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HOW TO INTRODUCE COURSE DYNAMIC INSTABILITY INTO SHIP MANOEUVRING MATHEMATICAL MODEL

ABSTRACT

The paper presents a method of conversion of the previously published full-mission time-domain manoeuvring mathematical model to an analytical model under steady turning conditions. The aim is to identify the parts of the model directly responsible for a given shape of the spiral test curve in case of directionally stable or unstable ships.

INTRODUCTION

The theory of ship course dynamic stability or instability has probably the longest history and greatest progress of all the topics generally covered by the ship manoeuvring research and science, and by the ship automatic control science as well – refer e.g. to [Norrbin, 1960], [Bech, 1972], [Lewis (ed.), 1989]. It is not purposeful to cite here all relevant publications, the number of which is really huge. The basic ideas are however the same. The possible approaches or concepts, in form of input manoeuvring equations or output stability relationships, can be quite different as dependent upon application goals.

The course stability is usually classified according to a direct (classical, Dieudonne's) or reversed (Bech's) spiral test data – a steady state turning response in terms of e.g. yaw velocity versus different rudder angles is examined. A valuable comparison is given e.g. in [Smitt, 1967]. These tests are very famous (and superior in the mathematical models tuning), although they are not normally included in the shipyard programme of sea trials, but are sometimes scheduled on free-running scale physical models during a ship prototype design. The IMO manoeuvring standards, mostly implemented within the design process (they are considered by ship designers as very weak and thus comfortable), ultimately set a major focus on zigzag data for some reasons. Nevertheless, it is recommended to execute spiral tests, and record as many variables as feasible, for manoeuvring mathematical model identification – this can prevent to some extent the occurrence of ambiguity and loss of physical meaning among some model parameters when such an identification (optimisation) is based on limited sea trials. One of interesting attempts to identify the parameters of rather simple manoeuvring equations just from spiral test data was provided e.g. by [Gill, 1976].

In [Artyszuk, 2003a] a practical four-quadrant ship manoeuvring mathematical model was introduced, where hull forces and moment nondimensional coefficients, as well as rudder lift and drag force coefficients, are linearly interpolated in two dimensions from lookup tables. Also, some course instability of the model was noticed when the hull-rudder interaction factors (rudder force augmentation and flow straightening parameters) were manipulated. The theory behind the identification of the hull hydrodynamics related lookup tables from sea trials was described in [Artyszuk, 2003b, 2005a]. Although the lookup tables ensure in general any level of flexibility to the model structure, due to common finite discretisation steps (e.g. for the sake of identification feasibility and low model complexity), they impose some stiffness, which can be advantageous or cause some adverse effects.

It is also rather known (though it may be sometimes not realised at a glance due to poor popularisation of the ship manoeuvring knowledge) that some steady state turning parameters (those of relative nature) are independent of the ship initial approach speed – naturally within a reasonable range of Froude numbers. Under this behaviour fall almost all merchant ships. A wider explanation of the steady state turning phenomena was provided in [Artyszuk, 2005b]. Hence some universal analytical equations can be established that take into account only hull and rudder parameters.

The present study aims at addressing systematically those parts of the manoeuvring mathematical model, as specified in [Artyszuk, 2003a], which are responsible for a prediction of steady state behaviour in spiral tests. If proper values of some parameters are set (e.g. by model tests in towing tanks), or at least their possible range of values, some other parameters can be often uniquely determined. For this reason, a special analytical model (algebraic equations) is to be derived from the general time-domain simulation model in concern, which shall support an identification of the latter. It describes a plot of nondimensional yaw velocity versus rudder angle (but for relatively low magnitudes e.g. up to 10 degrees), especially in the case when a ‘hysteresis’ appears for an unstable ship. Other steady-state parameters, like e.g. the absolute yaw velocity or drift angle, can be easily deduced from the nondimensional yaw velocity.

DERIVATION OF ANALYTICAL MODEL

The ship manoeuvring equations in the steady-state conditions (time derivatives are vanishing) are as follows:

$$\begin{cases} 0 = +(m + c_m m_{22})v_y\omega_z + F_{xH} + F_{xP} + F_{xR} \\ 0 = -(m + m_{11})v_x\omega_z + F_{yH} + F_{yR} \\ 0 = -(m_{22} - m_{11})v_x v_y + M_{zH} + M_{zR} \quad \text{where} \quad M_{zR} = F_{yR} \cdot x_R \end{cases} \tag{1}$$

where: m, m_{11}, m_{22} – ship mass and added masses; c_m – empirical factor; v_x, v_y, ω_z – surge, sway, and yaw velocities; F_x, F_y, M_z – surge, sway forces and yaw moment; x_R – rudder abscissa (negative); H, P, R – subscripts denoting hull, propeller, and rudder.

