likely losing a physical/hydrodynamic validity.

Let us write the basic coupled linear equations of drift and yaw in dimensionless form:

$$\begin{cases} \dfrac{d\beta}{ds'} = a_1\beta + b_1\omega'_z + c_1\delta \\ \dfrac{d\omega'_z}{ds'} = a_2\beta + b_2\omega'_z + c_2\delta \end{cases}, \qquad \ldots(1)$$

where:

$a_i$, $b_i$, $c_i$, $i \in \{1,2\}$ – hydrodynamic coefficients (constants);

$\beta$, $\omega'_z$ – drift angle and dimensionless yaw velocity;

$\delta$ – rudder angle;

$s'$ – dimensionless distance (or time).

Eq. (1) resolves into a decoupled second-order linear Nomoto model:

$$T_1T_2\frac{d^2\omega'_z}{ds'^2} + (T_1 + T_2)\frac{d\omega'_z}{ds'} + \omega'_z = K^*\left(\delta + T_3\frac{d\delta}{ds'}\right), \qquad \ldots(2)$$

where the four new parameters (time constants and gain) can be expressed by the former hydrodynamic coefficients that can be found in many references, also in [Artyszuk, 2016a].

The fully dimensionless second-order model, Eq. (2), is general in that it covers (is independent of) any advance speed and size/length of a ship. It involves evolution of a kinematical variable – dimensionless yaw velocity (and thus heading as its integral, as being the control input) – with dimensionless distance *s*'. The latter term is to be more preferred than equivalent 'dimensionless time', if nautical purposes are of primary concern (as such is often the case), and means the travelled distance counted in ship's length units. Also, the three 'time' constants in Eq. (2) should be interpreted identically, although the traditional symbol '*T*' is used for them.

The first-order Nomoto model is defined by:

$$T\frac{d\omega'_z}{ds'}+\omega'_z=K\delta\,, \qquad \ldots(3)$$

[Lewis (ed.), 1989] stipulates very low and thus negligible coupling terms $b_1$ and $a_2$ in Eq. (1), which i.a. directly degenerates to Eq. (3) with the following worth noting:

$$T=-\frac{1}{b_2}\quad,\quad K=-\frac{c_2}{b_2}\,, \qquad \ldots(4)$$

One can also use the expressions for second-order constants [Artyszuk, 2016a], which, in addition to *K* in Eq. (4), returns the condition investigated in [Artyszuk, 2016b], although different way – the equality/cancellation of $T_3$ and $T_1$ is obtained (instead of $T_2$):

$$T_1=T_3=-\frac{1}{a_1}\quad,\quad T_2=-\frac{1}{b_2}=T\,, \qquad \ldots(5)$$

Partly, this is attributable to the usual, arbitrary sequence (forced by appropriate expressions) of both time constants $T_1$ and $T_2$, such that $T_1>T_2$. One should note that the second-order model is symmetrical against $T_1$ and $T_2$ which means we can replace both constants each other and keep Eq. (2) identical. For an exemplary ship investigated in [Artyszuk, 2016a,b] we get:

$$K^*=4.90\,,\quad T_1=10.49\,,\quad T_2=0.30\,,\quad T_3=0.98\,, \qquad \ldots(6)$$

which by [Nomoto et al., 1957] turn into first-order constants:

$$K=4.90\,,\quad T=9.81\,, \qquad \ldots(7)$$

or by [Lewis (ed.), 1989] as follows (see details of Eq. (1) in [Artyszuk, 2016a]):

$$K^*=0.54\,,\quad T_1=1.61\,,\quad T_2=0.35\,,\quad T_3=1.61\,, \qquad \ldots(8)$$

$$K=0.54\,,\quad T=0.35\,, \qquad \ldots(9)$$

The difference between values of Eq. (7) and (9), and within the corresponding simulations of ship manoeuvres, especially zigzag tests, is remarkable. A closer look at the hydrodynamic assumptions themselves of [Lewis (ed.), 1989] enables to say they are against many experimental data.

Except for [Lewis (ed.), 1989], the approximation methods discussed above, let us say theoretical ones, return $K=K^*$ and work well if $T_3$ is very close to $T_2$, where the second-order model converges to first-order.

The other extreme approach goes far beyond the relationship with hydrodynamic (stability) derivatives of the background coupled linear equations of ship drift/sway and yaw, thus losing a direct link to Eq. (1). This can be called empirical or identification-based method that uses actual ship motions. Mostly, it assumes the classical, very elegant integral form of [Nomoto, 1960], as is strictly connected with the zigzag manoeuvre.

