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NETWORK STATIC DGPS POSITIONING FOR MEDIUM-LONG DISTANCES

ABSTRACT

The use of a network of reference stations instead of a single reference station allows to model some systematic errors in a region, and to increase the distance between the rover receiver and reference stations. GPS reference stations exist in some countries, and GPS observations are available for users in real-time mode and in post-processing. The paper presents DGPS post-processing positioning with the use of three reference stations at the same time. The traditional DGPS is based on one reference station. It has been shown that the accuracy of such positioning is about 1 – 2 meters, depending on the number of satellites and the value of PDOP (Position Dilution of Precision). The accuracy of DGPS positioning degrades when the distance between the rover and the receiver station is increased. The paper shows that when we use three reference stations simultaneously, we can create pseudorange corrections for a virtual reference station, located in the vicinity of an unknown station. Three reference stations give redundant observations and enable to reduce the values of some measurement errors and biases. Practical calculations and analysis of accuracy have been presented for medium-long distances between the rover station and references.

Keywords:
DGPS, network of reference station.

INTRODUCTION

The concept of using a multiple reference station network for GPS positioning has been investigated by several research groups over the last years. A review of various multi-reference station network approaches can be found in Fotopoulos and Cannon [2001] and in other original papers [e.g., Wanninger, 1995; Wübbena et al., 1996; Wanninger, 1997; Raquet, 1997; Lachapelle et al., 2000; Landau et al., 2002; Euler et al, 2001]. The models presented in these studies concern mostly carrier-phase positioning where high accuracy (cm or mm) is expected. To achieve this, the ambiguity resolution process should be first performed. However, in the case of DGPS we do not need to calculate ambiguities due to the fact that only pseudorange measurements are applied. Traditional DGPS positioning, based on the Coarse Acquisition (C/A) code, is a technique where at least two receivers are used. One GPS receiver is located at a reference site with known coordinates, and the other receiver is usually roving freely or is placed at an unknown point. The reference station calculates pseudorange corrections (PRC) that are applied to the remote receiver in near real time (real-time DGPS) or after GPS measurements (post--processing DGPS). In order to provide a differential positioning service for a large area, several reference stations have to be used [Wanninger, 2003]. This kind of network approach is known as wide-area DGPS (WADGPS), for example WAAS (Wide Area Augmentation System) or EGNOS (European Geostationary Navigation Overlay Service). Practical tests with WAAS and EGNOS systems presented by Chen and Li [2004] show that it is possible to achieve the accuracy of 1 – 2 m in the horizontal components and 2 – 4 m in the vertical components, i.e. at a level of 95%. The use of a virtual DGPS solution slightly improved standard solutions of WAAS/EGNOS systems; the tests were performed with a handheld DGPS receiver – Garmin12.

Due to the degradation of standard DGPS positioning by distance-dependent errors, this paper presents the approach of linear interpolation, and additionally – the smoothing process of pseudorange corrections to obtain better accuracy of the positions determined. The calculations presented in the paper are based on data from Ashtech μZ-CGRS receivers.

OBSERVATION EQUATION OF PSEUDORANGE MEASUREMENTS

The pseudorange equation corresponds to the geometric distance that would be traveled by the signal in the propagation medium, i.e. in a vacuum where there are no clock errors or other biases. Taking these errors and biases into account, the complete expression for the pseudorange takes the form [Kleusberg and Teunissen (eds), 1996]:

$$P_i^k(t) = \rho_i^k(t) + c[\delta_i(t) - \delta^k(t)] + I_i^k + T_i^k + d^{eph} + d_i(t) + d^k(t) + m_i(t) + \varepsilon_i^k \qquad (1)$$

where: $\rho_i^k(t)$ – geometric range between the satellite k (at transmit time) and the receiver i (at receiver time), computed from ephemeris data and station coordinates;

$\delta_i(t)$ – receiver clock error;

$\delta^k(t)$ – satellite clock error;

I_i^k – measurement delay due to ionosphere;

T_i^k – measurement delay due to troposphere;

d^{eph} – effect of ephemeris error;

$d_i(t)$ – receiver hardware delay;

$d^k(t)$ – satellite hardware delay;

$m_i(t)$ – multipath;

ε_i^k – pseudorange measurement error.