The hereafter efforts will concentrate upon the last two equations in (1), namely those controlling the sway force and yaw moment balance. There are three items that ought to be defined – hull sway force F_{yH} and yaw moment M_{zH}, and rudder sway force F_{yR}. Concerning the hull excitations, the four-quadrant expressions look like:

$$\begin{bmatrix} F_{yH} \\ M_{zH} \end{bmatrix} = 0.5\rho LT\left(v_{xy}^2 + \omega_z^2 L^2\right)\begin{bmatrix} c_{fyHm}(\beta, \Omega_m) \\ L \cdot c_{mzHm}(\beta, \Omega_m) \end{bmatrix} \tag{2}$$

where: ρ – water density; L, T – ship length and draft; v_{xy} – total linear velocity; β – drift angle; Ω_m – normalised (modified) nondimensional yaw velocity; c_{fyHm}, c_{mzHm} – hull hydrodynamic nondimensional coefficients supplied by lookup tables.

The particular motion parameters are represented by:

$$v_{xy} = \sqrt{v_x^2 + v_y^2}\,,\ \beta = \mathrm{arctg}\left(\frac{-v_y}{v_x}\right),\ \Omega_m = \frac{\omega_z L}{\sqrt{v_{xy}^2 + \omega_z^2 L^2}} \tag{3}$$

where the drift angle β ranges from –180° to +180° (positive for starboard turning), and the modified nondimensional yaw velocity Ω_m from –1 to +1 (positive for starboard turning).

The two-dimensional interpolating lookup tables, in the single region directly adjacent to values of input variables, for which the function is evaluated, are marked in tab. 1. Moreover, it is assumed that for the most essential inner part of the spiral test curve this four-value part will remain valid – thus an appropriate discretisation step is here needed.

Table 1. Extract from hull hydrodynamics lookup tables – coefficients c_{fyHm} and c_{mzHm}

$\Omega_m \backslash \beta$	β_1	β_2
Ω_{m1}	z_{11}	z_{12}
Ω_{m2}	z_{21}	z_{22}

A linear interpolation among nodes (values) of the lookup tables involves the following interpolating formula (note the first nonlinear term associated with 'a' parameter):

$$z = axy + bx + cy + d \tag{4}$$

where the arguments x and y are normalised to the range $\langle 0,1 \rangle$, and the constant coefficients are related to the corresponding part of the lookup table according to:

$$\begin{cases} z_{11} = d \\ z_{12} = b + d \\ z_{21} = c + d \\ z_{22} = a + b + c + d \end{cases} \quad \text{or} \quad \begin{cases} a = z_{11} - z_{12} - z_{21} + z_{22} \\ b = -z_{11} + z_{12} \\ c = -z_{11} + z_{21} \\ d = z_{11} \end{cases} \tag{5}$$

While interpolating the hull hydrodynamic coefficients, one can read:

$$z = a\frac{\beta}{\beta_2 - \beta_1} \cdot \frac{\Omega_m}{\Omega_{m2} - \Omega_{m1}} + b\frac{\beta}{\beta_2 - \beta_1} + c\frac{\Omega_m}{\Omega_{m2} - \Omega_{m1}} + d \tag{6}$$

Finally:

$$z = a'\beta\Omega_m + b'\beta + c'\Omega_m + d' \quad \text{where} \quad d' = d \tag{7}$$

Because the middle range of the lookup tables for hull hydrodynamic coefficients, i.e. around the origin (zero) of β and Ω_m, as of the most interest in the ship course instability modelling or identification (low drift angles and nondimensional yaw velocities), contains by nature some zeros, the above relationship (7) is simplified and marked by:

$$c_{fyHm} = a'_1\beta\Omega_m + b'_1\beta \tag{8}$$

$$c_{mzHm} = a'_2\,\beta\Omega_m + b'_2\,\beta + c'_2\,\Omega_m \tag{9}$$

To make the analysis easier and clearer, it is suggested to introduce in (8) and (9) the standard nondimensional yaw velocity $\overline{\omega_z}$ instead of the modified non-dimensional yaw velocity Ω_m:

$$\overline{\omega_z} = \frac{\omega_z L}{v_{xy}} = \frac{\Omega_m}{\sqrt{1-\Omega_m^2}}, \qquad \Omega_m = \frac{\overline{\omega_z}}{\sqrt{1+\overline{\omega_z}^2}} \tag{10}$$

