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## STUDY OF DIFFERENTIAL CODE GPS/GLONASS POSITIONING

### ABSTRACT

This paper presents the essential issues and problems associated with GNSS (Global Navigation Satellite System) code differential positioning simultaneously using observations from at least two independent satellite navigation systems. To this end, two satellite navigation systems were selected: GPS (Global Positioning System, USA) and GLONASS (GLObalnaya NAvigatsionnaya Sputnikovaya Sistema, Russia). The major limitations and methods of their elimination are described, as well as the basic advantages and benefits resulting from the application of the DGNSS (Differential GNSS) positioning method. Theoretical considerations were verified with the post-processed observations gathered during a six-hour measurement. The data from selected reference stations of the ASG-EUPOS (Active Geodetic Network — EUPOS) system located at different distances from the rover site was used. The study showed that the DGNSS positioning method achieves higher accuracy and precision, and improves the stability of coordinate determination in the time domain, compared to positioning which uses only one satellite navigation system. However, it was shown that its navigational application requires further studies, especially for long distances from the reference station.

### **Keywords:**

pseudorange, differential positioning, GPS, GLONASS, GNSS.

## INTRODUCTION

Differential positioning methods based on phase measurements are currently widely used in geodesy. They achieve the high accuracy often required across a wide range of applications in various fields. In contrast, differential code measurement is a subject which is much more rarely taken up considering the comparatively low (decimetre) accuracy. Despite this, the technology of DGPS (Differential GPS) measurements has successfully been applied in navigation [Vu A. et al., 2012], Geographical Information Systems (GIS) [Popielarczyk D., Templin T., 2013] and Location Based Services (LBS) [Jung S. et al., 2013] due to its relatively simple application, low device costs, as well as the fact that it is a method which meets the high performance requirements for integrity, continuity, availability and accuracy of coordinate determination. It was shown that in a short observation time code network DGPS positioning results can give even centimetre accuracy and can be more reliable than static relative phase positioning where gross errors often happen [Bakuła M., 2007].

Following the definition given in a document published by the United Nations [1998]: ‘The Global Navigation Satellite System (GNSS) is a space-based radio positioning system that includes one or more satellite constellations, augmented as necessary to support the intended operation, and that provides 24-hour three-dimensional position, velocity and time information to suitably equipped users anywhere on, or near, the surface of Earth (and sometimes off the Earth)’. Two core elements of this system indicated in 1998 were the Global Positioning System (GPS) operated by the United States of America and the Global Navigation Satellite System (GLONASS) operated by the Russian Federation. They were designed in the 1970s for military purposes. Therefore, problems connected with interoperability were not taken into account in the design of both systems. This topic at the present time seems to be one of the most important for researchers all over the world [Cai Ch., Gao Y., 2013; Li P., Zhang X., 2014; Montenbruck O. et al., 2014; Torre A. D., Caporali A., 2014]. Some of the interoperability issues were defined by Zinoviev [2005] and Januszewski [2011]. This paper deals with problems in user position determination using combined GPS/GLONASS observations and presents the background and results of code differential GPS (DGPS), GLONASS (DGLONASS) and GPS/GLONASS (DGNSS) positioning.

## THEORETICAL FOUNDATIONS

In the process of user position determination via satellite methods, one should first compute satellite vehicle (SV) coordinates. Unfortunately, unlike GPS

[GPS SPS Signal Specification, 1995], GLONASS does not use close analytical formulae for computing SV position. Instead, GLONASS uses a state vector referenced to a given epoch ( $t_b$ ). For computing SV coordinates at the moment  $t$  numerical integration has to be performed using 4th order Runge-Kutta method [GLONASS ICD, 2008], over time interval ( $t-t_b$ ). In the literature, however, analytical methods of solving this problem can be found [Góral W., Skorupa B., 2012]. Due to the different reference systems used in GPS (WGS 84 — World Geodetic System 1984) and GLONASS (PZ-90.02 — revised version of PZ-90 — rus. *Parametry Zemli*, 1990) (table 1), before calculating the pseudorange corrections and further user position, the transformation of satellite coordinates to a uniform reference system should be performed.