[Nomoto & Norrbin, 1969] state that linear range of motion, i.e. valid for the linear analysis of first-order, is in fact rarely encountered such that the so-called 'linear on the average' curve-fitting is actually exercised with Nomoto model and identification technique of 1960. And this is still very useful when determining various steering qualities of a ship. Both authors solely concentrate on the first-order model, robust one, but certainly we can naturally extend such concerns also to the second-order case. In the literature and research practice, this is sometimes referred to as the transition from local derivatives (mathematically and physically sound, geometrically represented by tangent) to global 'derivatives' (geometrically marked by chord). However, the cited reference focuses more on higher and different angles of rudder and heading deviation in the zigzag test than usual 10°/10°, when nonlinearity may increase. Nevertheless, since in general we are unable to discriminate in the zigzag record not only between the linear and nonlinear response, but as well between just the first- and second-order linear response, such an interpretation means *'linear averaging' of the second-order linearity* to the simpler one.

As stated before, [Lewis (ed.), 1989] returns the first-order model (of yaw) with some coefficients of only the yaw sub-equation in Eq. (1). The little more rational approach is presented by [Motora, 1960], which explains the existence of first-order model by means of introducing to the ship coupled motions a certain fixed empirical pivot point. This provides a 'better' proportionality of drift and yaw than is obtainable with the former rough approximation, and denotes keeping to some extent the contribution of $a_2$ (see Eq. (1)). However, also with this approach, as one may note, we cannot determine the $K$ and $T$ parameters from the coefficients of Eq. (1) or (2), if adequate simulation of the zigzag response is being aimed. For this purpose, one should still revert to their identification based on ship manoeuvring trials.

In the context of present research, [Motora, Fujino, 1969] made the biggest contribution to resolve the problem of some incompatibility of first- and second-order linear models, if the reduction of [Nomoto et al., 1957] is attempted. This was exemplified on a ship similar to the mentioned Mariner class ship (in terms of $T_3$ vs. $T_2$). Although both the authors are starting with nonlinear models of stable and unstable ships, a good deal of the performed analysis, to our interest, regards just a linear stable ship and provides a basic, simple insight to the phenomenon. Moreover, the authors go much far and show the difference if various combinations of rudder and (switching) heading angles are exercised in zigzag test. They prove that both first- and second-order models asymptotically converge at zero rudder and finite heading angles – a theoretical yet impractical case. Concentrating on another aspects (steering quality evaluation in various modes of control), they did not comment at all on the also shown standard case of 10°/10° (or other equal angle cases), where almost twice less values of *K* and *T* are obtained if one applies the *K*-*T* identification of [Nomoto, 1960] to the simulated second-order linear response. For other unequal combinations of rudder/heading, they found the differences versus input much more exaggerated, also without appropriately highlighting the huge inadequacy of first-order model. The ITTC contribution of [Motora, Fujino, 1969], a rather unique piece of research, is unfairly in the shadow of other more affirmative references (research papers and textbooks) dealing with transition from second- to first-order models, which have been partly mentioned before.

In summary, one must note that a derivation of the first-order model from the second-order one, based on various theoretical assumptions and aimed at practical application, surprisingly encounters significant difficulties. Both are quite different. The only way seems to (globally identify) curve-fit the first-order model to actual ship motion, best if the input motion (control) corresponds to the future operational area of the model. In particular, with the same first-order model parameters and within the range of low rudder angles (to involve linear dynamics), we are unable to accurately simulate both steady turning and course-keeping or changing manoeuvres. We can have either one or another. This is a serious drawback of the model (being not adequate/universal) in view of real and usual ship behaviour, unless the ship hydrodynamics undergoes the exceptional mentioned condition $T_3=T_2$. The first-order model, irretrievably losing some essential information, seems not useful in the 'reverse' engineering, when the other-way transition, towards second-order (and thus full mission model), is attempted. In the light of recent efforts [Artyszuk, 2018], and referring again to [Motora, Fujino, 1969], the best solution to the identification of second-order model is likely to use two or more different (in terms of rudder/heading combinations) zigzag records, as more or less supplemented with low rudder steady turning.

## 3. METHODOLOGY

In an attempt to solve the stated problem of approximating the second-order general model with first-order one, we may use analytical methods or numerical integration (simulation) methods. However, only the latter is of practical interest. Although both the linear models have their own known analytical solution, it is very complicated due to a specific steering control involved in the zigzag. The control is in the form of alternating, trapezoidal rudder angle being switched by (with feedback from) heading angle, thus leading to a multiple-piecewise and multiple-component complicated solution. For example, to reach the first overshoot angle, the zigzag consists of four rudder phases. The solution is illegible and not capable of easy use (e.g. to get a closed-form formula of models conversion, even using the simple method of [Nomoto, 1960]), also in that, it requires computation of the inverse function $s'(\psi)$ ($\psi$ – heading). The latter, much more complicated in case of the second-order model, is actually a solution of an exponential equation, which can be completed only numerically. That is why a direct numerical integration of Eq. (2) is preferred. Details of the ODE procedure implemented in hereafter computations are given in [Artyszuk, 2017].