However, for $\overline{\omega_z}$ up to 0.5 (Ω_m then equals 0.4472), valid for usual range of course stability problem, and being also the initial discretisation step of the lookup table, see later, the following practical equality can be established:

$$\overline{\omega_z} \approx \Omega_m \tag{11}$$

It implies that the hull excitations can now be rearranged according to:

$$F_{yH} = 0.5\rho L T v_{xy}^2\left(1+\overline{\omega_z}^2\right)\left(a'_1\,\beta\overline{\omega_z} + b'_1\beta\right) = v_{xy}^2\left(1+\overline{\omega_z}^2\right)\left(a_1\beta\overline{\omega_z} + b_1\beta\right) \tag{12}$$

$$\begin{aligned} M_{zH} &= 0.5\rho L^2 T v_{xy}^2\left(1+\overline{\omega_z}^2\right)\left(a'_2\,\beta\overline{\omega_z} + b'_2\beta + c'_2\,\overline{\omega_z}\right) \\ &= v_{xy}^2\left(1+\overline{\omega_z}^2\right)\left(a_2\beta\overline{\omega_z} + b_2\beta + c_2\overline{\omega_z}\right) \end{aligned} \tag{13}$$

The rudder lateral force F_{yR}, for small rudder incidence angles and local drift angles, can be approximately described as follows (refer also to [Artyszuk, 2003a, 2005b]):

$$\begin{aligned} F_{yR} &= 0.5\rho A_R v_{xy}^2 (1-w)^2 (1+c_{Th}) \frac{\partial c_L}{\partial \alpha}(1+a_H)\left(\delta + \frac{c_{12}\left(\beta + 0.5\overline{\omega_z}\cdot 57.3\right)}{(1-w)\sqrt{1+c_{Th}}}\right) = \\ &= 0.5\rho L T v_{xy}^2\left(b_{1R}\beta + c_{1R}\overline{\omega_z} + e_{1R}\delta\right) \end{aligned} \tag{14}$$

where: A_R – rudder area (in case of twin-rudder/twin-screw ships this shall be doubled); w – wake fraction; c_{Th} – thrust loading coefficient (average with regard to the usual speed loss in the range of small rudder angles); $\partial c_L/\partial\alpha$ – lift coefficient gradient corresponding to the aforementioned average c_{Th} (lift linearity assumed, the gradient relates to the incidence angle α expressed in degrees); a_H – rudder force augmentation empirical factor due to a hull interaction; δ – rudder angle in degrees (negative for starboard rudder); β – drift angle in degrees; c_{12} – rudder lateral inflow empirical factor.

The centrifugal term and Munk moment in the second (sway) and third (yaw) equation of (1), respectively, can be rewritten as:

$$-\left(m+m_{11}\right)v_x\omega_z \approx -\frac{\left(m+m_{11}\right)}{L}v_{xy}^2\,\overline{\omega_z} = 0.5\rho LTv_{xy}^2 c_1^*\,\overline{\omega_z} \tag{15}$$

$$-\left(m_{22}-m_{11}\right)v_x v_y \approx \left(m_{22}-m_{11}\right)v_{xy}^2\,\frac{\beta}{57.3} = 0.5\rho L^2 Tv_{xy}^2 b_2^{"}\beta \tag{16}$$

The most hydrodynamically convenient in the analysis of ship course stability is a chart of $\overline{\omega_z}$ versus rudder angle δ though in practical applications and full-scale measurements the absolute yaw velocity ω_z still dominates. Hence a proper conversion is necessary. Because the $\overline{\omega_z}(\delta)$ is not a unique function for a directionally unstable ship – for some (rather small) rudder angles δ there are two (direct spiral test curve) or three (reversed spiral test curve) different values of $\overline{\omega_z}$, all of them indicate the equilibrium of forces in the steady turning conditions – the best choice is to investigate the inverse relationship $\delta\left(\overline{\omega_z}\right)$. The latter is definitely unique.

Combining now both the sway and yaw equations (see also [Artyszuk, 2005b]) by rejecting the rudder sway force F_{yR}, the fundamental hull dependent only relationship $\beta\left(\overline{\omega_z}\right)$ is yielded in our new notations:

$$\beta = \frac{\left(-x'_R\,c_1^* + c_2\right)\overline{\omega_z} + c_2\,\overline{\omega_z}^3}{x'_R\left(b_1 + a_1\overline{\omega_z} + b_1\overline{\omega_z}^2 + a_1\overline{\omega_z}^3\right) - \left(b_2^* + a_2\overline{\omega_z} + b_2\overline{\omega_z}^2 + a_2\overline{\omega_z}^3\right)} \tag{17}$$

where: $b_2^* = b_2 + b_2^{"}$, and $x'_R = x_R / L$.