The right side of Eq. (1) can be written in another way [Raquet, 1999]:

$$P_i^k(t) = \rho_i^k(t) + \Delta\delta_I + \Delta\delta_{II} + \Delta\delta_{III} + \varepsilon_i^k\,, \tag{2}$$

The first term, $\Delta\delta_I$, includes the clock errors which are calculated and cancelled in DGPS positioning and satellite hardware delay. The term $\Delta\delta_{II}$ is a correlated error term and it includes all errors that are a function of the receiver position. They are called correlated errors because they are correlated between receivers that are close together. The correlated errors include ionospheric error, tropospheric error and satellite position errors. The third term, $\Delta\delta_{III,}$ is the uncorrelated error term. It includes all errors that are not cancelled in the process of generating pseudorange corrections, and which are not a function of the receiver position. That is why they are uncorrelated. The uncorrelated errors include multipath, receiver hardware delay and measurement noise. In order to reduce the influence of correlated and partly uncorrelated errors, linear interpolation of pseudorange corrections, calculated with the use of three reference stations, was applied in the study.

PSEUDORANGE CORRECTIONS FOR A VIRTUAL REFERENCE STATION

In order to reduce distance-dependent errors, with three reference stations in the vicinity of an unknown control point, virtual reference pseudorange corrections can be calculated using linear interpolation. Additionally, smoothing of pseudorange corrections for every satellite can be used to reduce some non-systematic errors. The flowchart of that process is shown in fig. 1. Three pseudorange corrections generate a correlation plane for every satellite, thus the pseudorange correction for every satellite can be presented as the following equations:

$$a(t)x_{REF1} + b(t)y_{REF1} + c(t) = PRC_{REF1}(t) \tag{3a}$$

$$a(t)x_{REF2} + b(t)y_{REF2} + c(t) = PRC_{REF2}(t) \tag{3b}$$

$$a(t)x_{REF3} + b(t)y_{REF3} + c(t) = PRC_{REF3}(t) \tag{3c}$$

The factors $a(t), b(t), c(t)$ are calculated for every epoch according to the matrix equation:

$$\begin{bmatrix} a(t) \\ b(t) \\ c(t) \end{bmatrix} = \begin{bmatrix} x_{REF1} & y_{REF1} & 1 \\ x_{REF2} & y_{REF2} & 1 \\ x_{REF3} & y_{REF3} & 1 \end{bmatrix}^{-1} \begin{bmatrix} PRC_{REF1}(t) \\ PRC_{REF2}(t) \\ PRC_{REF3}(t) \end{bmatrix} \tag{4}$$

where x_{REF}, y_{REF} are plane coordinates.

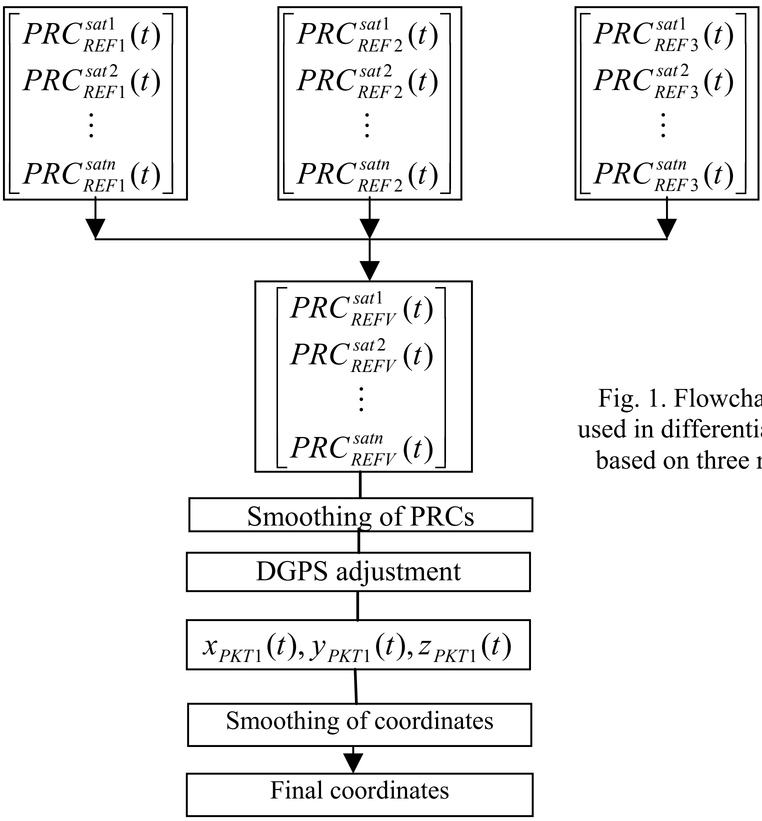

Fig. 1. Flowchart of the strategy used in differential GPS positioning based on three reference stations

