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MANOEUVRING TO REQUIRED APPROACH PARAMETERS - DISTANCE AND TIME ON COURSE

ABSTRACT

The predicted object distance on course D_c and the time interval to its occurrence T_c are sometimes used additional criteria for collision threat. They are calculated in some ARPAs as BCD - the bow crossing distance and BCT - the bow crossing time. The scope of this paper is aimed at the problem which although it can be and it is connected with collision avoidance manoeuvres, but it is rather reversed and can be applied for intentional approaches or in naval tactical manoeuvres - what own speed and/or course manoeuvre should be undertaken to achieve the required distance on course D_c and /or time to this distance T_c?

ASSUMPTIONS AND INPUT PARAMETERS

The part "Assumptions and input parameters" is the same as in the previous paper: "Manoeuvring to required approach parameters - CPA distance and time".

For the purposes of this analysis, own vessel and extraneous objects of interest are regarded as if the mass of each object was concentrated at a point. It will be assumed that all moving external objects are travelling at constant speed and course. In the movable plane tangential to the Earth's surface Cartesian coordinates system Ox, Oy (Fig. 1) with Oy pointing North O is at the present position of own vessel. It will also be assumed that manual plots or the radar processing and tracking has yielded the present relative position of the extraneous object X, Y and components of its true V_{tx}, V_{ty} or relative V_{rx}, V_{ry} speed. The relationship of the own and the object speeds can be described by equations

$$V_{tx} = V_{rx} + V_x \qquad (1)$$

$$V_{ty} = V_{ry} + V_y \qquad (2)$$

where: V_x, V_y - own speed components,

$$V_x = V \sin \psi \tag{3}$$

$$V_y = V \cos \psi \tag{4}$$

$$V = \sqrt{V_x^2 + V_y^2} \tag{5}$$

where: ψ - own course (the angle measured clockwise from Oy to V).

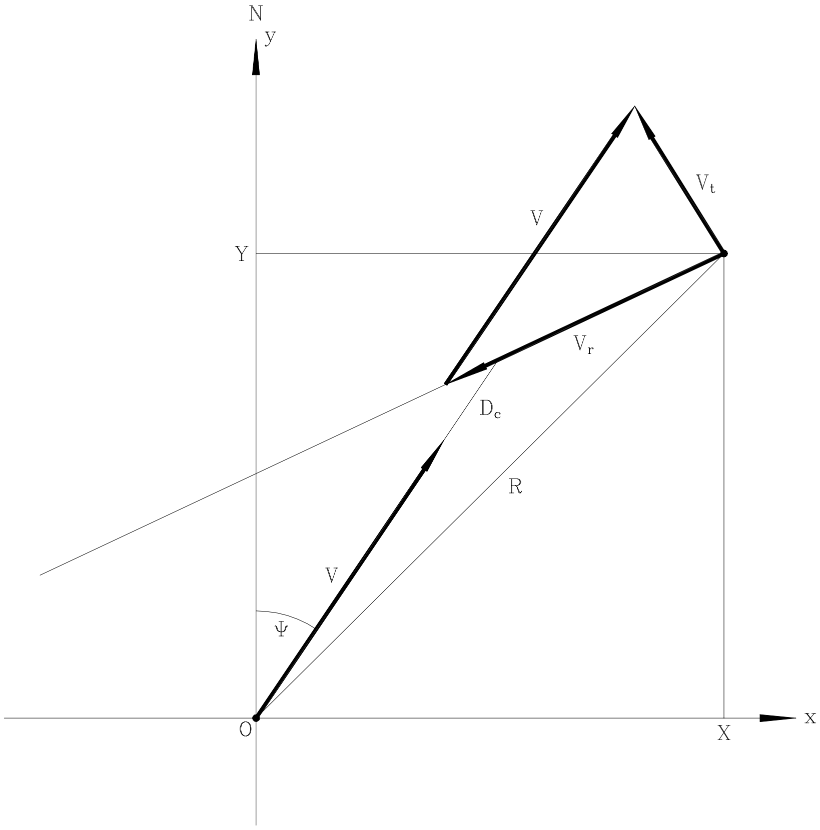

Fig. 1. Input parameters

The own and the object's motion parameters should be either ground or sea referenced and a drift angle is assumed to be zero. The relative position of an extraneous object, at time t, is given by:

$$X(t) = X + V_{rx}\, t \tag{6}$$

$$Y(t) = Y + V_{ry}\, t \tag{7}$$

and then [Lenart, 1986]

$$D_c = \frac{XV_{ry} - YV_{rx}}{V_{ry}\sin\psi - V_{rx}\cos\psi} \tag{8}$$

$$T_{Dc} = \frac{X\cos\psi - Y\sin\psi}{V_{ry}\sin\psi - V_{rx}\cos\psi} \tag{9}$$

$D_c>0$ means that the object will cross the course of own vessel ahead (i. e. D_c is the bow crossing distance) and if $D_c<0$ the object will cross the course astern.