Taking into account the sway equation from (1) for example, and discarding the common terms, the rudder angle as independent variable finally reads:

$$\delta = \delta\left(\overline{\omega_z}\right) = -\frac{1}{e_{1R}}\left[\beta\left(b_1 + b_{1R} + a_1\overline{\omega_z} + b_1\overline{\omega_z}^2 + a_1\overline{\omega_z}^3\right) + \left(c_1^* + c_{1R}\right)\overline{\omega_z}\right] \tag{18}$$

where for β the expression (17) shall be substituted. It shall be reminded that equation (18) is valid for the same range (positive or negative) of the nondimensional yaw velocity, for which the values of the hull nondimensional coefficients are taken from the background lookup tables, refer to tab.1 and formulas (8) and (9). The negative (starboard) rudder has to always follow the positive yaw velocity, and vice versa. The solution of (18) is not symmetric, as opposed to what happens in many spiral tests, though it seems that changing the sign of $\overline{\omega_z}$ in (18) would automatically reverse the sign of δ. The latter is not true. The usual central symmetry among values of the hull lookup tables for positive and negative yaw velocities (accompanying the same sign of the drift angle), does not imply the same signs of all the constants in (8) or (9). Both the terms associated with the product $\beta\Omega_m$ – namely a'_1 and a'_2 – will just assume the opposite values.

GENERAL PROPERTIES OF SPIRAL TEST DATA

Since the subject literature does not provide a lot of complete information concerning ship turning behaviour in steady state conditions – both motion data and force components are important for ship design improvement or mathematical model identification, figures from 1 to 3 are intended to partially fill in these gaps. They are based on a direct spiral test simulation with two ship manoeuvring mathematical models – a chemical tanker (with two versions of rudder parameters) and twin-screw/twin-rudder ferry – refer to [Artyszuk, 2003a] for the model structure (and model parameters of the first ship).

Both models were more or less accurately optimised (tuned) against available full-scale trials. Though figs. from 1 to 3 do not present the force information as well (for a reason of restraining the paper's volume), both tanker and ferry models are essentially quite different. Within the rudder angle range up to 10 degrees – the tanker experiences the rudder yaw moment M_{zR} in the order of 25% of the hull yaw moment M_{zH} (both are surprisingly negative for starboard turning and vice versa), while for the ferry the rudder yaw moment is almost zero (the hull yaw moment itself compensates the Munk moment). It means that for the ferry low sensitivity of spiral test data prediction upon rudder coefficients shall be experienced. The sub-charts of figs. 1 – 3 contain in sequence: surge velocity v_x, yaw velocity ω_z, nondimensional yaw velocity $\overline{\omega_z}$, drift angle β, and thrust loading coefficients c_{Th}. The plots of ω_z, $\overline{\omega_z}$ or β are similar (correlated) to each other, also any instability loop occurs in all these three parameters – thus they are nearly equivalent in the analysis – as stated before the nondimensional yaw velocity $\overline{\omega_z}$ is preferred.

For a stable ship, both spiral (direct, classical) and reversed spiral tests produce the same curve of steady state response, e.g. in terms of $\overline{\omega_z}$ – see fig. 4C, though the reversed type is considered faster but more difficult.

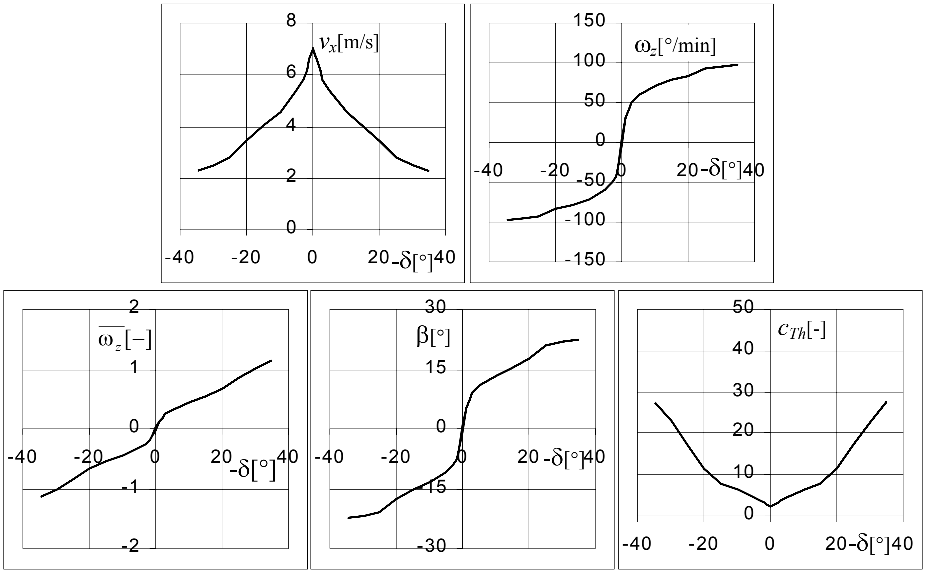

Fig. 1. Spiral test data for marginally unstable chemical tanker L = 97.4m ('chem100') – simulation with final model [Artyszuk, 2003] – a_H = 0.6, c_{12} = 1.0 (version '00n')