The intended simulation experiment performed within this study consisted of simulating a second-order response with Eq. (2) and, based on this output, subsequent identification of $K$ and $T$ by [Nomoto, 1960] method. This means fitting a first-order response to the former one, but with regard to two curves – heading and (dimensionless) yaw velocity. The adopted definitions of $K$ and $T$ are:

$$K = \frac{\psi_{OS1}}{\Delta\left(s'_{OS1}\right)}, \qquad \dots(10)$$

$$T = \frac{K \cdot \Delta\left(s'_{CR2}\right) + \frac{\pi}{18}}{\omega'_z\big|_{CR2}}, \qquad \dots(11)$$

where:

$$\Delta\left(s'\right) = \int_0^{s'} \delta ds', \qquad \dots(12)$$

The above are based on the first overshoot angle ('$OS_1$') and the second counter-rudder order ('$CR_2$', when heading assumes the first opposite/negative 10°), accordingly. With regard to $K$, the second and consecutive overshoot angles may also be used, resulting in more or less different magnitude. Likewise, the constant $T$ can be freely linked to other instances of heading as far as we know the dimensionless yaw velocity at those points.

Eq. (11) is a minor modification of [Nomoto, 1960] formula, in which the yaw velocity was originally taken at the zero heading, and thus the last term in the numerator ($\pi/18$) did not appear. The undertaken action serves keeping the number of additional points in the zigzag analysis, and in the computation algorithm as well, to minimum. The counter-rudder points, as fundamental, have just been existing (being determined automatically) therein. In addition, the used point is close to the original (heading) zero-crossing point. However, searching for 'zero occurrence' is not problematic in the algorithm since it already takes place while looking for the overshoot angles, at which the yaw velocity comes to zero.

In view of the rudder trapezoidal run, Eq. (12) can easily be solved analytically. In this context, ready-to-use formulas were published, e.g. in [Artyszuk, 2018]. However, due to a wide range of the second-order model being planned and a risk of getting the overshoot at the rudder slope (not within the usual constant rudder phase), the numerical integration is performed.

One shall notice in the above that, partly for simplicity of this somehow 'pilot' study, we turn with Eqs. (10-12) to the first half of heading cycle.

The magnitude of coefficients in Eq. (2), of hydrodynamic sense, mainly depends on the shape geometry of a ship and her propulsion-steering equipment, in particular on the ratios of main dimensions within and between the hull, propeller, rudder. The other minor factor is the specific flow conditions corresponding to the Froude and/or Reynolds number.

It is difficult to settle limits for the four coefficients – $K^*$, $T_1$, $T_2$, $T_3$ – as wide as possible, to encompass all ship cases, yet finite enough to keep the extent of simulation experiment practical. There is little information or such is being considerably scattered in literature. Due to variety of ship designs, also their assessment based on hydrodynamic derivatives in Eq. (1), through *subcomponents thereof*, published or computable with empirical methods, creates a challenge. The present author made a rough, fast theoretical evaluation based on formulas of [Artyszuk, 2016a] and feasible values of all the factors therein, and this resulted in rather wide ranges of the model parameters. To much extent, the parameters are even independent of each other. For example, if we assume values for the basic two coefficients, $K^*$ and $T_1$, the other two factors responsible for the second-order linearity, expressed through the meaningful ratios, can be calculated by:

$$\frac{T_2}{T_1} = \frac{1}{-(a_1+b_2)T_1 - 1} \quad , \quad \frac{T_3}{T_2} = \frac{c_2}{\left(\dfrac{K^*}{T_1}\right)}, \qquad \ldots(13)$$

For a ship examined by [Artyszuk, 2016a] $c_2$=1.54 was reported. Taking the considered therein structure for this coefficient and simulating the worst (most extreme), rather unlikely case, we would get $c_2$=0.07 and 42, which means a spread 600 times.

The other approach to estimating the boundaries of four model parameters can be based on IMO (International Maritime Organisation) manoeuvring standards for zigzag test 10°/10°, which regulate overshoot angles and initial turning ability index. However, only the latter ($s'_{10}$ – dimensionless distance at the first change of heading 10° or counter-rudder) would have rather definite effect on parameter limits, through $K^*/T_1$ control, see [Artyszuk, 2017], although this is partly obscured by $T_2$ and $T_3$.

Therefore, somehow arbitrarily, mostly symmetrical and largely spaced values were chosen for $K^*$ and $T_1$ (giving) around the reference value $\{K^*,T_1\}=\{5,10\}$, see Eq. (6), to investigate basic relationship. This gives nearly 3x3 experimental matrix and is illustrated in Figure 1 with cross-references to subsequent figures with output data presentation. On the other hand, the following dense discrete points were selected for $T_2$ and $T_3$ due to their role in the second-order linearity:

$T_2/T_1$ = 0.01, 0.02, 0.05, 0.1, 0.2, 0.4 (6 values),

$T_3/T_2$ = 0, 0.2, 0.5, 1, 1.5, 2, 4, 6, 10 (9 values).