DERIVATION OF EQUATION V = F(ψ, D$_C$)

From equations (1) through (4)

$$V_{rx} = V_{tx} - V\sin\psi \tag{10}$$

$$V_{ry} = V_{ty} - V\cos\psi \tag{11}$$

Substitution in equation (8) and rearranging yields

$$V = \frac{A_{Dc}V_{tx} - V_{ty}}{A_{Dc}\sin\psi - \cos\psi} \tag{12}$$

where:

$$A_{Dc} = \frac{Y - D_c\cos\psi}{X - D_c\sin\psi} \tag{13}$$

Equation (12) gives the speed V which own vessel must adopt to achieve the required distance on course D_c (in respect to the selected object) for different assumed own courses ψ, but we should search for solution

$$V \geq 0 \tag{14}$$

and ψ for which

$$T_c \geq 0 \tag{15}$$

Condition (15) means that the approach on course is at present or will be in the future and not in the past. Equation (15) (from equations (9), (10) and (11)) can be rearranged to the form

$$(X\cos\psi - Y\sin\psi)(V_{ty}\sin\psi - V_{tx}\cos\psi) \geq 0 \tag{16}$$

A conventional PPI displays the position of each object by plotting them in polar (r, ψ) or Cartesian (x, y) coordinates. If we apply a scaling factor τ to the speed coordinates (V, ψ) or (V_x, V_y) such that

$$r = V\,\tau \qquad (17)$$

$$x = V_x\,\tau \qquad (18)$$

$$y = V_y\,\tau \qquad (19)$$

then the position and speed coordinates can be plotted on a common display.

In the combined coordinates frame for plotting position and speed can also be plotted positions and speed vectors of objects and the own speed vector (real or simulated). Figure 2 illustrates a family of lines (12) for various required D_c and an exemplary object.

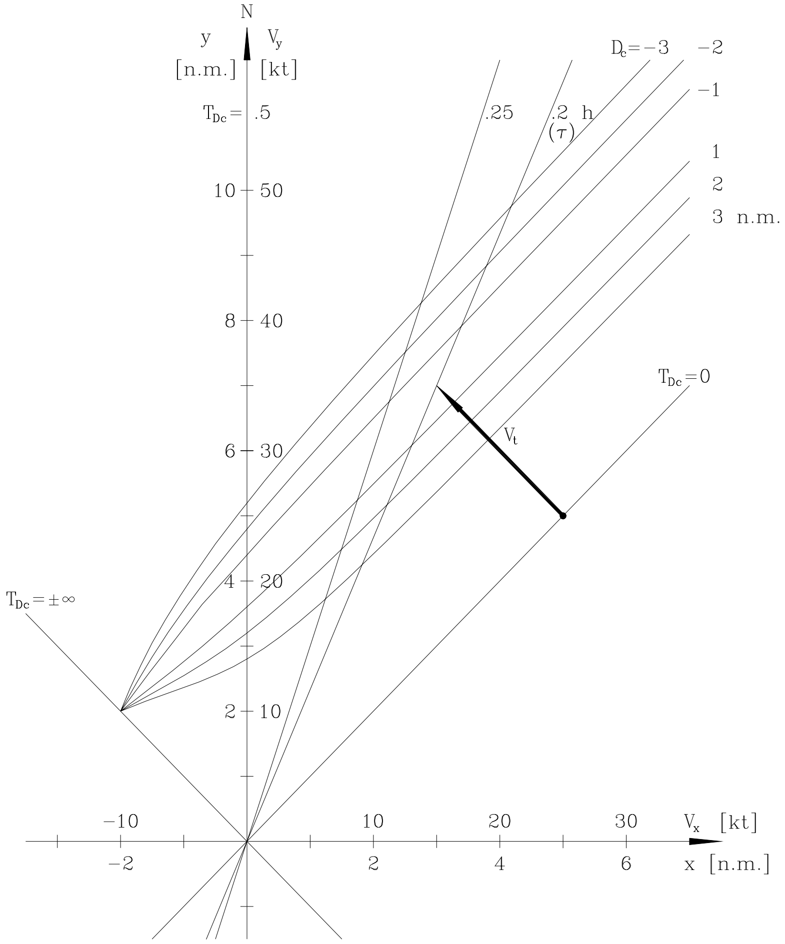

Fig. 2. Lines D_c=const. and T_{Dc}=const.
τ=0.2 h, X=Y=5 n.m., V_{tx}= -10 kt, V_{ty}=10 kt

DERIVATION OF EQUATION ψ= G(V, D_C)