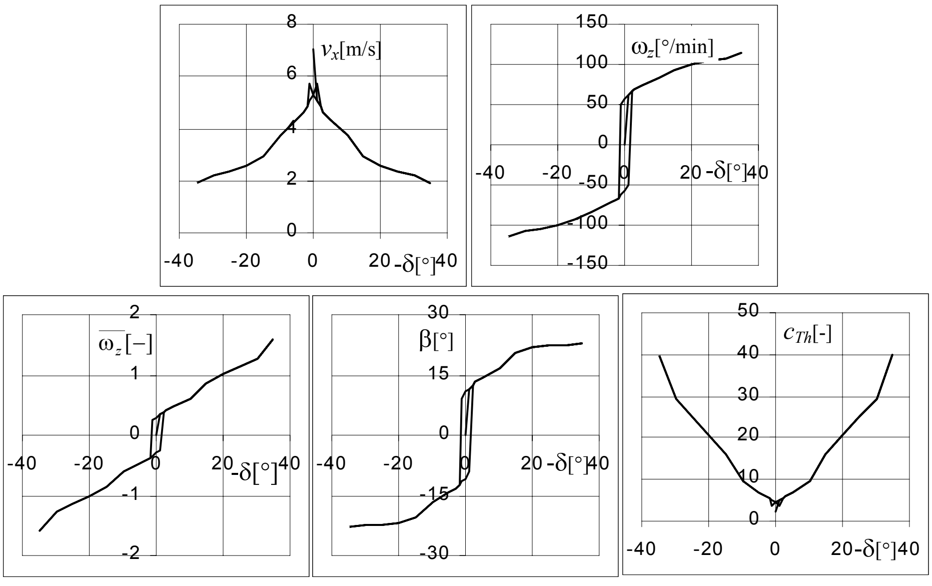

Fig. 2. Spiral test data for moderately unstable chemical tanker L = 97.4m ('chem100') – simulation with modified final model [Artyszuk, 2003] – a_H = 1.0, c_{12} = 0.5 (version '21')

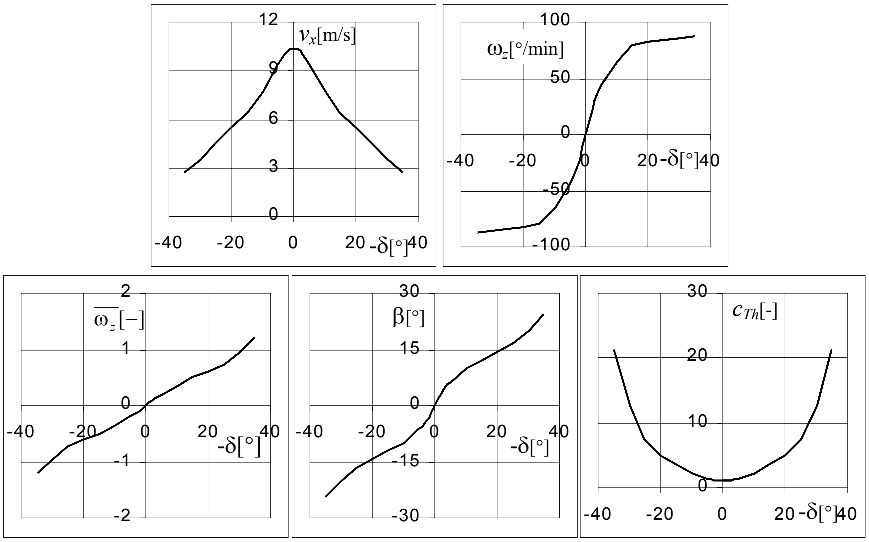

Fig. 3. Spiral test data for stable 2-screw ferry L = ca.150m ('ferry150') – preliminary simulation with partially tuned model

However, for an unstable ship the situation rapidly changes, refer to fig. 4A and 4B – the complete curve of steady turning $\overline{\omega_z}$, i.e. associated with the equilibrium of all involved forces and moments, can be obtained only through the reversed spiral test. Such a curve is just expressed by (18). The most critical is a piece between the points *D* and *D'*, as crossing the axes origin *O*, that can not be achieved by means of a direct spiral procedure – the accessible points in a direct spiral test are marked by the solid line in fig. 4A. The same points are obviously also acquired in a reversed spiral test, which is a bit wider and more general test. The points *D* and *D'* are the extrema of the $\delta\left(\overline{\omega_z}\right)$ relationship given by (18).