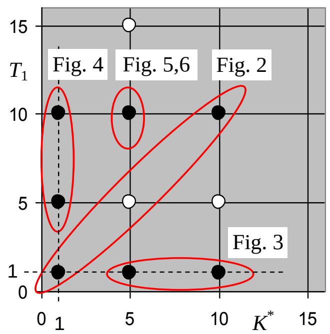


Figure 1. Simulation experimental design for the study
(black/white dots – included in/excluded from paper's presentation).
Source: Author.

The rudder speed is uniformly and arbitrarily set to 23°/$L$, where $L$ is ship's length, or dimensionless 0.435 [-] (=rad/$L$), unless otherwise stated.

The numerical accuracy of the first-order approximation (with the identified *K*, *T*) for the given second-order input, in terms of the zigzag heading 'reverse' simulation, is to be left for future investigations. Both first- and second-order zigzag curves are practically/visually close to each other, however, at the present stage being of relatively less interest.

## 4. THE RESULTS

Fig. 2 presents the situation when both $K^*$ and $T_1$ are equal. The intersection point of all curves in this and subsequent Figs. 3-6 denotes the mentioned condition $T_3$=$T_2$, where the second-order model becomes first-order.

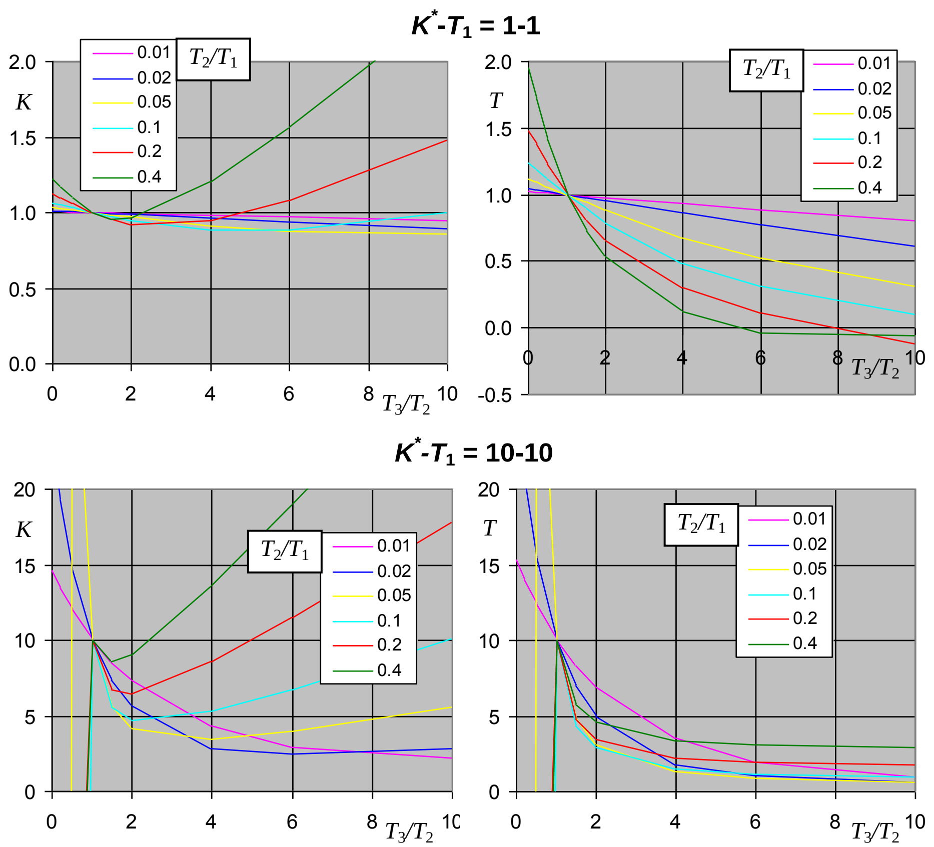


Figure 2. The *K*, *T* values for equal ($K^*$=$T_1$) 2nd-order input of 1-1 and 10-10. Source: Author.

However, since $T_2$ is being settled in relation to $T_1$, as fraction less than 1, if it is high enough and combined with relatively high $T_3/T_2$, one may get $T_3$ suddenly higher than $T_1$. This generally happens for $T_2/T_1$=0.2 or 0.4 while $T_3/T_2$>2÷4, see the red and green lines in Fig. 2 and in later ones. It is also leading in extreme situations to a negative equivalent time constant by [Nomoto et al., 1957] definition and partly to a negative fitted constant $T$ (see the top right sub-chart of this figure). Additionally, this denotes achieving meantime $T_1$=$T_3$ that works same way as the above directly considered state $T_3$=$T_2$. In such a case, where $T_1$ and $T_3$ 'cancel' each other, $T_2$ resumes the role of $T_1$.