In general, the coordinates of the GLONASS SVs are transformed to the WGS 84 system, mainly due to its high accuracy and stability with regard to the ITRS (International Terrestrial Reference System). The conversion is performed by tri-dimensional similarity transformation [Boucher C., Altamimi Z., 2001]:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{WGS\ 84} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{PZ-90.02} + \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix} + \begin{pmatrix} D & -R3 & R2 \\ R3 & D & -R1 \\ -R2 & R1 & D \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{PZ-90.02}, \quad (1)$$

where:

$T1, T2, T3$  — translation parameters;

$D$  — scale factor;

$R1, R2, R3$  — rotation angels.

Many publications have dealt with the determination of the transformation parameters. They differ by methods used, as well as the site location: global [FANA, 2009; IGEX-98, 1999] or regional [Bazlov Y. A. et al., 1999; Roßbach U. et al., 1996].

Knowing the SVs coordinates in a uniform WGS 84 reference system, it is possible to calculate a pseudorange correction ( $PRC$ ), which for a satellite  $s$  consists of the two elements:

$$PRC^S(t) = PRC^S(t_0) + RRC^S(t_0)(t - t_0), \quad (2)$$

where:

$PRC^S(t_0)$  — the pseudorange correction for satellite  $s$  at reference epoch  $t_0$ ;

$RRC^S(t_0)$  — the range rate correction;

$t$  — an epoch;

$t - t_0$  — the latency.

Table 1. Parameters defining the WGS 84 and PZ-90.02 systems

Parameter	WGS 84	PZ-90.02
Semi-major axis ( $a$ )	6378137.0 m	6378136.0 m
Flattening ( $f$ )	1 / 298.257223563	1 / 298.25784
Angular velocity of the Earth ( $\omega$ )	7292115*10 <sup>-11</sup> rad/s	7292115*10 <sup>-11</sup> rad/s
Earth's Gravitational Constant ( $GM$ )	3986004.418*10 <sup>8</sup> m <sup>3</sup> /s <sup>2</sup>	3986004.418*10 <sup>8</sup> m <sup>3</sup> /s <sup>2</sup>

Equation 2 refers to a real time positioning. In the post-processed calculations, latency does not occur. It amounts to 0 seconds and eliminates the  $RRC^S$  element for the equation.

Therefore, in further equations, for the clarity of the record, the epoch marking was aborted. Following Hoffman-Wellenhof et al. [2008], the generalized equation of the measured pseudorange is as follows:

$$R_{REF}^S = \varrho_{REF}^S + \Delta\varrho_{REF}^S + \Delta\varrho^S + \Delta\varrho_{REF}, \quad (3)$$

where:

$\Delta\varrho_{REF}^S$  — the range biases depending on the terrestrial base position and satellite position as well;

$\Delta\varrho^S$  — the satellite-dependent range bias;

$\Delta\varrho_{REF}$  — the receiver-dependent range bias.

The  $PRC$  for the reference station is calculated as the difference:

$$PRC_{REF}^S = \varrho_{REF}^S - R_{REF}^S, \quad (4)$$

where:

$\varrho_{REF}^S$  — the geometric range obtained from the known position of the reference station and the broadcast ephemerides;

$R_{REF}^S$  — the measured quantity.

Substituting (3) into (4) gives:

$$PRC_{REF}^S = -\Delta\varrho_{REF}^S - \Delta\varrho^S - \Delta\varrho_{REF}. \quad (5)$$

Adding the  $PRC_{REF}^S$  value to the pseudorange measured by the rover receiver ( $ROV$ ) results:

$$R_{ROV_{corr}}^S = \varrho_{ROV}^S + \Delta\varrho_{ROV}^S + \Delta\varrho^S + \Delta\varrho_{ROV} + (-\Delta\varrho_{REF}^S - \Delta\varrho^S - \Delta\varrho_{REF}). \quad (6)$$

Re-arranging (6) and additionally using the shorthand notations leads to:

$$R_{ROV_{corr}}^S = \varrho_{ROV}^S + \Delta\varrho_{RR}^S + \Delta\varrho_{RR}, \quad (7)$$

where:

$$\begin{aligned} \Delta\varrho_{RR}^S &= \Delta\varrho_{ROV}^S - \Delta\varrho_{REF}^S && \text{— the distance-dependent bias;} \\ \Delta\varrho_{RR} &= \Delta\varrho_{ROV} - \Delta\varrho_{REF} && \text{— the receiver-specific bias.} \end{aligned}$$