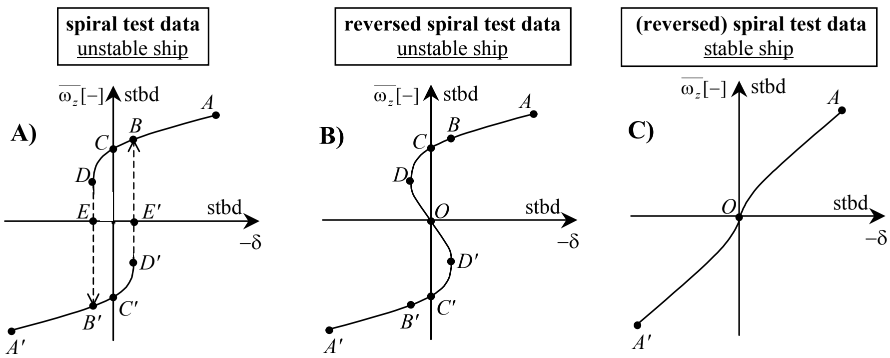

Fig. 4. Steady turning data and course stability via direct and reversed spiral test

Normally in practice, both direct and reversed spiral tests start from points *A* or *A'* (representing the highest rudder angle in concern) and are advanced towards lower rudder angles. However, this is not necessary to reproduce the whole curve specific to a particular spiral test. Any point can be chosen on the segment *A-D* or *A'-D'* as the initial one, and any direction of rudder change is allowed, for example from *A* to *D* or the opposite one. Moreover, if the current point is *D* and higher port rudder angles are ordered, then a sudden jump occurs to the second opposite leg of the spiral test curve – point *B'*, and vice versa.

Regarding the special region *D-O-D'* characteristic for the reversed spiral test, it shall be noted that a ship has always a tendency to leave this area and assume a location between points *A* and *D*, or *A'* and *D'* – this fact is rarely known in the literature. For example, if the turning equilibrium is somewhere between *D* and *O*, and the helm is put amidships, then a ship will enter the point *C*, unless a special steering by an experienced helmsman and with an available rate of turn indicator is performed as demanded in the principles of the reversed spiral test.

The area restricted by points *B-C-D-B'-C'-D'-B* is usually called a instability loop (or briefly a hysteresis). The basic parameters are the (nondimensional) yaw velocity corresponding to points *C* and *C'*, and the boundary rudder deflection arising in points *D* and *D'*. For a lot of ships, the spiral test curves are symmetrical against the origin of coordinates.

DEFINITION OF SPECIFIC POINTS ON SPIRAL TEST CURVE

Limiting our considerations to the positive domain of $\overline{\omega_z}$, the spiral test curve (see fig. 4A,B) intersects the $\overline{\omega_z}$ axis in <u>one (*O*) or two points (*O*, *C*)</u> that are defined by roots of the following equation (refer to (18)):

$$\delta\left(\overline{\omega_z}\right) = 0 \tag{19}$$

or

$$f\left(\overline{\omega_z}\right) = \overline{\omega_z}\left(A\overline{\omega_z}^5 + B\overline{\omega_z}^4 + C\overline{\omega_z}^3 + D\overline{\omega_z}^2 + E\overline{\omega_z} + F\right) = 0 \tag{20}$$

where

$$\begin{cases} A = c_2 a_1 \\ B = c_2 b_1 \\ C = 2c_2 a_1 - c_1^* a_2 + c_{1R} x'_R\, a_1 - c_{1R} a_2 \\ D = 2c_2 b_1 + c_2 b_{1R} - c_1^* b_2 + c_{1R} x'_R\, b_1 - c_{1R} b_2 \\ E = c_2 a_1 - c_1^* a_2 + c_{1R} x'_R\, a_1 - c_{1R} a_2 \\ F = -x'_R\, c_1^* b_{1R} + c_2\left(b_1 + b_{1R}\right) - c_1^* b_2^* + c_{1R} x'_R\, b_1 - c_{1R} b_2^* \end{cases} \tag{21}$$

The mutual magnitude and sign among particular contributions appearing on the right side of (21) are very important while checking an existence of roots or modelling the course instability. For a stable ship all coefficients in (20) from A to F are negative (the only root is $\overline{\omega_z} = 0$). Some of them are of course always negative independent of the level of ship course stability – they result from the common ship hydrodynamics. On the other hand, the unstable ship will get at least one of the above six polynomial coefficients as positive. The most efficient way of finding or testing a root in (20) under known coefficients is however a numerical solution.