EQUATIONS OF DGPS ADJUSTMENT

In order to determine the coordinates of an unknown point in in the global geocentric coordinate system, we must perform linearization of the observed equation [Leick, 1995; Parkinson and Spilker (eds), 1996; Hofmann-Wellenhof et al, 1997]. The linearization process is carried out for each of the observed satellites. Let us denote the vector of observation by L, the vector of unknowns by X and the design matrix by A. Then, employing the least-squares method, the estimator of the unknown vector $\hat{X}$ equals:

$$\hat{X} = (A^T PA)^{-1} A^T PL, \tag{5}$$

where P – a weight matrix.

It is well known that the values of $\hat{X}$ depend on functional and stochastic models. Many different approaches can be found in literature, corresponding to various special applications of GPS methods used in practice. In the present DGPS post-processing positioning, using the least-squares solution, the matrices of Eq. (5) are as follows:

$$A = \begin{bmatrix} -\dfrac{x^1(t) - x_{i0}}{\rho_{i0}^1(t)} & -\dfrac{y^1(t) - y_{i0}}{\rho_{i0}^1(t)} & -\dfrac{z^1(t) - z_{i0}}{\rho_{i0}^1(t)} \\ \vdots & \vdots & \vdots \\ -\dfrac{x^n(t) - x_{i0}}{\rho_{i0}^n(t)} & -\dfrac{y^n(t) - y_{i0}}{\rho_{i0}^n(t)} & -\dfrac{z^n(t) - z_{i0}}{\rho_{i0}^n(t)} \end{bmatrix} \tag{6}$$

$$L = \begin{bmatrix} P_i^1(t) - \rho_{i0}^1 + \Delta\delta t^1 + PRC_{REFV}^{sat1}(t) \\ \vdots \\ P_i^n(t) - \rho_{i0}^n + \Delta\delta t^n + PRC_{REFV}^{satn}(t) \end{bmatrix} \tag{7}$$

$$P = I \tag{8}$$

where: $PRC_{REFV}^{satn}(t)$ – represents pseudorange corrections for the virtual reference station;

x_{i0}, y_{i0}, z_{i0} – the approximate coordinates for the unknown station;

$x^k(t), y^k(t), z^k(t)$ – the coordinates of the satellite k, and

$$\rho_{i0}^{k}(t) = \sqrt{[x^{k}(t) - x_{i0}]^{2} + [y^{k}(t) - y_{i0}]^{2} + [z^{k}(t) - z_{i0}]^{2}} \tag{9}$$

Additionally, the values of $PRC_{REFV}^{satn}(t)$ were smoothed by the *supsmooth* function of Mathcad 11 software. The *supsmooth* function uses a symmetric k nearest neighbor linear least-squares fitting procedure to make a series of line segments through the data, and adaptively chooses different bandwidths for different portions of the data. Additionally, the *supsmooth* function was implemented to smooth the coordinates obtained from Eq. (5).

In the present DGPS positioning we used a unity weight matrix, following the assumption that the accuracy of pseudorange measurements obtained from particular satellites is the same [Bakuła, 2004]. However, when multipath affects pseudorange measurements, the presented model will not eliminate the influence of gross errors.

NUMERICAL EXAMPLE

The GPS data used in this chapter were collected at four known permanent reference stations of the ASG-PL (e.g. Polish Active Geodetic Network) network (fig. 2): KLOB in Kłobuck, KATO in Katowice, WODZ in Wodzisław Śląski, and TARG in Tarnowskie Góry, Poland [Kryński et al, 2003]. The reference stations are equipped with Ashtech µZ-CGRS (Continuous Geodetic Reference Station) receivers and ASH701945C_M SNOW antennas. The GPS observations were performed on 7 March 2003, for half an hour – from 13:00 to 13.30 UTC; the sampling interval was 5 second, the values of PDOP during the measurements were between five and seven (fig. 3). Five satellites were used in the calculations. In the calculations presented in the paper, the station TARG was treated as unknown, whereas the other three: WODZ, KLOB and KATO were treated as reference stations. Differential positioning was carried out as follows:

1) Traditional DGPS positioning:
   - WODZ-TARG – results in fig. 4,
   - KLOB-TARG – results in fig. 5,
   - KATO-TARG – results in fig. 6;
2) DGPS positioning with the use of three reference stations, fig. 7.