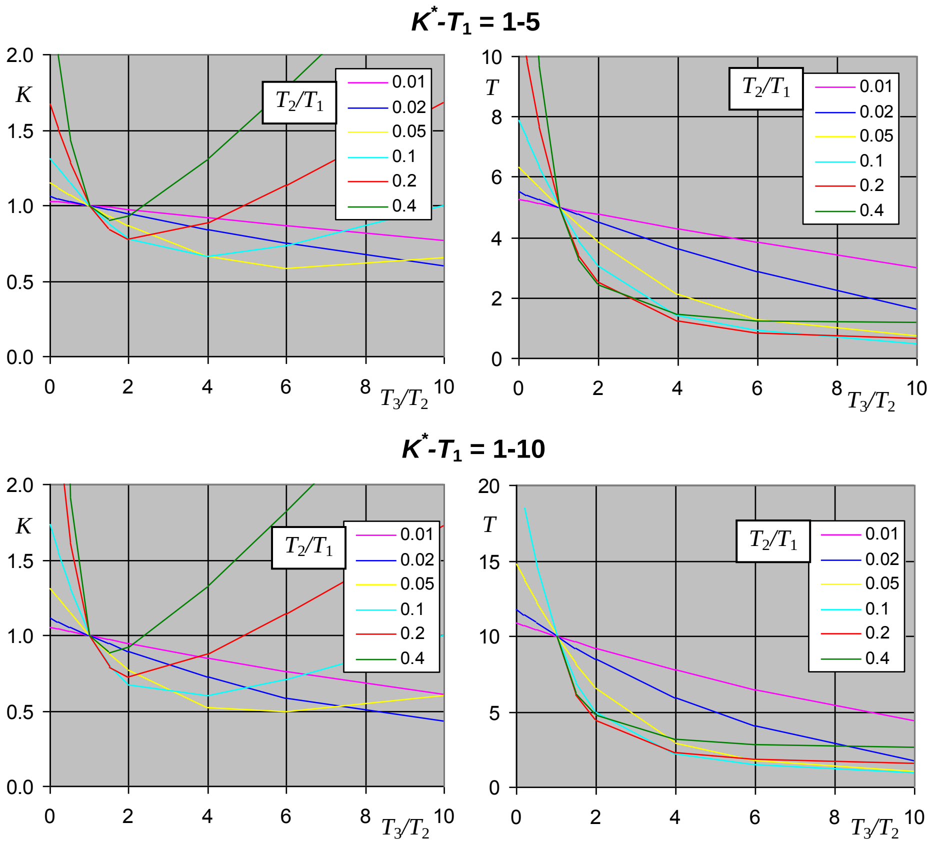


Figure 3. The $K$-$T$ values for unequal ($K^*$<$T_1$) 2nd-order input of 1-5 and 1-10. Source: Author.

Although both cases of $T_1$ in Fig. 2, 1 or 10, are equally subject to relatively high $T_3$ values, as inherent to the present simulation experimental setup, only for higher $T_1$ (or better, for higher $K^*$) one always gets the positive $T$ as of a stable ship. The latter condition just enables us to numerically simulate the zigzag with the first-order linear model. However, the negative $T$ combined with positive $K$ is surprisingly only specific to this case $K^*=T_1$. This is not observable within the other instances of $K^*$-$T_1$ as reported further. One can also notice that the case of Fig. 1 is rather interesting since it contains a 'whole spectrum' of behaviour for $K$ and $T$, in which they can vary differently for various $T_2/T_1$: stabilising and converging in $K$ and spreading in $T$ with increase of $T_3/T_2$, or vice versa.

The identified $K$ and $T$ are generally lower than the input $K^*$ and $T_1$ if $T_3/T_2>1$. This trends inverses for $T_3/T_2>1$. However, for $K^*$-$T_1$=1-1 one can practically read $K=K^*$ (=const).

Moreover, for $T_3/T_2<1$ one can get very high values (in absolute magnitude) of $K$ and $T$, which are not being seen in Figures 3 and 4. Examples of that are (for $K^*$-$T_1$=10-10):

$$T_2/T_1=0.05,\ T_3/T_2=0.2 \Rightarrow \{K,\ T\}=\{-388,\ -458\}$$
$$T_2/T_1=0.05,\ T_3/T_2=0.5 \Rightarrow \{K,\ T\}=\{+34,\ +38\}$$