The equilibrium condition for maximum positive (port) rudder angle (point D) is governed by:

$$\delta'\left(\overline{\omega_z}\right) = 0 \tag{22}$$

where (see eq. (20))

$$\delta'\left(\overline{\omega_z}\right) = -\frac{1}{e_{1R}} \frac{f'\left(\overline{\omega_z}\right) f_4\left(\overline{\omega_z}\right) - f\left(\overline{\omega_z}\right) f_4'\left(\overline{\omega_z}\right)}{f_4^2\left(\overline{\omega_z}\right)} \tag{23}$$

with

$$f_4\left(\overline{\omega_z}\right) = \left(x'_R\, a_1 - a_2\right)\overline{\omega_z}^3 + \left(x'_R\, b_1 - b_2\right)\overline{\omega_z}^2 + \left(x'_R\, a_1 - a_2\right)\overline{\omega_z} + \left(x'_R\, b_1 - b_2^*\right) \tag{24}$$

One of the additional parameters of the spiral test curve, fig. 4A or B, is its slope at the axes origin represented by the derivative (23) at $\overline{\omega_z} = 0$. In general, it reads:

$$\delta'(0) = -\frac{1}{e_{1R}} \cdot \frac{F}{x'_R b_1 - b_2^*} \tag{25}$$

where the numerator F in the last fraction is given by (21). The course dynamic instability occurs when $\delta'(0)$ is positive – the same signs of yaw velocity and rudder angle occur. The denominator of the last fraction is always negative, therefore the sign of $\delta'(0)$ is defined by the sign of F – a positive F value is required for an unstable ship. This is a significant improvement over the previous conditions related to the existence of nonzero roots in the equation (20).

Finally

$$\delta'(0) = -\frac{-x'_R\, c_1^* b_{1R} + c_2\left(b_1 + b_{1R}\right) - c_1^* b_2^* + c_{1R} x'_R\, b_1 - c_{1R} b_2^*}{e_{1R}\left(x'_R b_1 - b_2^*\right)} \tag{26}$$

It shall be noted that the all terms in the numerator of (26) are negative except for the third element '$-c_1^* b_2^*$' (hull related only), which is positive. Now everything concerning the sign of F depends upon the particular magnitudes of all the terms in F, including specially those connected with the rudder. No impact of the so-called mixed partial derivatives connected with a_1 or a_2 parameters is here observed.

CASE STUDIES – NUMERICAL COMPUTATIONS

In view of the above derivations and for exemplary purposes, the selected data of ships, for which figs. 1 – 3 have been constructed based on the full-mission simulation, are presented in tab. 2. The optimised hull hydrodynamic coefficients in the form of lookup tables (extracts only) are shown in tab. 3 (chemical tanker) and in tab. 4 (ferry). The final parameters that appear in (17) and (18) are collected in tab. 5. The polynomial coefficients in (20) and (21) are demonstrated in tab. 6 (positive values in bold).

The A parameter is practically non-positive (relying on the hull sway force dependence upon the yaw velocity), the B coefficient is always negative. An uncertainty often exists with regard to the sign of a_1 or a_2 hull parameters for an arbitrary hull, and b_2 or b_2^* parameters. The latter are highly dependent upon the loading conditions i.e. a trim by stern – b_2^* is positive for the even keel or medium trim by stern while b_2 is the difference of b_2^* and positive $b_2^{''}$, refer to (16) and (17). In tab. 7 are indicated the particular contributions (components) of coefficients from C to F, see (21) – positive values are in bold.

The agreement between the developed analytical steady turning model, see (18), and the time-domain simulation with full-mission manoeuvring mathematical models is excellent – fig. 5. The vanishing instability loop in fig. 1 (version '00n' of the chemical tanker) however disappeared in the analytical approach – note the negative values of the polynomial coefficients from A to F in tab. 6.

Table 2. Ship particulars

	chemical tanker – stable '00n'	chemical tanker – unstable '21'	ferry
$A_R[\mathrm{m}^2]$	12.2		24 (both rudders)
$\dfrac{\partial c_L}{\partial\alpha[°]}(1+a_H)$	0.0544	0.0680	0.0330
$\tilde{c}_{12}[]$	1.0	0.5	0.5
$\tilde{w}[]$	0.33		0.15
$\tilde{c}_{Th}[]$	4		1.5
$L[\mathrm{m}]$	97.4		150
$T[\mathrm{m}]$	7.1		5

Table 3. Hull sway force and yaw moment coefficients for chemical tanker

c_{fyHm}

$\Omega_m \backslash \beta$	0°	+10°
0.0000	0.0000	0.0434
+0.4472	0.0000	0.0556

c_{mzHm}

$\Omega_m \backslash \beta$	0°	+10°
0.0000	0.00000	-0.01847
+0.4472	-0.02520	-0.03998

Table 4. Hull sway force and yaw moment coefficients for ferry

c_{fyHm}

$\Omega_m \backslash \beta$	0°	+10°
0.0000	0.0000	0.0470
+0.4472	0.0000	0.0793

c_{mzHm}

$\Omega_m \backslash \beta$	0°	+10°
0.0000	0.00000	-0.01254
+0.4472	-0.02000	-0.03003