In both approaches real errors were calculated (DX, DY, DZ) as the differences between the fixed coordinates of the reference station – TARG and the coordinates obtained in successive epochs of differential positioning.

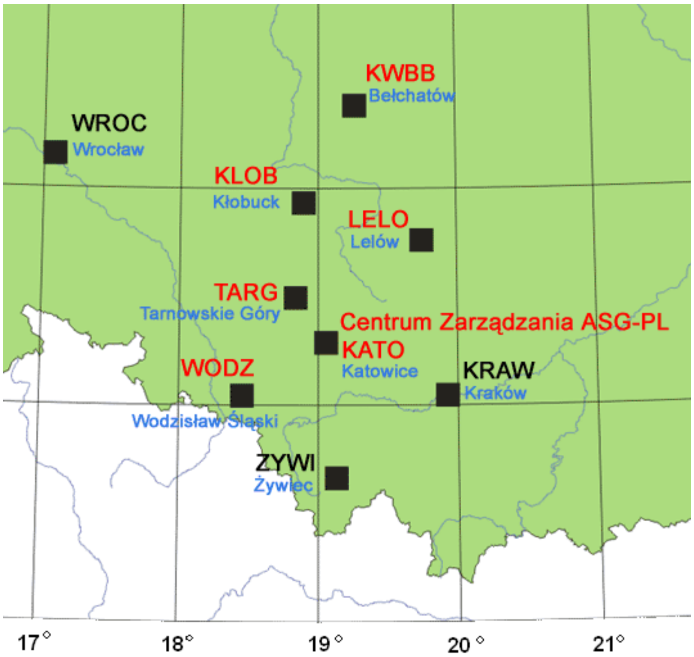

Fig. 2. Deployment of permanent reference stations in southern Poland, as a part of the ASG-PL network (the picture from the website www.asg-pp.pl/rozmieszczenie.html)

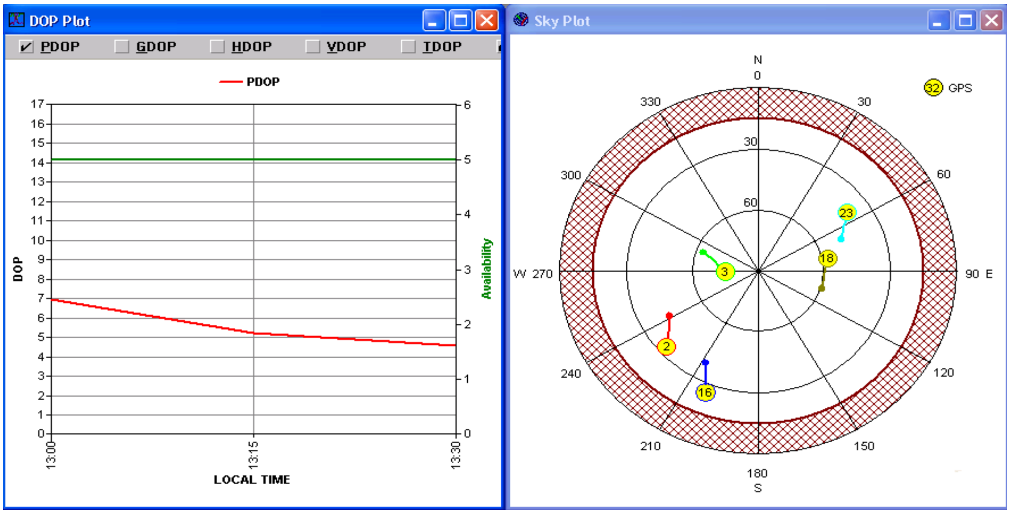

Fig. 3. Satellites and the values of PDOP during GPS measurements (obtained from the Mission Planning Program of Magellan Corp.)