Substitution in equation (12) equation (13) and rearranging yields

$$A_{VDc} \sin\psi - B_{VDc} \cos\psi + C_{VDc} = 0 \tag{20}$$

where:

$$A_{VDc} = YV - D_c V_{ty} \tag{21}$$

$$B_{VDc} = XV - D_c V_{tx} \tag{22}$$

$$C_{VDc} = XV_{ty} - YV_{tx} \tag{23}$$

If we search for own course (which will lead to the required distance on course D_c at an assumed own speed V then we can get an inverse function ψ=g(V, D_c) to the function V=f(ψ, D_c) by substitution in equation (20) trigonometric identities

$$\sin\psi = \frac{2\tan\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{24}$$

$$\cos\psi = \frac{1-\tan^2\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{25}$$

and after solving

$$\tan\frac{\psi}{2} = \frac{-A_{VDc} \pm \sqrt{A_{VDc}^2 + B_{VDc}^2 - C_{VDc}^2}}{B_{VDc} + C_{VDc}} \tag{26}$$

Real solutions exist if

$$A_{VDc}^2 + B_{VDc}^2 \geq C_{VDc}^2 \tag{27}$$

and equation (26) can give up to two own courses (which will lead to the required distance on course D_c at an assumed own speed V if they additionally fulfil condition (16).

DERIVATION OF EQUATION ψ= G(T_C)

Substitution in equation (9) equations (10) and (11) results in equation

$$T_{Dc} = \frac{-Y\sin\psi + X\cos\psi}{V_{ty}\sin\psi - V_{tx}\cos\psi} \tag{28}$$

This equation reveals that the time to distance on course T_{Dc} is independent of own speed V. Therefore from the above

$$\tan\psi = \frac{X + V_{tx}T_{Dc}}{Y + V_{ty}T_{Dc}} \tag{29}$$

and equation (42) gives own course (which will lead to the required time to distance on course T_{Dc}.

A graphical interpretation of solutions given by equation (29) can be obtained in Cartesian coordinates of own speed V_x, V_y since then

$$V_y = \frac{X + V_{tx}T_{Dc}}{Y + V_{ty}T_{Dc}} V_x \tag{30}$$

and the locus of points for which T_{Dc} is a constant is a straight line crossing the origin of coordinates. Figure 2 illustrates a family of lines (30) for various values of T_{Dc}.

EQUATIONS V, ψ= F(D_C, T_{DC})

If we search for own speed V and own course (which will lead to the required distance on course D_c at the required time to this distance T_{Dc}, then T_{Dc} by equation (29) determinates own course ψ, and D_c with (by equations (12) and (13) yields own speed V.

POSITION OF DISTANCE ON COURSE

When the object is on our course then the relative position of the object (in respect to our vessel) is (X_c, Y_c) or in polar coordinates (D_c, β_c) or (D_c, β'_c) where β_c and β'_c are true and relative bearings to the object at distance on course respectively. These parameters are given by equations

$$X_c = X + V_{rx} T_{Dc} = X + (V_{tx} - V \sin\psi)\, T_{Dc} \tag{31}$$

$$Y_c = Y + V_{ry} T_{Dc} = Y + (V_{ty} - V \cos\psi)\, T_{Dc} \tag{32}$$

$$D_c = \sqrt{X_c^2 + Y_c^2} \tag{33}$$

$$\beta'_c = 0° \qquad \text{and } \beta_c = \psi \qquad \text{if } D_c > 0 \tag{34}$$

$$\beta'_c = 180° \qquad \text{and } \beta_c = \psi + 180° \qquad \text{if } D_c < 0 \tag{35}$$

and D_c and T_{Dc} are either required or calculated, T_{Dc} from equation (28) and D_c from equation (33) or (8) transformed to true speeds by substitution equations (10) and (11)

$$D_c = \frac{XV_{ty} - YV_{tx} - (X\cos\psi - Y\sin\psi)V}{V_{ty}\sin\psi - V_{tx}\cos\psi} \tag{36}$$

TIME TO MANOEUVRE

It has to be emphasized that the calculated above manoeuvres are kinematic and should be undertaken immediately. If we require to have the time lapse Δt for calculations, for the decision to initiate a manoeuvre and for the execution of the calculated manoeuvre then X, Y in the above equations should be replaced by $X_{\Delta t}$, $Y_{\Delta t}$ respectively, given by equations:

$$X_{\Delta t} = X + V_{rx}\Delta t \tag{37}$$

$$Y_{\Delta t} = Y + V_{ry}\Delta t \tag{38}$$

REFERENCES

1. Lenart A. S. Some selected problems in analysis and synthesis of shipboard collision avoidance systems. Zeszyty Naukowe Politechniki Gdańskiej Nr 405 - Budownictwo Okrętowe Nr XLIV, Gdańsk 1986 (in Polish).