Table 5. Values of equilibrium equation basic parameters

	chemical tanker – stable '00n'	chemical tanker – unstable '21'	ferry
a_1	0.00273		0.00722
b_1	0.00434		0.00470
c_1^*	-0.27347		-0.20542
a_2	0.000825		0.000561
b_2	-0.001847		-0.001254
b_2^*	0.002428		0.001252
c_2	-0.056351		-0.044720
b_{1R}	0.00145	0.00091	0.00070
c_{1R}	0.04164	0.02603	0.01993
e_{1R}	0.00219	0.00274	0.00187

Table 6. Values of polynomial coefficients (21)

	chemical tanker – stable '00n'	chemical tanker – unstable '21'	ferry
A	-0.000154	-0.000154	-0.000323
B	-0.000245	-0.000245	-0.000210
C	-0.000173	-0.000139	-0.000613
D	-0.001090	-0.001054	-0.000731
E	-0.000019	***0.000015***	-0.000291
F	-0.000053	***0.000124***	-0.000127

Table 7. Values and signs of particular contributions to polynomial coefficients (21)

	term no.				
	1	2	3	4	5
chemical tanker (stable, '00n')					
C	-0.000307	***0.000226***	-0.000057	-0.000034	
D	-0.000489	-0.000082	-0.000505	-0.000090	***0.000077***
E	-0.000154	***0.000226***	-0.000057	-0.000034	
F	-0.000199	-0.000326	***0.000664***	-0.000090	-0.000101
chemical tanker (unstable, '21')					
C	-0.000307	***0.000226***	-0.000036	-0.000021	
D	-0.000489	-0.000051	-0.000505	-0.000056	***0.000048***
E	-0.000154	***0.000226***	-0.000036	-0.000021	
F	-0.000124	-0.000296	***0.000664***	-0.000056	-0.000063
ferry					
C	-0.000646	***0.000115***	-0.000072	-0.000011	
D	-0.000420	-0.000031	-0.000258	-0.000047	***0.000025***
E	-0.000323	***0.000115***	-0.000072	-0.000011	
F	-0.000071	-0.000241	***0.000257***	-0.000047	-0.000025

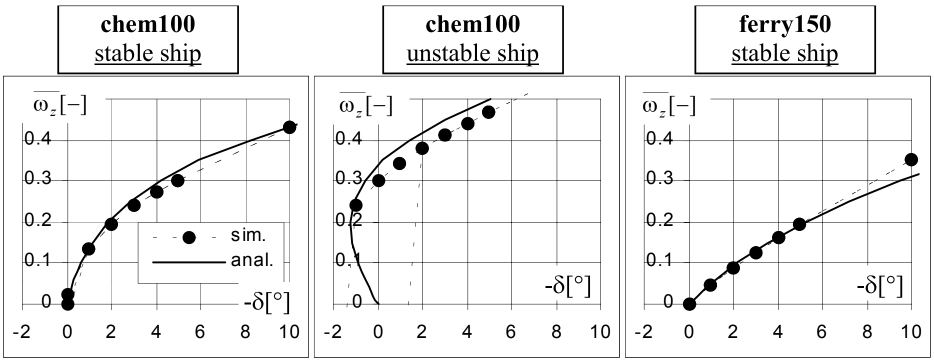

Fig. 5. Analytical approximate results vs. full-mission simulation

FINAL REMARKS

In respect of the derivations undertaken in the present study, the biggest challenge in the future research seems to be how to establish the most sensitive parameters and execute a unique identification of them based on the parameters of the spiral test curve. For a stable ship there is only one parameter, the curve slope at the origin of coordinates, that can be evaluated from the full scale manoeuvring trials. For an unstable ship, one can easily specify at least additional three parameters – point *C* (the nondimensional yaw velocity at null rudder angle) and point *D* with two parameters (both the nondimensional yaw velocity and rudder angle). Such a sensitivity analysis is not a simple task since the sensitivity, in general terms as the output versus input increment, always depends upon the initial (reference) conditions. Concerning the full-scale based unique identification, there is a need to decide what parameters can be estimated by other methods or left unattended.

It is also really very interesting to build an analytical model, in a way quite similar to the presented in the paper, which 'controls' the shape of the pull-out manoeuvre curve. The pull-out test indicates a transient behaviour of course stability – in the steady state conditions it converges to point *C*.

As aforementioned, the reversed spiral test proves to display some essential parameters of an unstable ship behaviour. Just for testing the effect of various combinations of parameters in the full-mission manoeuvring simulation models, working in the off-line (fast-time) mode, it seems indispensable to design a special automatic controller to perform this trial.

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