Please note that the technology of code differential positioning eliminates systematic errors associated with satellites  $\Delta\varrho^S$  [Hofmann-Wellenhof B. et al., 2008]. Some errors are correlated over a certain area and they are called the distance-dependent errors  $\Delta\varrho_{RR}^S$ , although the differential technique eliminates this group of errors [Seeber G., 2003]. The  $\Delta\varrho_{RR}$  error is mainly associated with the multipath effect and the receiver noise. These errors are uncorrelated at the reference and user receiver, and cannot be corrected using DGNSS. In fact, a user inherits the errors incurred at the reference station [Monteiro L. S. et al., 2005]. Therefore, it is important to minimize these errors by careful siting and equipment selection at both the reference and user stations [Misra P., Enge P., 2011]. The effect of multipath and receiver noise on code differential positioning was discussed extensively by Shuxin et al. [2002].

Since the discussed satellite navigation systems (GPS and GLONASS) use different time systems, all observations should be expressed in a uniform time system. Both GPS System Time (GPST) and GLONASS System Time (GLONASSST) are real versions of the various UTC (Coordinated Universal Time) laboratories they reflect [Januszewski J., 2010]. Since the offset between time systems [GPST — GLONASSST] changes slowly, thus it can be considered constant over short enough time intervals, or it can be predicted [Zinoviev A. E., 2005]. Furthermore, time difference remains stable within one day [Cai Ch., Gao Y., 2009]. Nevertheless, when using GPS and GLONASS code measurements jointly, the difference in system times has to be taken into account. This can be done in several ways. One way to obtain the value of time offset is by downloading the BIPM (fr. Bureau International des Poids et Mesures) Circular T data document [BIPM, 2013] and calculating the [GPST — GLONASSST] value. This data is not available in real-time and its global

uncertainty is in the order of hundreds of nanoseconds. Another source of [GPST — GLONASS] time offset is the navigational message broadcast by the GLONASS SV as the word  $\tau_{GPS}$ . It was first broadcast by the GLONASS-M SV generation (first launched in 2003). The difference between the GPS and GLONASS time is emitted with a specified maximum deviation of 30 ns [Hoffman-Wellenhof B. et al., 2008]. The time offset between any satellite systems can also be calculated by ‘spending’ an observation from one SV [Januszewski J., 2010]. The estimated time offset have an accuracy of more than 10 ns [Cai Ch., Gao Y., 2009]. This leads to the requirement to have a minimum of five GPS + GLONASS SVs for computing a solution. However, there does not seem to be a problem with the existence of two full SVs constellations, even when obstructions are present. Some authors introduced so called quasi-observable [Cai Ch., Gao Y., 2009] or pseudo-measurement [Angrisano A. et al., 2013] to compensate the sacrifice.

Finally, to achieve three-dimensional coordinates a non-linear set of equations should be formulated and solved by one of the known techniques: closed form solutions [Bancroft S., 1985; Kleusberg A., 2003], or iterative solutions based on either linearization (Least Squares Solution) [Leick A., 2004] or on Kalman filtering [Strang G., Borre, 1997 K.; Welch G., and Bishop G., 2006].

## MEASUREMENT AND DATA ANALYSIS

The GNSS measurements were conducted in Olsztyn on 31st July 2013, between 8:00 a.m. and 2:00 p.m. local time and the results were analysed. The data was gathered using a TOPCON HiPer pro receiver, recording code and phase observations for two satellite navigation systems (GPS and GLONASS) simultaneously. The measurements were taken in an unobstructed area. Post-processed calculations were performed using data downloaded from the ASG-EUPOS system [Bosy J. et al., 2008]. Three (whenever possible) evenly-distributed stations were used, located to the south and west of the rover site (fig. 1).