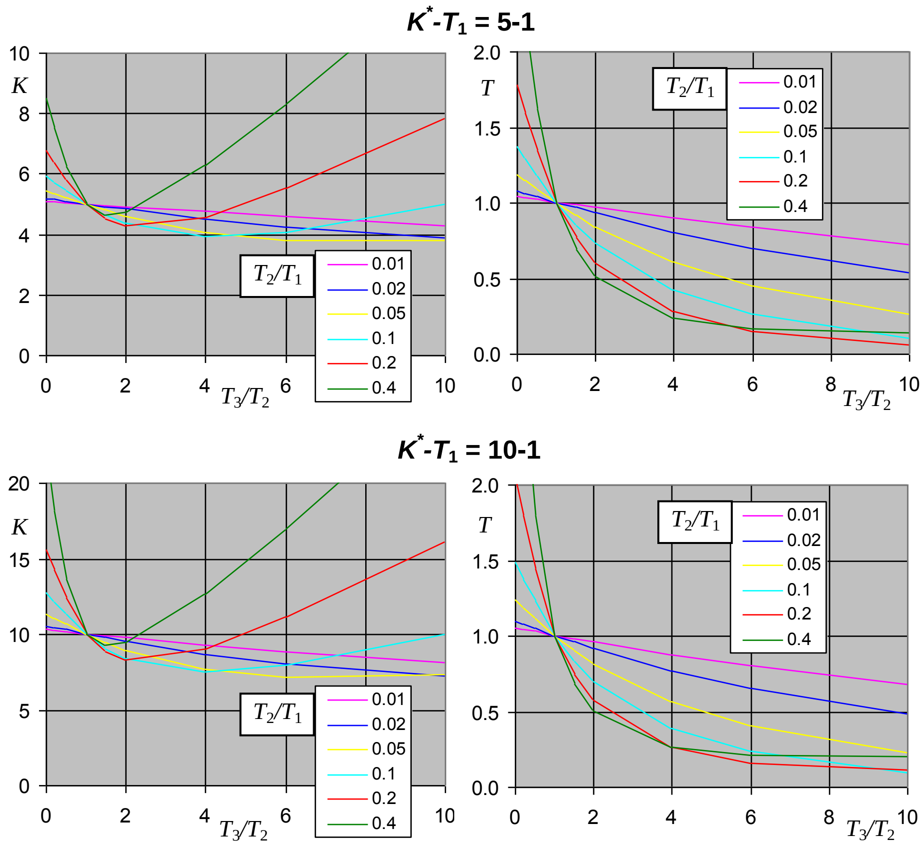


Figure 4. The $K$-$T$ values for unequal ($K^*>T_1$) 2nd-order input of 5-1 and 10-1. Source: Author.

However, they appear in the case of Fig. 5 and in the cases, not shown in the paper, corresponding to $K^{*}$-$T_1$={5-5; 5-15; 10-5}. However, for both negative $K$ and $T$ values, as is always herein the case, the zigzag test is still well and adequately simulated.

In Fig. 3 ($K^{*}/T_1$<<1, $K^{*}$=const) one achieves moderate spread both in $K$ and $T$, yet with clear first signs of stabilisation and convergence within $T$, as becoming much lower than $T_1$.