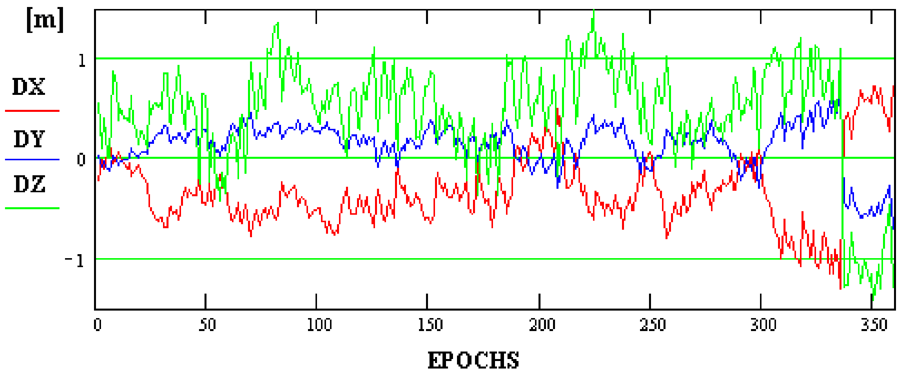

Fig. 4. Values of horizontal (DX, DY) and vertical (DZ) errors in traditional DGPS positioning for the baseline WODZ-TARG, 58 km in length, obtained in the local topocentric coordinate system

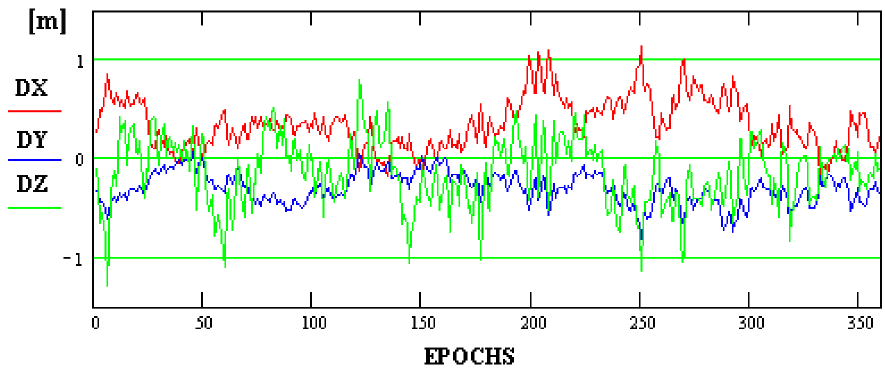

Fig. 5. Values of horizontal (DX, DY) and vertical (DZ) errors in traditional DGPS positioning for the baseline WODZ-TARG, 50 km in length, obtained in the local topocentric coordinate system

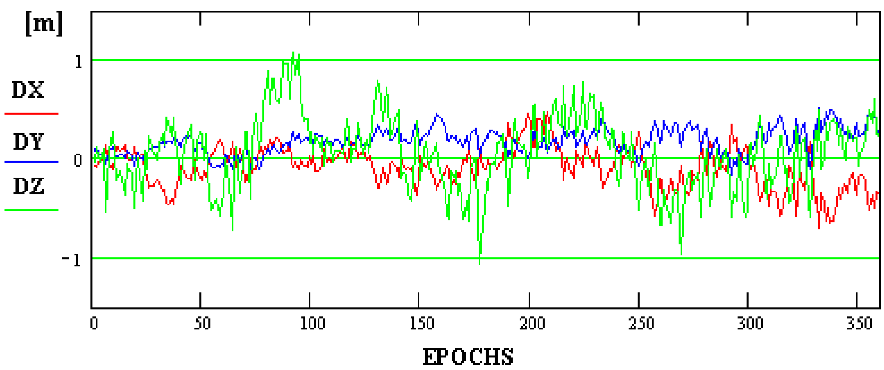

Fig. 6. Values of horizontal (DX, DY) and vertical (DZ) errors in traditional DGPS positioning for the baseline WODZ-TARG, 26 km in length, obtained in the local topocentric coordinate system

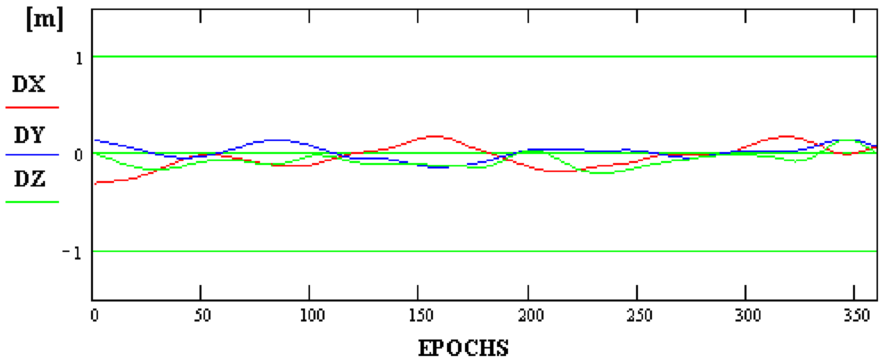

Fig. 7. Values of horizontal (DX, DY) and vertical (DZ) errors in differential GPS positioning, obtained in the local topocentric coordinate system, when three reference stations were used simultaneously with the linear interpolation and smoothing process