The achieved results, based on the nearby LAMA station (approx. 21 km in a straight line from the rover site), were used as comparative values for results achieved using distant stations (171–411 km). Additionally, in order to show the influence of the multipath and receiver noise in the final result, the KROL station was used for calculations and comparisons. It was located less than 140 metres from the rover site. For the transparency of the drawing, KROL was not placed in figure 1.

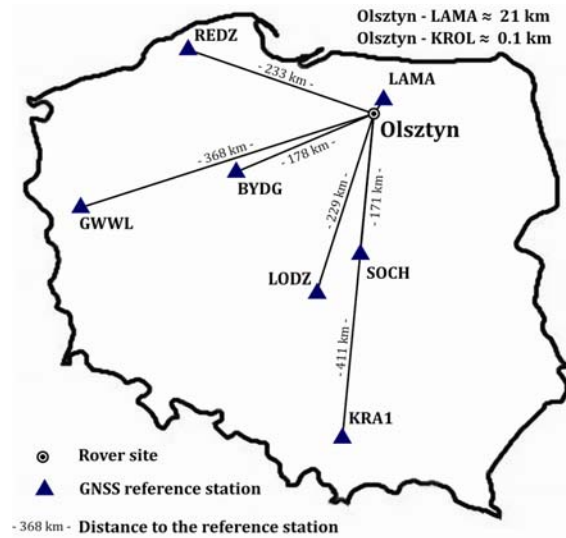


Fig. 1. Location of the reference stations

The calculations were made by RTKLib v 2.4.2, using code C/A observations (L1 frequency) of the SVs located at least 15° above the horizon. Using each of the GNSS reference stations, the following solutions were obtained: DGPS, DGLONASS and DGNSS (GPS + GLONASS). Reference coordinates (*REF*) were calculated utilizing phase observations, recorded over a six hour measurement period. All results were provided in the uniform WGS 84 reference system and the GPS time system.

For analysis of the three-dimensional positions obtained in each solution (DGPS, DGLONASS and DGNSS) a standard deviation ( $\sigma$ ) (characterizing the measurement precision) and a value of the mean error (*RMS*) (referring to the reference coordinates) according to the following formulas were calculated:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (8)$$

and

$$RMS = \sqrt{\frac{\sum_{i=1}^n (x_i - x_{REF})^2}{n}} \quad (9)$$

where:

- $\bar{x}$  — a mean value of the data set;
- $x_{REF}$  — a reference value;
- $x_i$  — *i*-th element of the data set;
- $N$  — a number of measurements.

Additionally, using TEQC (Estey and Merteens, 1999), the mean value of the *MPI* coefficient was calculated for every station. *MPI* is a linear combination of pseudorange (only L1 frequency) and phase measurements (L1 and L2 frequencies), and reflects the effect of a multipath plus receiver noise on GNSS measurements [Rocken C. et al., 1995]. The detailed formulas used in TEQC software and their derivation along with the description were shown by Leick [2004] and Rocken et al. [1995]. In table 2:  $\sigma$ , *RMS* and mean *MPI* coefficient value were shown with the receiver and antenna model used on a given reference station. The mean *MPI* coefficient value for the receiver at the rover site was 0.22 m.

Analysing the results presented in table 2 an improvement in the precision and accuracy of DGNSS positioning compared to differential GPS positioning is observed. The accuracy of the obtained coordinates was improved by the 20% for the horizontal component and by 28% for the vertical component. The worst, even metre accuracy was obtained with DGLONASS method. Comparing the results from the nearest stations: KROL (less than 140 m) and LAMA (ca. 21 km), the effect of multipath plus receiver noise (mean *MPI* value) on the final measurement results is observable. Coordinates obtained utilizing the data from the nearby station (KROL) are characterized by much lower precision and accuracy. The group of six reference stations located to the west and the south from the measurement site was equipped with the same receivers and similar antenna models. Described situation has reflected in the *MPI* coefficient. These values oscillated between 0.35 and 0.39 m for the first five stations. The slightly higher coefficient value for the KRA1 station (0.44 m) can correlate with a lack of a radome on the antenna or with high cranes situated nearby, which could affect the multipath effect.