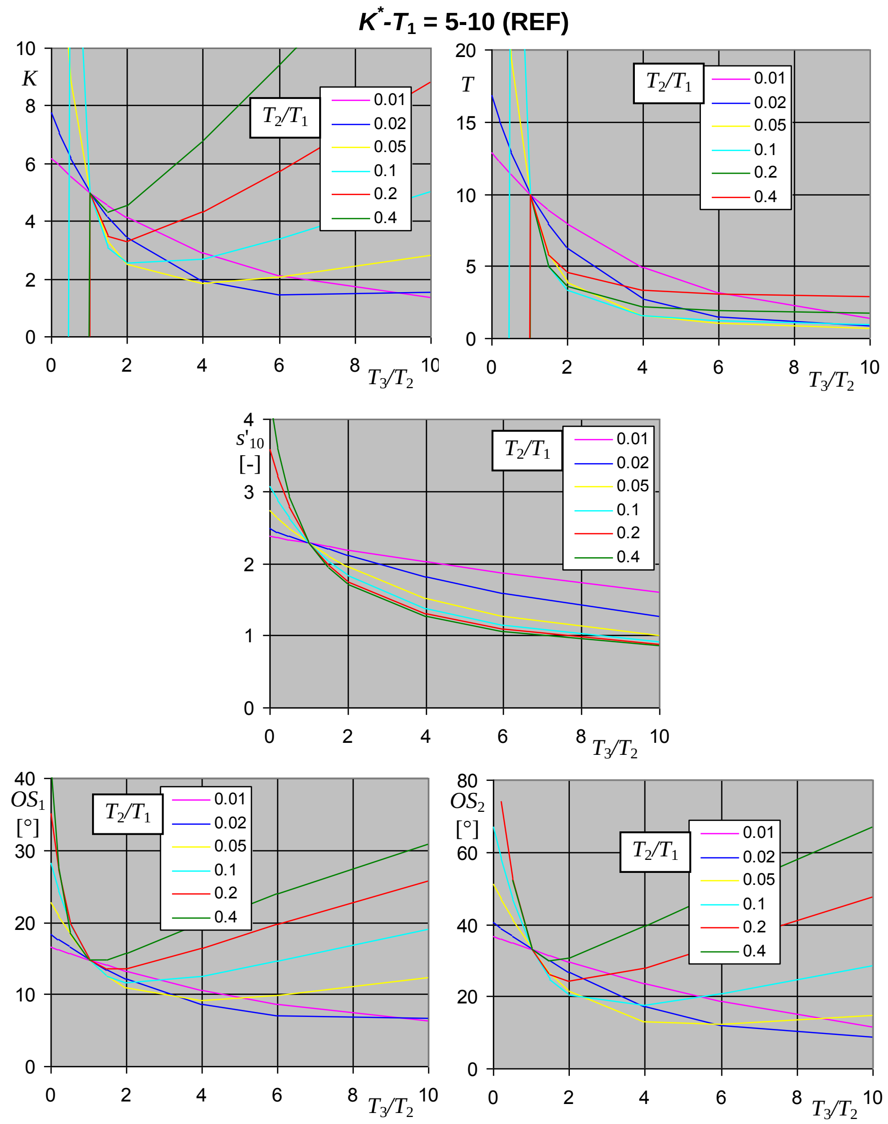


Figure 5. The $K$-$T$ values for reference case ($K^{*}$-$T_1$ = 5-10), with its zigzag parameters. Source: Author.

In Fig. 4 ($K^*/T_1$>>1, $T_1$=const), a good stabilisation/convergence within $K$ is actual, together with a wide spread for $T$.

In both Figs. 3 and 4, the top and bottom paired $K$-$T$ charts are similar in pattern to each other. Where appropriate, a scaling factor has to be only applied.

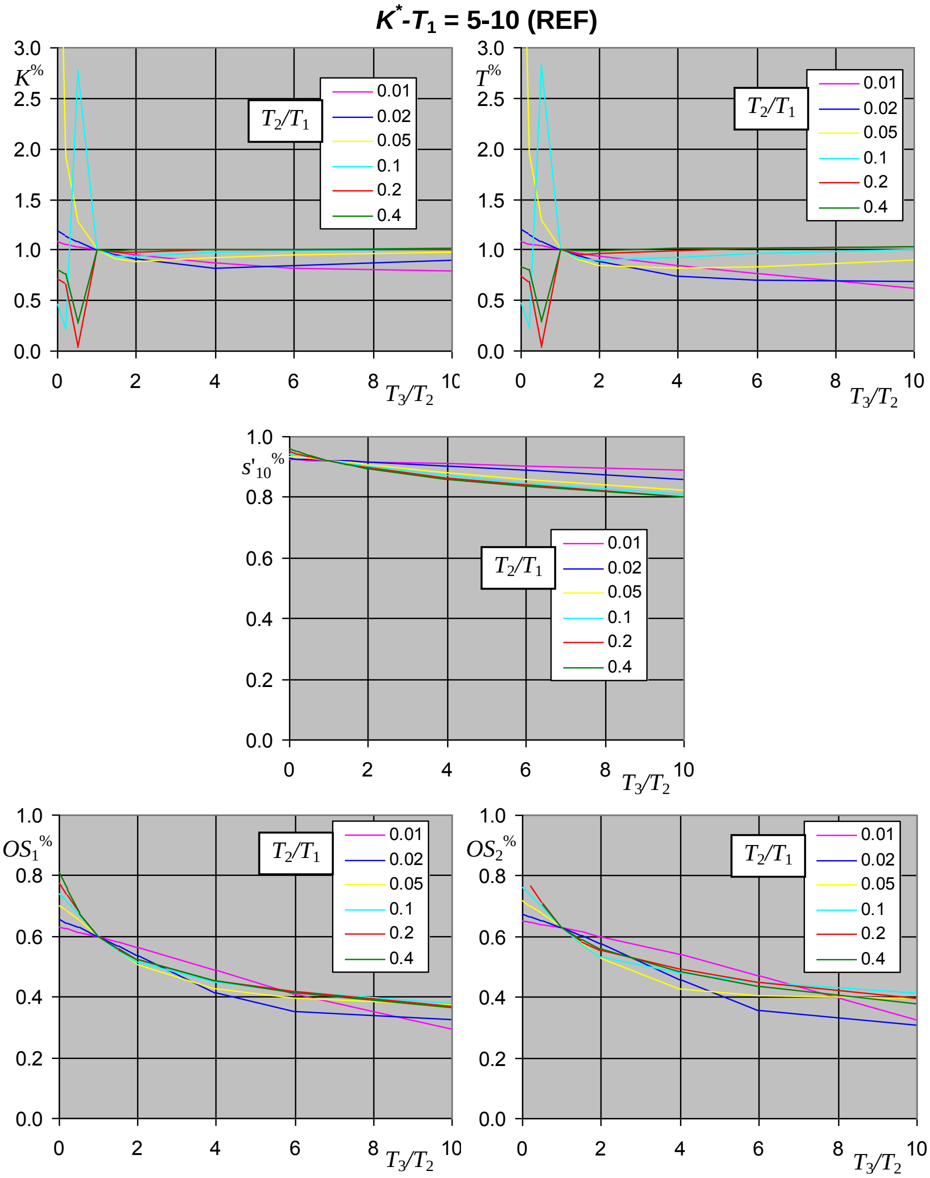


Figure 6. The relative $K$-$T$ values for reference case ($K^*$-$T_1$ = 5-10), with its zigzag parameters, for rudder speed 140°/L. Source: Author.