CONCLUSIONS

In this paper the linear interpolation approach was applied to generate pseudorange corrections of differential positioning with the use of the C/A code. Additionally, the smoothing of pseudorange corrections and final coordinates from adjustment was implemented to reduce some non-systematic errors. Test data from the ASG-PL network were used to evaluate the performance of this method. The numerical results obtained in the study show that the use of linear interpolation of pseudorange corrections for medium distances can reduce distance-dependent errors in DGPS positioning. When the traditional method of DGPS positioning was used, the accuracies were different for every reference station. For the longest distances: WODZ – TARG and KLOB – TARG the real accuracy was in the range of 1 – 1.5 m. Whereas, the application of the linear interpolation and smoothing process of pseudorange corrections significantly improved the accuracy of DGPS positioning, and enabled to achieve accuracy below 0.3 m for every epoch. Accuracy higher than 0.3 m can be used not only for navigation or any location-based service, but also for many mapping applications of the Geographic Information System.

REFERENCES

[1] Bakuła M., An Approach of DGPS Positioning Based on Two Reference Stations, Artificial Satellites, 2004, vol. 39, no. 4.

[2] Chen R., Li X., Virtual Differential GPS Based on SBAS Signal, GPS Solutions, 2004, vol. 8, no. 4.

[3] Euler H. J., Keenan C. R., Zebhauser B. E., Wübbena G., Study of Simplified Approach in Utilizing Information from Permanent Reference Station Arrays, Paper presented at ION GPS 2001, Salt Lake City, Utach 2001.

[4] Fotopoulos G., Cannon M. E., An Overview of Multi-Reference Station Methods for Cm-Level Positioning, GPS Solutions, 2001, vol. 4, no. 3.

[5] Hofmann-Wellenhof B., Lichtenegger H., Collins J., Global Positioning System: Theory and Practice, 4th ed. Springer, Berlin Heidelberg – New York 1997.

[6] Kleusberg A., Teunissen P. J. G., GPS For Geodesy, Springer, Berlin Heidelberg – New York 1996.

[7] Kryński J., Rogowski J. B., Zieliński J. B., National Report of Poland to EUREF 2003, EUREF Symposium, Toledo, Spain, 4 – 7 June 2003.

[8] Lachapelle G., Alves P., Fortes L. P., Cannon M. E., Townsend B., DGPS RTK Positioning Using a Reference Network, Proceedings of GPS2000 (Session C3, Salt Lake City, 19 – 22 September), The Institute of Navigation, Alexandria, VA, 2000.

[9] Landau H., Vollath U., Chen X., Virtual Reference Stations Systems, Journal of Global Positioning Systems, 2002, vol. 1, no. 2.

[10] Leick A., GPS Satellite Surveying, 2nd ed., John Wiley & Sons, New York 1995.

[11] Parkinson B., Spilker J. J., GPS Theory and Applications, vol. 1 and 2, AIAA, Washington, DC, 1996.

[12] Raquet J., A New Approach to GPS Carrier Phase Ambiguity Resolution Using a Reference Receiver Network, Proceedings of National Technical Meeting, Santa Monica, January 14 – 16), The Institute of Navigation, Alexandria, VA., 1997.

[13] Raquet J., Development of a Method for Kinematic GPS Carrier Phase Ambiguity Resolution Using Multiple Reference Receiver, UCGE no. 20116, (PhD thesis), 1999.

[14] Wanninger L., Improved Ambiguity Resolution by Regional Differential Modelling of the Ionosphere, Proc. of ION GPS 95, Palm Springs 1995.

[15] Wanninger L., Real-Time Differential GPS-Error Modelling in Regional Reference Station Networks in: Brunner, F. K. (Hg.): Advances in Positioning and Reference Frames, Proceedings of the IAG Scientific Assembly, Rio de Janeiro 1997.

[16] Wanninger L., GPS on the Web: Virtual reference stations (VRS), GPS Solutions, 2003, vol. 7, no. 2.

[17] Wübbena G., Bagge A., Seeber G., Böder V., Hankemeier P., Reducing Distance Dependent Errors for Real-Time Precise DGPS Applications by Establishing Reference Station Networks, Proceedings of the International Technical Meeting, ION GPS-96, Kansas City 1996.

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