A parameter  $\Delta = RMS - \sigma$  was introduced. It assumes only positive values, whereas the case  $\Delta > 0.00$  m should be interpreted as the occurrence of a systematic error. Please note that it is evidence of a phenomenon's occurrence rather than its quantity. The values of  $\Delta$  parameters obtained from every station and for individual solutions are presented in table 3.

For the stations located to the west (BYDG, REDZ, GWWL) from the measurement site, the systematic error of the eastern component increases along with the increase in distance. An analogous situation appears for the group from the 'southern' stations (SOCH, LODZ, KRA1). Apart from the accuracy and precision improvement, the differential code GNSS positioning also had improved stability of the coordinate determination in the time domain (fig. 2).



Table 2. The collation of the DGPS, DGLONASS and DGNSS code positioning results

Station name	Solution	X		Y		h		Mean MPI [m]	Receiver / antenna model
		$\sigma$ / RMS [m]		$\sigma$ / RMS [m]		$\sigma$ / RMS [m]			
KROL	DGPS	0.73 / 0.73		0.44 / 0.44		1.19 / 1.19		0.75	JAVAD TRE_G3TH SIGMA / JAV_GRANT- G3T JAVC
	DGLONASS	1.00 / 1.00		0.71 / 0.81		2.08 / 3.21			
	DGNSS	0.48 / 0.48		0.32 / 0.33		1.03 / 1.06			
LAMA	DGPS	0.49 / 0.49		0.30 / 0.30		0.78 / 0.78		0.09	Leica GRX1200GG+G NSS / Leica L1/L2 Choke Ring
	DGLONASS	0.49 / 0.49		0.40 / 0.48		0.93 / 1.62			
	DGNSS	0.36 / 0.36		0.22 / 0.22		0.50 / 0.52			
BYDG	DGPS	0.60 / 0.61		0.36 / 0.41		1.02 / 1.03		0.37	
	DGLONASS	0.97 / 0.98		0.62 / 0.81		1.43 / 2.73			
	DGNSS	0.44 / 0.45		0.26 / .039		0.77 / 0.77			
REDZ	DGPS	0.64 / 0.64		0.38 / 0.46		1.09 / 1.10		0.37	
	DGLONASS	0.98 / 0.98		0.59 / 0.86		1.45 / 2.91			
	DGNSS	0.47 / 0.47		0.28 / 0.46		0.81 / 0.81			
GWWL	DGPS	0.64 / 0.66		0.37 / 0.55		1.11 / 1.15		0.39	Trimble NetR5 / Trimble Zephyr GNSS Geodetic II w/Radome (TRM55971.00 TZGD or TRM57971.00 TZGD*) and Trimble Zephyr GNSS Geodetic II** (TRM57971.00 NONE)
	DGLONASS	1.12 / 1.14		0.71 / 1.00		2.03 / 3.21			
	DGNSS	0.47 / 0.50		0.29 / 0.57		0.79 / 0.79			
SOCH*	DGPS	0.64 / 0.65		0.36 / 0.37		1.03 / 1.06		0.39	
	DGLONASS	1.20 / 1.20		0.73 / 0.88		2.06 / 3.36			
	DGNSS	0.45 / 0.46		0.28 / 0.31		0.84 / 0.85			
LODZ	DGPS	0.63 / 0.67		0.38 / 0.40		1.06 / 1.09		0.35	
	DGLONASS	1.00 / 1.06		0.69 / 0.75		2.21 / 3.27			
	DGNSS	0.44 / 0.52		0.28 / 0.33		0.77 / 0.77			
KRA1**	DGPS	0.69 / 0.81		0.42 / 0.42		1.17 / 1.21		0.44	
	DGLONASS	1.05 / 1.14		0.63 / 0.72		2.47 / 3.14			
	DGNSS	0.50 / 0.65		0.29 / 0.32		0.83 / 0.84			