Fig. 5 provides more data on the second-order zigzag response undergoing the investigated transformation to first-order, however, only for the reference case. We can notice therein a drop of the 'initial' distance $s'_{10}$ with increasing $T_3/T_2$ or $T_2/T_1$ (conditioned here by $T_3/T_2>1$). Both the overshoots angles, $OS_1$ and $OS_2$, have a similar pattern to the chart of $K$. On the other hand, the image of $s'_{10}$ is more resembling, though not exactly, the plot of $T$.

Fig. 6 presents the change of particular quantities of Fig. 5, given here as ratio of the new to old (reference) value, when the rudder speed is increased more than 6 times, from 23°/$L$ to 140°/$L$. Such quantities are marked therefore with '%'. Although both the overshoot angles are largely decreased with $T_3/T_2$, by abt. 65%, with little effect from $T_2/T_1$, the rudder speed generally affects $s'_{10}$-distance to lesser degree, as well as the identified $K$ and $T$ constants.

## CONCLUSIONS

The first-order model has weaknesses related to determination of its parameters from hydrodynamic derivatives, if considerably transient manoeuvres are to be simulated. In the latter, such a model loses its adequacy, or exactly, in better words – the adequacy of its parameters. It sounds strange yet true. To rectify this problem, we shall use the first-order model only with its identifications based on actual motions. However, the model (its parameters) taken by identification, in the light of [Motora, Fujino, 1969] research or the present research on standard zigzag with various instances of second-order model, is of little usefulness. This is because the application of such an identified first-order model cannot be extended towards arbitrary manoeuvres as should be the case with (universal) models, and shall be limited to similar manoeuvres as those used in the identification. The observed 'redefinition' of $K^*$-$T_1$ (of second-order model) to arrive at $K$-$T$ (of first-order model, approximate one) is essentially the effect of weighing of all the four parameters of second-order model, $K^*$-$T_1$-$T_2$-$T_3$, based on actual manoeuvre. We may even say that the latter two parameters, $T_2$ and $T_3$, more or less modify the role played by the primary parameters, $K^*$ and $T_1$, if compared to the first-order model.

Therefore, the reported $K$-$T$ values elsewhere in literature shall be always supplemented with the input manoeuvre data. This is completely a new look and revision proposal for the first-order linear model practice. In same way, although easily obtainable, the *K-T* model may not be used for conversion (of its parameters) to second-order models, unless it is considered in parallel with the input manoeuvre. However, such a conversion also poses itself a really difficult task in that it can only provide some limiting relationships imposed upon the four parameters of second-order model, often not resolving their ambiguity. However, at the present stage of research, they cannot be provided analytically, even for a single zigzag manoeuvre. The charts presented in the paper render some assistance in this context. The earlier mentioned potential advantage of two or more different zigzag manoeuvres (in terms of rudder/heading), or other types of less nautically demanding manoeuvres, is going to be researched in the future.

Although the presented research is based on second-order linear model, the same or even much worse problems – related to vulnerability of *K-T* identification to input manoeuvre – would happen if a real, more or less nonlinear zigzag trial is to be processed.

A second-order model, besides also some its drawbacks, shall be thus a basis of scientific and engineering interest. This is partly seen in the widely investigated so-called nonlinear extensions of Nomoto models, also with zigzag tests, that is rarely seen in case of the first order model. However, there is little discussion in literature on the significance of all coefficients in such a nonlinear model, where the four parameters of the second-order linear model are playing a significant role in the overall performance of the model.

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**JAROSŁAW ARTYSZUK**
Maritime University of Szczecin
Wały Chrobrego 1-2, 70-500 Szczecin, Poland
e-mail: j.artyszuk@am.szczecin.pl

**STRESZCZENIE**

Artykuł zawiera systematyczne wyniki identyfikacji liniowego modelu Nomoto pierwszego rzędu na podstawie symulowanej próby wężowej różnych wariantów modelu drugiego rzędu. W toku analizy stwierdzono ogólną nieadekwatność (strukturalną) modelu pierwszego rzędu do symulacji dowolnych manewrów, w przeciwieństwie do modelu drugiego rzędu, nawet w granicach liniowości dynamiki statku. Wyniki identyfikacji silnie bowiem zależą od rozpatrywanego manewru.