Table 3. The collation of  $\Delta$  parameter values

Station group	Station name	Solution								
		DGPS			DGLONASS			DGNSS		
		$\Delta_x$ [m]	$\Delta_y$ [m]	$\Delta_h$ [m]	$\Delta_x$ [m]	$\Delta_y$ [m]	$\Delta_h$ [m]	$\Delta_x$ [m]	$\Delta_y$ [m]	$\Delta_h$ [m]
Reference	KROL	0.00	0.00	0.00	0.00	0.10	1.13	0.00	0.01	0.03
	LAMA	0.00	0.00	0.01	0.00	0.07	0.69	0.00	0.01	0.02
Western	BYDG	0.00	0.05	0.02	0.01	0.19	1.30	0.01	0.13	0.00
	REDZ	0.01	0.09	0.02	0.00	0.27	1.46	0.01	0.18	0.00
	GWWL	0.01	0.18	0.04	0.02	0.30	1.18	0.03	0.28	0.00
Southern	SOCH	0.01	0.00	0.02	0.00	0.15	1.30	0.01	0.03	0.01
	LODZ	0.04	0.02	0.03	0.06	0.06	1.07	0.08	0.05	0.00
	KRA1	0.13	0.00	0.04	0.09	0.09	0.68	0.16	0.03	0.01

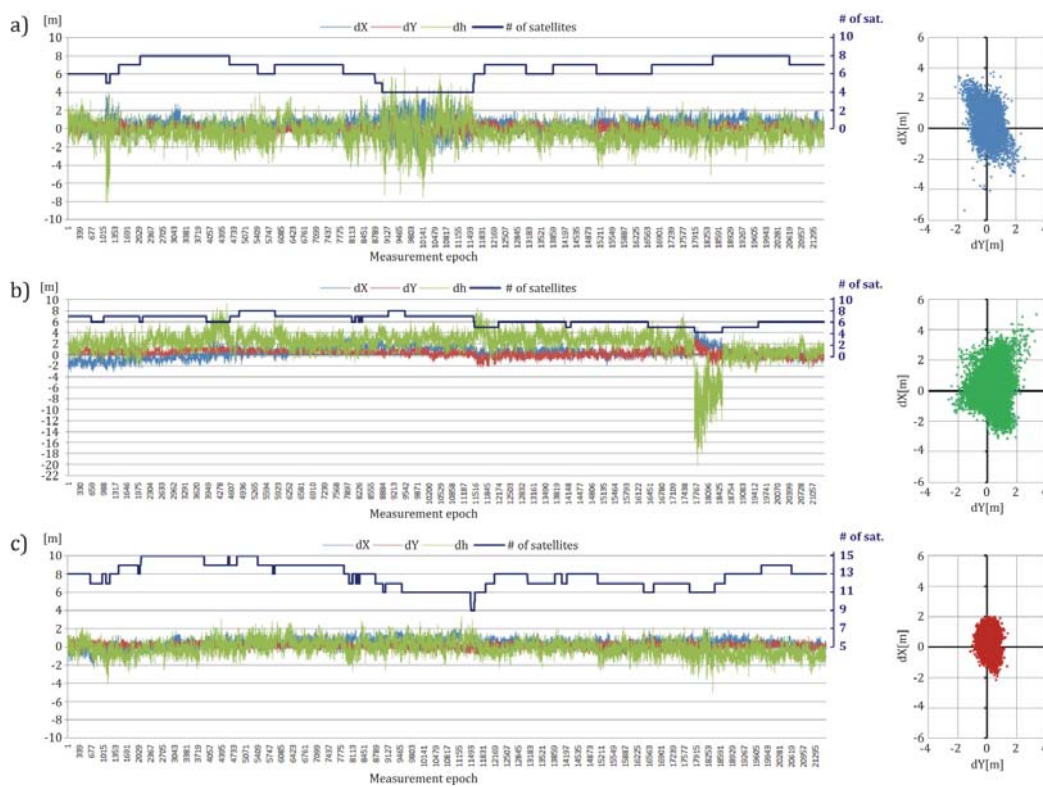


Fig. 2. The position distribution in the time domain (left) and on the plane (right) of DGPS (a), DGLONASS (b) and DGNSS (c) positioning for the KRA1 station

The improved stability of coordinate determination with the DGNSS method is mainly related to the significant number of observed SVs (9-16). In the case of the DGPS positioning, 4 to 8 satellites were tracked in the course of measurement. A similar situation occurred for DGLONASS positioning, although with one difference: between 11:02:31 and 11:06:18 of the local time, a gap occurred in the continuity of the position determination due to an insufficient number of SVs. This break was not taken into account in figure 2b. Increased number of SVs is especially important when high obstructions are present and GPS accuracy is degraded. While analysing the planar position distribution (fig. 2) for individual solutions obtained relative to the most distant station (KRA1), it is observable that the deviations from reference coordinates for the DGNSS positioning do not exceed 2.12 m for the northern component, 1.39 m for the eastern component and 5.02 m for the height determination. In the case of the DGPS and DGLONASS positioning, the deviations are much greater and reach 4 metres or more for horizontal components and 8 metres or more for height. Presented results of the DGPS measurements coincide with the error growth equation introduced by Monteiro et al. [2005] i.e. the DGPS error [95%] is equal 0.5 m to 1.0 m near the reference station plus 0.4 m for each 100 nm (1 nautical mile = 1.852 km) distance from the reference station. Specht [2011] showed that results based on long-term observations exhibit even greater accuracy. Other augmenting systems like EGNOS (European Geostationary Navigation Overlay System) show slightly worse accuracy (horizontal accuracy for 95% level and for one hour periods do not exceed 2.3 m) [Felski A., Nowak A., 2013]. However, test results for the EGNOS positioning system during the period of solar maximum activity revealed that its performance is sometimes worse than in the autonomous positioning mode [Grzegorzewski M. et al., 2012].

The entire six-hour measurement material was equally divided into ten one-minute sessions. Final result was calculated as an average value of each session. It was aimed at examining the described solution in short time spans and its practical use in different applications, e.g. in navigation or GIS systems. In figure 3, RMS errors obtained in the course of short sessions for the GWWL station are presented (the station located the most to the west). Additionally, the average value of the RMS error was given in brackets for individual components from ten one-minute sessions. The horizontal lines emphasize the values of RMS errors obtained from the entire six-hour session.

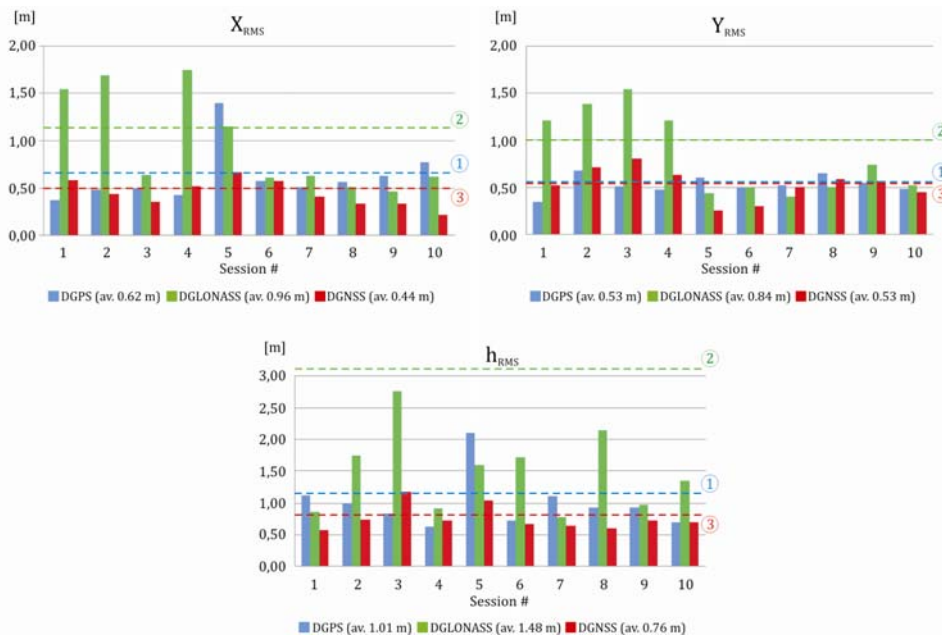


Fig. 3. Errors obtained in the course of short sessions along with horizontal lines determining RMS errors of the DGPS (1), DGLONASS (2) and DGNSS (3) positioning from the entire measurement session for the GWWL station

The average RMS error from short sessions is at the level of the results given in the table 2. The only significant improvement of accuracy is noticeable in case of determining the vertical component in the DGLONASS solution. It is caused by a gross error which occurred at the end of the six-hour session (fig. 2b) and was placed in none of the short sessions. Most of the RMS errors from one-minute sessions are located within the limits of values obtained in the course of the entire measurement, or they are located below. In this case, the most accurate was DGNSS positioning, particularly for determining the high component, whereas DGLONASS positioning was the least accurate. Only in one case the DGPS method was less accurate and precise than DGLONASS (fig. 4).

In the course of session No. 5, the number of the GPS SVs amounted only to 4, which was automatically transferred into the position determination quality. During the short sessions, especially at long distances from the reference station, the occurrence of systematic errors in the determined positions is noticeable. It caused a shift of the measurement results with regard to the reference coordinates (fig. 4). It is an important aspect in the context of the practical application of presented solution. This disadvantage may be eliminated by applying a network solution for pseudorange correction determination [Bakuła M., 2010].

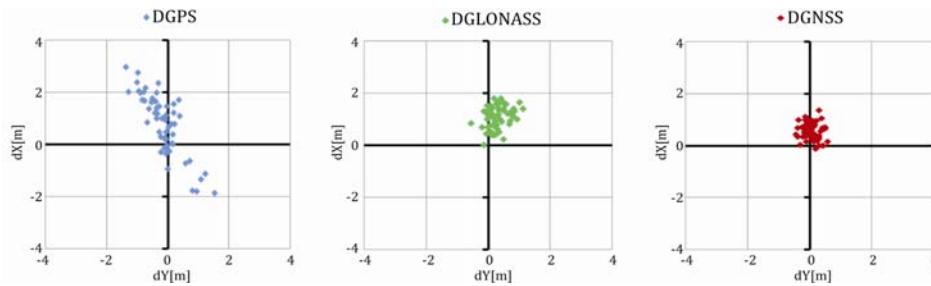


Fig. 4. Examples of scatter plots for the 5th one-minute session for the GWWL station

## CONCLUSIONS

This paper presents the essential issues and problems associated with DGNSS code positioning, i.e. the positioning which simultaneously uses observations from at least two independent satellite navigation systems. The measurement results, conducted to confirm the theoretical considerations, showed that the DGNSS positioning largely improves the accuracy of determining coordinates towards the DGPS positioning (ca. 20–28%). Deviations from the reference position did not exceed 2.12 m for the northern component, 1.39 m for the eastern component and 5.02 m for the height determination for long distances from a reference station. The DGNSS positioning also increases the stability of coordinate determination in the time domain — over six hours, no measurement fluctuations were observed. The method presented in the paper, during short sessions (one-minute long), was confirmed to be the most effective. Nevertheless, there are still systematic errors in the determined coordinates, especially for long distances from the reference station.

A lack of fluctuations in coordinate determination process, ensured positioning continuity and a redundancy of systems in the process of estimating user position are essential issues in the context of the code DGNSS positioning application for precise navigation, as well as in Geographic Information Systems and Location Based Services.

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**STRESZCZENIE**

W artykule przedstawiono zasadnicze zagadnienia i problemy odnoszące się do pozycjonowania opartego na różnicowych pomiarach kodowych GNSS przy równoczesnym wykorzystywaniu sygnałów od co najmniej dwóch niezależnych systemów nawigacji satelitarnej. W tym celu wybrano amerykański GPS oraz rosyjski GLONASS. W artykule opisano najważniejsze ograniczenia oraz metody ich eliminacji, jak również podstawowe zalety i korzyści będące wynikiem stosowania DGNSS (różnicowa wersja GNSS) dla pozycjonowania. Teoretyczne rozważania zostały zweryfikowane metodą opracowania w post-processingu obserwacji zebranych podczas sześciogodzinnej sesji pomiarowej. Do obliczeń użyto danych od wybranych stacji odniesienia ASG EUPOS (aktywna sieć geodezyjna). Badania wykazały, że metoda pozycjonowania DGNSS pozwala osiągnąć wyższą dokładność i precyzję, a także poprawić stabilność wyznaczanych współrzędnych w domenie czasu w porównaniu do metod opartych na wykorzystaniu sygnałów tylko od jednego systemu satelitarnego.